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Combinatorics and Graph Theory

Second Edition

Springer

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To

Priscilla, Sophie, and Will,

Holly,

Kristine, Amanda, and Alexandra

Preface to the Second Edition

There are certain rules that one must abide by in order to create a successful sequel.

— Randy Meeks, from the trailer to *Scream 2*

While we may not follow the precise rules that Mr. Meeks had in mind for successful sequels, we have made a number of changes to the text in this second edition. In the new edition, we continue to introduce new topics with concrete examples, we provide complete proofs of almost every result, and we preserve the book's friendly style and lively presentation, interspersing the text with occasional jokes and quotations. The first two chapters, on graph theory and combinatorics, remain largely independent, and may be covered in either order. Chapter 3, on infinite combinatorics and graphs, may also be studied independently, although many readers will want to investigate trees, matchings, and Ramsey theory for finite sets before exploring these topics for infinite sets in the third chapter. Like the first edition, this text is aimed at upper-division undergraduate students in mathematics, though others will find much of interest as well. It assumes only familiarity with basic proof techniques, and some experience with matrices and infinite series.

The second edition offers many additional topics for use in the classroom or for independent study. Chapter 1 includes a new section covering distance and related notions in graphs, following an expanded introductory section. This new section also introduces the adjacency matrix of a graph, and describes its connection to important features of the graph. Another new section on trails, circuits, paths, and cycles treats several problems regarding Hamiltonian and Eulerian paths in graphs, and describes some elementary open problems regarding paths in graphs, and graphs with forbidden subgraphs.

Several topics were added to Chapter 2. The introductory section on basic counting principles has been expanded. Early in the chapter, a new section covers multinomial coefficients and their properties, following the development of the binomial coefficients. Another new section treats the pigeonhole principle, with applications to some problems in number theory. The material on Pólya's theory of counting has now been expanded to cover de Bruijn's more general method of counting arrangements in the presence of one symmetry group acting on the objects, and another acting on the set of allowed colors. A new section has also been added on partitions, and the treatment of Eulerian numbers has been significantly expanded. The topic of stable marriage is developed further as well, with three interesting variations on the basic problem now covered here. Finally, the end of the chapter features a new section on combinatorial geometry. Two principal problems serve to introduce this rich area: a nice problem of Sylvester's regarding lines produced by a set of points in the plane, and the beautiful geometric approach to Ramsey theory pioneered by Erdős and Szekeres in a problem about the existence of convex polygons among finite sets of points in the plane.

In Chapter 3, a new section develops the theory of matchings further by investigating marriage problems on infinite sets, both countable and uncountable. Another new section toward the end of this chapter describes a characterization of certain large infinite cardinals by using linear orderings. Many new exercises have also been added in each chapter, and the list of references has been completely updated.

The second edition grew out of our experiences teaching courses in graph theory, combinatorics, and set theory at Appalachian State University, Davidson College, and Furman University, and we thank these institutions for their support, and our students for their comments. We also thank Mark Spencer at Springer-Verlag. Finally, we thank our families for their patience and constant good humor throughout this process. The first and third authors would also like to add that, since the original publication of this book, their families have both gained their own second additions!

May 2008

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Preface to the First Edition

> *Three things should be considered: problems, theorems, and applications.*
>
> — Gottfried Wilhelm Leibniz,
> *Dissertatio de Arte Combinatoria*, 1666

This book grew out of several courses in combinatorics and graph theory given at Appalachian State University and UCLA in recent years. A one-semester course for juniors at Appalachian State University focusing on graph theory covered most of Chapter 1 and the first part of Chapter 2. A one-quarter course at UCLA on combinatorics for undergraduates concentrated on the topics in Chapter 2 and included some parts of Chapter 1. Another semester course at Appalachian State for advanced undergraduates and beginning graduate students covered most of the topics from all three chapters.

There are rather few prerequisites for this text. We assume some familiarity with basic proof techniques, like induction. A few topics in Chapter 1 assume some prior exposure to elementary linear algebra. Chapter 2 assumes some familiarity with sequences and series, especially Maclaurin series, at the level typically covered in a first-year calculus course. The text requires no prior experience with more advanced subjects, such as group theory.

While this book is primarily intended for upper-division undergraduate students, we believe that others will find it useful as well. Lower-division undergraduates with a penchant for proofs, and even talented high school students, will be able to follow much of the material, and graduate students looking for an introduction to topics in graph theory, combinatorics, and set theory may find several topics of interest.

Chapter 1 focuses on the theory of finite graphs. The first section serves as an introduction to basic terminology and concepts. Each of the following sections presents a specific branch of graph theory: trees, planarity, coloring, matchings, and Ramsey theory. These five topics were chosen for two reasons. First, they represent a broad range of the subfields of graph theory, and in turn they provide the reader with a sound introduction to the subject. Second, and just as important, these topics relate particularly well to topics in Chapters 2 and 3.

Chapter 2 develops the central techniques of enumerative combinatorics: the principle of inclusion and exclusion, the theory and application of generating functions, the solution of recurrence relations, Pólya's theory of counting arrangements in the presence of symmetry, and important classes of numbers, including the Fibonacci, Catalan, Stirling, Bell, and Eulerian numbers. The final section in the chapter continues the theme of matchings begun in Chapter 1 with a consideration of the stable marriage problem and the Gale–Shapley algorithm for solving it.

Chapter 3 presents infinite pigeonhole principles, König's Lemma, Ramsey's Theorem, and their connections to set theory. The systems of distinct representatives of Chapter 1 reappear in infinite form, linked to the axiom of choice. Counting is recast as cardinal arithmetic, and a pigeonhole property for cardinals leads to discussions of incompleteness and large cardinals. The last sections connect large cardinals to finite combinatorics and describe supplementary material on computability.

Following Leibniz's advice, we focus on problems, theorems, and applications throughout the text. We supply proofs of almost every theorem presented. We try to introduce each topic with an application or a concrete interpretation, and we often introduce more applications in the exercises at the end of each section. In addition, we believe that mathematics is a fun and lively subject, so we have tried to enliven our presentation with an occasional joke or (we hope) interesting quotation.

We would like to thank the Department of Mathematical Sciences at Appalachian State University and the Department of Mathematics at UCLA. We would especially like to thank our students (in particular, Jae-Il Shin at UCLA), whose questions and comments on preliminary versions of this text helped us to improve it. We would also like to thank the three anonymous reviewers, whose suggestions helped to shape this book into its present form. We also thank Sharon McPeake, a student at ASU, for her rendering of the Königsberg bridges.

In addition, the first author would like to thank Ron Gould, his graduate advisor at Emory University, for teaching him the methods and the joys of studying graphs, and for continuing to be his advisor even after graduation. He especially wants to thank his wife, Priscilla, for being his perfect match, and his daughter Sophie for adding color and brightness to each and every day. Their patience and support throughout this process have been immeasurable.

The second author would like to thank Judith Roitman, who introduced him to set theory and Ramsey's Theorem at the University of Kansas, using an early draft

of her fine text. Also, he would like to thank his wife, Holly (the other Professor Hirst), for having the infinite tolerance that sets her apart from the norm.

The third author would like to thank Bob Blakley, from whom he first learned about combinatorics as an undergraduate at Texas A & M University, and Donald Knuth, whose class *Concrete Mathematics* at Stanford University taught him much more about the subject. Most of all, he would like to thank his wife, Kristine, for her constant support and infinite patience throughout the gestation of this project, and for being someone he can always, well, count on.

September 1999

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