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Symmetries in Atomic Nuclei

From Isospin to Supersymmetry

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Foreword

One of the main objectives of research in physics is to find simple laws that give rise to a deeper understanding and unification of diverse phenomena. A less ambitious goal is to construct models which, in a more or less restricted range, permit an understanding of the physical processes involved and lead to a systematic analysis of the available experimental data while providing insights into the complex systems being studied. A necessary condition to reach these goals is the development of experimental instruments and methods that give access to the relevant information needed to test these models and guide the introduction of new concepts.

The problem of understanding and predicting the behavior of nuclei is one of the most difficult tasks encountered by scientists of the past and present century. From the beginning of the twentieth century until today startling discoveries have been made in nuclear physics, new elements have been synthesized, novel phenomena have been elucidated and important applications have been established. The history of nuclear physics has been characterized by a steady increase toward today's understanding of the atomic nucleus, starting from Rutherford's famed experiment, passing through the discovery of the neutron by Chadwick and culminating in the formulation of the liquid-drop and shell models, among the most important benchmarks. In spite of the impressive advances that have been made over a period of nearly one century, the field of nuclear physics is currently at a cross roads: the advent of radioactive-ion beams will vastly expand our observational capabilities and first results of this new generation of experiments already indicate that our current understanding of nuclei, which is mainly based on experiments along the line of stability, is in need of modification. The field has always been fertilized through the very strong interaction between experimental observations and theoretical modelling. Indeed, in very few fields of physics the development of experiment and theory is so closely intertwined. This interconnection of experiment and theory is a significant part of what is fascinating in nuclear physics: new ideas can be often proposed and verified or falsified in short succession and experimental observations can quickly lead to new theoretical concepts. A historic example of the latter is Heisenberg's introduction of isospin only a few months after the discovery of the neutron by Chadwick.

The nature of the strong force among nucleons that binds nuclei together, which is still not fully understood, coupled to the many-body characteristics of these systems, has given rise to a rich and complex field of scientific inquiry,

bringing forth a very creative research area. The fact that nuclei contain many particles but not nearly enough to treat them statistically, explains why they can be alternatively described both as a collection of individual nucleons and as a single object akin to a charged, dense liquid drop. These two representations of the nucleus reflect its collective and single-particle features, both of which are prominently displayed by nuclei. The following questions then arise: How do collective effects arise from individual particle behavior? How can we reconcile these properties that seemingly exclude each other? The solution of this paradox was outlined by Elliott in 1958 who indicated how nuclear collective deformation may arise from single-particle excitations using symmetry arguments based on $SU(3)$.

In 1975 Arima and Iachello followed a similar line of argument again using symmetry methods by proposing the Interacting Boson Model (IBM). This model and its extensions have proved remarkably successful in providing a bridge between single-particle and collective behavior, based on the approximately bosonic nature of nucleon-pair superpositions that dominate the dynamics of valence nucleons and that arise from the underlying nuclear forces. This is in close analogy to the Bardeen–Cooper–Schrieffer (BCS) theory of semi-conductors with its coupling of electrons to spin-zero Cooper pairs which gives rise to collective behavior we know as superconductivity. From this conceptual basis a unified framework for even–even and odd-mass nuclei has resulted. One of the most attractive features of the IBM is that it gives rise to a simple algebraic description, where so-called dynamical symmetries play a central role, both as a way to improve our basic understanding of the role of symmetry in nuclear dynamics and as starting points from which more precise calculations can be carried out. More specifically, this approach has, in a first stage, produced a unified description of the properties of medium-mass and heavy even–even nuclei, which are pictured in this framework as belonging (in general) to transitional regions between the various dynamical symmetries. Later, odd-mass nuclei were also analyzed from this point of view, by including the degrees of freedom of a single fermion of the nuclear shell model. The bold suggestion was then made by Iachello in 1980 that a simultaneous description of even–even and odd-mass nuclei was possible through the introduction of a superalgebra, with energy levels in both nuclei belonging to the same (super)multiplet. In essence, this proposal was based on the fact that even–even nuclei behave as (composite) bosons while odd-mass ones behave as (approximate) fermions. At the appropriate energy scales their states can then be viewed as elementary. The simple but far-reaching idea was then put forward that both these nuclei can be embedded into a single conceptual framework, relating boson–boson and boson–fermion interactions in a precise way. These concepts were subsequently tested in several regions of the nuclear table. The final step of including odd–odd nuclei into this unifying framework was then made by extending these ideas to the neutron–proton boson model, thus formulating a supersymmetric theory for quartets of nuclei.

Symmetry and its mathematical framework—group theory—play an increasingly important role in physics. Both classical and quantum many-body systems usually display great complexity but the analysis of their symmetry properties often gives rise to simplifications and new insights which can lead to a deeper understanding. In addition, symmetries themselves can point the way toward the formulation of a correct physical theory by providing constraints and guidelines in an otherwise intractable situation. It is remarkable that, in spite of the wide variety of systems one may consider, all the way from classical ones to molecules, nuclei, and elementary particles, group theory applies the same basic principles and extracts the same kind of useful information from all of them. This universality in the applicability of symmetry considerations is one of the most attractive features of group theory.

Most people have an intuitive understanding of symmetry, particularly in its most obvious manifestation in terms of geometric transformations that leave a body or system invariant. This interpretation, however, is not enough to grasp its deep connections with physics, and it thus becomes necessary to generalize the notion of symmetry transformations to encompass more abstract ideas. The mathematical theory of these transformations is the subject matter of group theory.

Over the years many monographs have been written discussing the mathematical theory of groups and their applications in physics [1, 2, 3, 4, 5, 6, 7, 8]. The present book attempts to give a pedagogical view of symmetry methods as applied to the field of nuclear-structure physics. The authors have collaborated for many years in this field but have also independently studied diverse aspects of nuclei and molecules from a symmetry point of view. Two of us have written a previous text on algebraic methods [7]. The present volume has a different focus as it concentrates on the theory and applications of symmetries in nuclear physics, stressing the underlying physical concepts rather than the mastery of methods in group theory. The discussion starts from the concept of isospin, used to this day in the elucidation of nuclear properties, to arrive at the ideas and methods that underlie the discovery of supersymmetry in the atomic nucleus. We emphasize here crucial experimental verification of these symmetries, explaining some of the experimental methods and adopt a more intuitive physical approach, dispensing of mathematical rigor and attempting to focus on the physical arguments that are at the core of new discoveries and breakthroughs. We also have aimed to give a modern account of the current state of this exciting field of research. This we hope has been achieved through the many boxes and examples with which we have illustrated the ideas explained in the main text.

We apologize for the amount of references to our own work, which we can only attempt to justify by stating our belief that we needed to rely on our own experience in order to have an inside look on the way symmetries can be a guide to study nuclear structure.

It should be clear that a book of this kind has not been written in isolation. We will not embark on the perilous exercise of mentioning physicists with

whom we have collaborated or discussed in the course of writing for fear of leaving out some deserving colleague. Nevertheless, we would like to thank Richard Casten and Stefan Heinze, for their careful reading of a draft of this text and their many constructive comments on it. Finally, we wish to dedicate this book to our wives: Annelies (Jolie) and Vera (Van Isacker), and to our children: Dan (Frank), David (Van Isacker), Joke (Jolie), Marieke (Jolie), Pablo (Frank), Thomas (Van Isacker) and Wouter (Jolie), who have lovingly tolerated us (and our physics) for so long.

Contents

1	Symmetry and Supersymmetry in Quantal Many-Body Systems	1
1.1	Symmetry in Quantum Mechanics	2
1.1.1	Some Definitions	2
1.1.2	Symmetry Transformations	4
1.1.3	Symmetry	5
1.1.4	Degeneracy and State Labeling	6
1.1.5	Dynamical Symmetry Breaking	7
1.1.6	Isospin in Nuclei	8
1.1.7	Selection Rules	16
1.2	Dynamical Symmetries in Quantal Many-Body Systems	18
1.2.1	Many-Particle States in Second Quantization	18
1.2.2	Particle-Number Conserving Dynamical Algebras	19
1.2.3	Particle-Number Non-conserving Dynamical Algebras	22
1.2.4	Superalgebras	24
1.3	The Algebraic Approach	26
2	Symmetry in Nuclear Physics	29
2.1	The Nuclear Shell Model	29
2.1.1	The SU(2) Pairing Model	31
2.1.2	The SU(3) Rotation Model	39
2.1.3	A Symmetry Triangle for the Shell Model	50
2.2	The Interacting Boson Model	51
2.2.1	Dynamical Symmetries	54
2.2.2	Geometry	57
2.2.3	Partial Dynamical Symmetries	64
2.2.4	Core Excitations	69
2.3	A Case Study: ^{112}Cd	71
2.3.1	Early Evidence for Vibrational Structures and Intruder Configurations	71
2.3.2	The $^{110}\text{Pd}(\alpha, 2n\gamma)^{112}\text{Cd}$ Reaction and its Interpretation	71
2.3.3	Studies of ^{112}Cd Using the $(n, n'\gamma)$ Reaction	75

3	Supersymmetry in Nuclear Physics	79
3.1	The Interacting Boson–Fermion Model	80
3.2	Bose–Fermi Symmetries	82
3.3	Examples of Bose–Fermi Symmetries	84
3.4	Nuclear Supersymmetry	87
3.5	A Case Study: Detailed Spectroscopy of ^{195}Pt	89
3.5.1	Early Studies of ^{195}Pt	90
3.5.2	High-Resolution Transfer Studies Using (p,d) and (d,t) Reactions	93
3.6	Supersymmetry without Dynamical Symmetry	98
4	Symmetries with Neutrons and Protons	105
4.1	Pairing Models with Neutrons and Protons	106
4.2	Interacting Boson Models with Neutrons and Protons	111
4.2.1	s Bosons Only	112
4.2.2	s and d Bosons	114
4.3	The Interacting Boson Model-2	117
4.4	A Case Study: Mixed-Symmetry States in ^{94}Mo	123
4.4.1	The Discovery of Mixed-Symmetry States in Deformed Nuclei	123
4.4.2	Mixed-Symmetry States in Near-Spherical Nuclei	124
4.4.3	Mixed-Symmetry States in ^{94}Mo	125
5	Supersymmetries with Neutrons and Protons	133
5.1	Combination of F Spin and Supersymmetry	133
5.2	Examples of Extended Supersymmetries	136
5.3	One-Nucleon Transfer in Extended Supersymmetry	138
5.4	A Case Study: Structure of ^{196}Au	141
5.4.1	First Transfer Reaction Experiments	141
5.4.2	New Experiments at the PSI, the Bonn Cyclotron and the Munchen Q3D Spectrometer	143
5.4.3	Recent High-Resolution and Polarized-Transfer Experiments	147
5.4.4	Comparison with Theory	148
5.4.5	Two-Nucleon-Transfer Reactions	150
6	Supersymmetry and Supersymmetric Quantum Mechanics	155
6.1	The Supersymmetric Standard Model	155
6.2	Strings and Superstrings	156
6.3	Supersymmetric Quantum Mechanics	158
6.3.1	Potentials Related by Supersymmetry	159
6.3.2	The Infinite Square-Well Potential	161
6.3.3	Scattering Off Supersymmetric Partner Potentials	162
6.3.4	Long-Range Nucleon–Nucleon Forces and Supersymmetry	164

6.3.5	Matrix Approach to Supersymmetric Quantum Mechanics	167
6.3.6	Three-Dimensional Supersymmetric Quantum Mechanics in Atoms	168
7	Conclusion	171
	References	173
	Index	183

List of Boxes

Isoscalar factors and the Wigner–Eckart theorem	12
Solution of the Richardson model	34
The collective model of nuclei	39
Permutation symmetry and Young diagrams	42
Shape phase transitions and Landau theory	60
The dynamical symmetries of the SO(8) model	109