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Hyperbolic Partial Differential Equations

Springer

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ISBN 978-0-387- 87822-5 e-ISBN 978-0-387-87823-2
DOI 10.1007/978-0-387-87823-2
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2009928133

Mathematics Subject Classification (2000): 35Lxx

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

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Introduction

The aim of this book is to present hyperbolic partial differential equations at an elementary level. In fact, the required mathematical background is only a third year university course on differential calculus for functions of several variables. No functional analysis knowledge is needed, nor any *distribution theory* (with the exception of shock waves mentioned below). All solutions appearing in the text are piecewise classical C^k solutions. Beyond the simplifications it allows, there are several reasons for this choice: First, we believe that all main features of hyperbolic partial differential equations (PDE) (well-posedness of the Cauchy problem, finite speed of propagation, domains of determination, energy inequalities, etc.) can be displayed in this context. We hope that this book itself will prove our belief. Second, all properties, solution formulas, and inequalities established here in the context of smooth functions can be readily extended to more general situations (solutions in Sobolev spaces or temperate distributions, etc.) by simple standard procedures of functional analysis or distribution theory, which are "external" to the theory of hyperbolic equations: The deep mathematical content of the theorems is already to be found in the statements and proofs of this book. The last reason is this: We do hope that many readers of this book will eventually do research in the field that seems to us the natural continuation of the subject: nonlinear hyperbolic systems (compressible fluids, general relativity theory, etc.). In this area, a large part of the work is devoted to prove global existence in time of classical solutions, in which case the whole work is about understanding the behavior and decay of smooth solutions.

There are of course many excellent books and textbooks partially or completely devoted to the subject of hyperbolic equations, some of which are quoted in the References at the end. But having discarded the books clearly too difficult to read for a first approach or that use abundantly distribution theory and Sobolev spaces, we found it somewhat hard to indicate references providing an easy introductory exposition of such subjects as, for instance, inequalities for variable coefficient equations, geometrical optics, etc. (Or, the references were scattered in many different books.)

The content of this book can be roughly divided into two parts. The first part includes all aspects of the theory having to do with vector fields and integral curves:

i) Cauchy problem for vector fields and (linear) method of characteristics (Chapter 1);

ii) Differential operators or systems in the plane, which reduce to systems of coupled vector fields (Chapter 2);

iii) Quasilinear scalar equations and eikonal equations, solved by nonlinear methods of characteristics, involving weaving by vector fields (Chapter 3).

We believe this part especially intuitive and *easy to visualize*: It is what makes hyperbolic PDE so attractive. Chapter 4 is a short introduction to conservation laws in one space dimension (shocks, simple waves, rarefaction waves, Riemann problem, etc.), which uses the language of vector fields and characteristics. This is the only place where the concept of solution "in the sense of distribution" is needed, but it is easy to understand in the special case of shock waves.

The second part describes the world of the wave equation and its perturbations for space dimensions two or three. Our treatment here, though completely elementary, emphasizes concepts proved useful by recent research developments: Lorentz fields and Klainerman inequality, weighted inequalities, conformal energy inequalities, etc. Following this orientation, we insisted more on inequalities than on explicit or approximate solutions. Chapter 5 presents the classical solution formula, along with the geometry of Lorentz fields, null frames, etc. In Chapter 6, we teach the reader how to prove an energy inequality, starting from the simplest case of a strip to proceed to inequalities in domains of determination; we also include an improvement of the standard inequality, Morawetz and KSS inequalities, and conformal inequality. Finally, Chapter 7 is devoted to variable coefficient equations or symmetric systems: We present the available inequalities with their amplification factors, Klainerman "energy method," and we touch upon geometrical optics and parametrics.

The natural readership for this book comprises senior or graduate students in mathematics interested in PDE; But the book can also be used by researchers of other fields of mathematics or sciences seeking to learn the basic facts about techniques they have heard of. The chapters are essentially independent, the language of vector fields or submanifolds, which is widely used throughout the book, being presented in two short Appendices.

In some chapters, Notes at the end explain the sources and references for further learning. At many places (and especially for energy inequalities in Chapters 6 and 7), instead of writing the proofs of the Theorems in the traditional formal way, we have presented them as “do it yourself” instructions with clearly identified steps. Finally, about 100 exercises are proposed, so that this book may be a useful textbook.