Dissipative Ordered Fluids

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Theories for Liquid Crystals

Springer

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ISBN 978-0-387-87814-0 e-ISBN 978-0-387-87815-7
DOI 10.1007/978-0-387-87815-7
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2012930001

Mathematics Subject Classification (2010): 76A15, 82D30

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

This book envisages liquid crystals as particular examples of dissipative ordered fluids. While it may be unique in taking this special perspective, it is not the only mathematical book on liquid crystals, and so one should have more than one good reason to read it. We can only give the reasons that made us write it: the reader will decide whether they suffice.

First, we felt the need to formulate a unified mathematical framework within which dynamical theories for liquid crystals can be phrased, a framework that is general enough also to incorporate dynamical theories for other ordered fluids. Our general topic is the evolution of order in fluids and its interaction with flow. Liquid crystals are the ideal arena for testing such a general theory for dissipative ordered fluids, because they are perhaps the best understood incarnation of these fluids. The established dynamical theories for liquid crystals have passed the tests of time and experimental scrutiny. Although we chose to concentrate on this special class of ordered fluids, we also highlight the opportunities that our general method offers in other closely related fields.

Since liquid crystals are here only examples of a wider family of ordered fluids, they are not treated in the full generality of all their condensed phases. Although our study is not limited to the traditional *uniaxial* nematics, since it also embraces the newly discovered (and still disputed) *biaxial* phases, it does not cover *smectic* liquid crystals. This large class of fluids, closer indeed to solids, is too complex to be included in an introductory book such as this. However, we interpret *nematics* in a broad sense, incorporating *chiral* nematics, often also called *cholesterics*.

Our narrative starts from a molecular description of the order that gives rise to the condensed phases of liquid crystals, and it moves on to the construction of continuum theories capable of describing their evolution. We sought secure guidance in such an endeavor and found it in a *dissipation principle*, which can be traced back to both the work and vision of RAYLEIGH. We interpret this principle in precise, mathematical terms and phrase it within a thermodynamic context, though most of the theories we review are purely mechanical in nature.

We have deliberately chosen to talk about theories in the plural. Order in liquid crystals appears in various guises and can be described in different ways, each more appropriate than others for certain purposes or in certain contexts. Theories broadly fall into two classes, depending on how the molecular order is described on larger length scales: there are *director* theories and *tensor* theories. The way in which theories in these large classes are established and how they are related is the leitmotif of the core of this book.

We do not limit our scope to harmonizing in a unified setting existing theories, but we also venture into hitherto unexplored territory. In doing so, we derive a new theory for the acoustic actions in nematic liquid crystals that is capable of explaining quantitatively experiments performed almost half a century ago that cannot be completely understood within the classical dynamical theories.

Since this is a mathematical book, we strive for rigor and precision. However, though we use the languages of analysis, algebra, and geometry, this is not a book in any of these mathematical disciplines. This is a book on *mechanics*, the mathematical science of *motion*, which is the archetype of all dynamical processes.

Although we tried to be as comprehensive as the scope of an introductory book allowed us to be, we could not cover all aspects of nematic order evolution. In particular, defect dynamics and dynamics of thin nematic films on surfaces remain untreated. Given the body of theoretical results available in the literature and the interest in their practical applications, these related subjects would actually deserve to fill a whole book by themselves.

This is a book on theories and their conceptual interplay. We have therefore, apart from rare exceptions, not included excercises or assignments. It is our hope that the reader will learn from this book how to phrase a continuum theory for the dissipative dynamics of ordered fluids that could stand the scrutiny of experimental physics, as did the celebrated theories of ERICKSEN–LESLIE and LANDAU–DE GENNES.

Glasgow, Pavia
September 2011

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