Stochastic Analysis of Computer Storage

Mathematics and Its Applications

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SERIES EDITOR'S PREFACE

Approach your problems from the right end and begin with the answers. Then one day, perhaps you will find the final question.

'The Hermit Clad in Crane Feathers' in R. van Gulik's *The Chinese Maze Murders.*

It isn't that they can't see the solution. It is that they can't see the problem.

G.K. Chesterton. *The Scandal of Father Brown* 'The point of a Pin'.

Growing specialization and diversification have brought a host of monographs and textbooks on increasingly specialized topics. However, the "tree" of knowledge of mathematics and related fields does not grow only by putting forth new branches. It also happens, quite often in fact, that branches which were thought to be completely disparate are suddenly seen to be related.

Further, the kind and level of sophistication of mathematics applied in various sciences has changed drastically in recent years: measure theory is used (non-trivially) in regional and theoretical economics; algebraic geometry interacts with physics; the Minkowsky lemma, coding theory and the structure of water meet one another in packing and covering theory; quantum fields, crystal defects and mathematical programming profit from homotopy theory; Lie algebras are relevant to filtering; and prediction and electrical engineering can use Stein spaces. And in addition to this there are such new emerging subdisciplines as "experimental mathematics", "CFD", "completely integrable systems", "chaos, synergetics and large-scale order", which are almost impossible to fit into the existing classification schemes. They draw upon widely different sections of mathematics. This programme, Mathematics and Its Applications, is devoted to new emerging (sub)disciplines and to such (new) interrelations as exempla gratia:

- a central concept which plays an important role in several different mathematical and/or scientific specialized areas;
- new applications of the results and ideas from one area of scientific endeavour into another;
- influences which the results, problems and concepts of one field of enquiry have and have had on the development of another.

The Mathematics and Its Applications programme tries to make available a careful selection of books which fit the philosophy outlined above. With such books, which are stimulating rather than definitive, intriguing rather than encyclopaedic, we hope to contribute something towards better communication among the practitioners in diversified fields.

New technology generates new mathematical problems. After all, as the David report remarked, the currently so-celebrated high technology is essentially mathematical technology. Many of these problems can, of course, be handled by employing or adapting more-or-less standard techniques. Occasionally however - and this seems to happen more frequently of late - the problems ask completely different questions than ever before and call for new and unexplored mathematical models. Two recent examples are provided by flexible manufacturing systems and (efficient) allocation problems in computers. Actually there are two such problems (at least): allocation of processors, a set of problems falling well into the established field of queueing theory and allocation of memory space. The latter presents many new aspects, e.g. involving spatial phenomena such as fragmentation, which is notorious for slowing down things. As every computer buff knows, memory, or more pre-

cisely, efficient use of memory storage space, remains a vastly important matter. This will also remain so.

Thus a new young and vigorous area of research has sprung up together with the usual host of heuristic algorithms and principles demanding systematic study and evaluation, the development of the mathematics to do so, and research monographs presenting matters in a coherent and integrated fashion. That is precisely what this book does for (stochastic) models for computer memory storage allocation.

The unreasonable effectiveness of mathematics in science ...

Eugene Wigner

Well, if you know of a better 'ole, go to it.

Bruce Bairnsfather

What is now proved was once only imagined.

William Blake

As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited.

But when these sciences joined company they drew from each other fresh vitality and thenceforward marched on at a rapid pace towards perfection.

Joseph Louis Lagrange.

Bussum, April 1987 Michiel Hazewinkel

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