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(continued after index)

J. Chakrabarty

Applied Plasticity, Second Edition

 Springer

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Editor-in-Chief

The Mechanical Engineering Series features graduate texts and research monographs to address the need for information in contemporary mechanical engineering, including areas of concentration of applied mechanics, biomechanics, computational mechanics, dynamical systems and control, energetics, mechanics of materials, processing, production systems, thermal science, and tribology.

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Series Preface

Mechanical engineering, an engineering discipline forged and shaped by the needs of the industrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of productivity and competitiveness that require engineering solutions, among others. The Mechanical Engineering Series features graduate texts and research monographs intended to address the need for information in contemporary areas of mechanical engineering.

The series is conceived as a comprehensive one that covers a broad range of concentrations important to mechanical engineering graduate education and research. We are fortunate to have a distinguished roster of consulting editors on the advisory board, each an expert in one of the areas of concentration. The names of the consulting editors are listed on the facing page of this volume. The areas of concentration are applied mechanics, biomechanics, computational mechanics, dynamic systems and control, energetics, mechanics of materials, processing, production systems, thermal science, and tribology.

Austin, Texas

Frederick F. Ling

*This book is humbly dedicated to the loving
memory of MA INDIRA who continues to be the
source of real inspiration to me.*

Preface

The past few years have witnessed a growing interest in the application of the mechanics of plastic deformation of metals to a variety of engineering problems associated with structural design and technological forming of metals. Written several years ago to serve as a companion volume to the author's earlier work under the title *Theory of Plasticity*, which comprehensively expounds the fundamentals of plasticity of metals, the present work seems to have stood the test of time and has established itself as a comprehensive reference work that is equally useful for classroom purposes. While the earlier work is mainly concerned with the application of the theory to the solution of elastic/plastic problems, limit analysis of framed structures, and problems in plane plastic strain involving slipline fields, several important areas of plasticity related to the analysis of multidimensional structures and various metal-forming processes had to be left out for obvious reasons. The present text is intended to fill this gap and to make available to the reader in a single volume a detailed account of a wide range of useful results that are scattered in numerous periodicals and other sources.

The fundamentals of the mathematical theory of plasticity are discussed in Chapter 1 with sufficient details, in order to eliminate the need for frequent references to the author's earlier volume. The theory of plane plastic stress and its applications to structural analysis and sheet metal forming are presented in Chapter 2. The axially symmetrical plastic state, as well as a few three-dimensional problems of plasticity, is treated in Chapter 3. The plastic behavior of plates and shells, mainly from the point of view of limit analysis, is discussed with several examples in Chapters 4 and 5. The plasticity of metals with fully developed orthotropic anisotropy and its application to the plastic behavior of anisotropic sheets are presented in Chapter 6. The generalized tangent modulus theory of buckling in the plastic range for columns, plates, and shells is treated in Chapter 7 from the point of view of the bifurcation phenomenon. Chapter 8 deals with a wide range of topics in dynamic plasticity, including the wave propagation, armor penetration, and structural impact in the plastic range. The fundamentals of the rigid/plastic finite element method, with special reference to its application to metal-forming processes, are presented in Chapter 9, where several examples are included for illustration.

The publication of the revised second edition of *Applied Plasticity* is deemed necessary not only for the obvious need for updating the book but also for the purpose

of making it more suitable for the teaching of appropriate courses on plasticity at the graduate level. During the preparation of the second edition, several parts of the text have been extensively revised in the light of the recent developments of the subject, and new references to the published literature have been made in appropriate places. The discussion of the finite element method in plasticity, previously relegated to an appendix in the first edition, has now been expanded into a new chapter to permit a more complete treatment of the subject. A new section has been added in Chapter 4 to discuss the yield line theory for plate bending, not only for the derivation of complete solutions but also for the estimation of upper bounds on the limit load. A set of homework problems has been included at the end of each chapter for the benefit of both the student and the instructor, many of these problems having been designed to supplement the text. The references to the published literature have now been collected together and placed at the very end of the book for the sake of the expected convenience of the reader.

The book in its present form would be suitable for teaching advanced graduate level courses on plasticity and metal forming to students of mechanical and manufacturing engineering, as well as on structural plasticity to students of civil and structural engineering. The book will also be found useful for teaching courses on dynamic plasticity to both the mechanical and civil engineering students. Though intended primarily for research workers in the field of plasticity, senior undergraduate students and practicing engineers are also likely to benefit from this book to a large extent.

I take this opportunity to express my gratitude to the late Professor J. M. Alexander, formerly of Imperial College, London, who not only stimulated my interest in plasticity but also encouraged me to undertake the task of writing this book. I am also grateful to Dr. Frederick F. Ling, the Editor-in-Chief of this Series, for his encouragement and support for the publication of the second edition of *Applied Plasticity*. It is a pleasure to offer my sincere thanks to Ms. Jennifer Mirski, the Assistant Engineering Editor of Springer for her helpful cooperation and support during the preparation of the manuscript. Finally, I am deeply indebted to my wife Swati, who gracefully accepted the hardship of many lonely hours to enable me to complete this work in a satisfactory manner.

J. Chakrabarty

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