PATH PLAYER GAMES

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Aims and Scope
Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.
The Springer Series in Optimization and Its Applications publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multiobjective programming, description of software packages, approximation techniques and heuristic approaches.

PATH PLAYER GAMES

Analysis and Applications

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To Torsten,
I am glad that we play on the same path.

Contents

Symbols and Abbreviations

General Symbols

$\mathbf{0}_n$ ..... n-dimensional zero vector
$\mathbf{1}_n$ ..... n-dimensional one vector
$\mathbb{I}_n$ ..... $n \times n$ identity matrix
$\mathbb{N}_0$ .. natural numbers including zero
$\mathbb{R}_+$ ..... nonnegative real numbers
$\mathbb{R}^n$ .. n-dimensional Euclidian space
$\mathbb{R}^n_+$ ..... nonnegative n-dimensional Euclidian space
$sgn(a)$ ..... signum of a

Symbols: Path Player Game

$b(f)$ ..... vector of benefit functions
$b_P(f)$ ..... benefit of player P
$\tilde{b}_P(f)$ .... one-dimensional benefit of player P
$c_e(f_e)$ ..... cost on edge e
$c_P(f)$ ..... cost of path P
$\tilde{c}_P(f)$ .... one-dimensional cost of P
$d_P(f_{-P})$ ..... decision limit
e ..... edge
e_m ..... class of edges shared by player set m
E ..... set of edges e
E_P^{com} ... set of commonly used edges
E_P^{exc} ... set of exclusively used edges
f ..... network flow
f_e ..... flow on edge e
f_P ..... flow on path P
f_{-P} ..... network flow excluding P
$f_P^{\max}(f_{-P})$ ... best reaction set of P
$\mathbb{F}$ ..... set of feasible flows
Γ ..... game instance
G ..... game network
$G(n)$ .standard network of n players
$I(\varphi)$ ..... cost of sequence φ
κ_P ..... security payment of path P
$l(\varphi)$ ..... length of sequence φ
M ... real number, sufficiently large
$ND(\Gamma)$ ... set of nondominated flows
$NE(\Gamma)$ ..... set of equilibria
ω_P ..... security limit of path P
$\Pi(f)$ ..... potential function
φ ..... strategy sequence
P ..... path
$\mathcal{P}$ ..... set of paths P
$\mathbb{P}(n)$ ..... power set of set of players
r ..... flow rate
s ..... source vertice
t ..... sink vertice
v ..... vertice
v_i ..... ith unit vector
V ..... set of vertices v

Symbols: Game on Polyhedra

A ..... coefficient matrix
b ..... coefficient vector
$c_i(x)$ ..... cost of player i

h_i ..... strategy set of player i
$H(A,b)$ ..... hypercuboid
i ..... player
n ..... number of players
$S(A,b)$ ..... polyhedron of feasible solutions
$S_i(x_{-i})$ feasible strategies of player i
x ..... solution
x_i ..... strategy of player i
x_{-i} ..... solution excluding x_i
$z_m(x_m)$ ..... cost function of set m

Symbols: Line Planning Game

$b_P(f)$ ..... payoff of line P
$c_P(f)$ ..... cost of line P
$d_P^1(f_{-P})$ ..... lower decision limit
$d_P^2(f_{-P})$ ..... upper decision limit
f ..... network frequency
f_e ..... frequency on edge e
f_P ..... frequency on path P
f_P^{br} ..... best reaction set of P
$f^{\min}$ ..... minimal frequency
$f_q^{\min}$ .. minimal frequency of $\{s_q, t_q\}$
$f_e^{\max}$ .. maximal frequency on edge e
$\mathbb{F}^{\mathrm{LPG}}$ ..... set of feasible frequencies
$\mathbb{F}^{ILPG}$ ..... set of feasible integer frequencies
N .... real number, sufficiently large
P ..... line
Q ..... number of OD pairs
$\mathcal{P}$ ..... line pool
$\mathcal{P}_q$ ..... line pool of $\{s_q, t_q\}$
s_q ..... qth origin
t_q ..... qth destination

Abbreviations

AFIP ..... approximate finite improvement property
FBRP .... finite best-reply property
FIP .... finite improvement propery
GNE ..... generalized equilibria
ILPG ... integer line planning game
LPG ..... line planning game
NCS ..... noncompensative-security
OD ..... origin–destination
PPG ..... path player game
QVI ... quasi-variational inequalities