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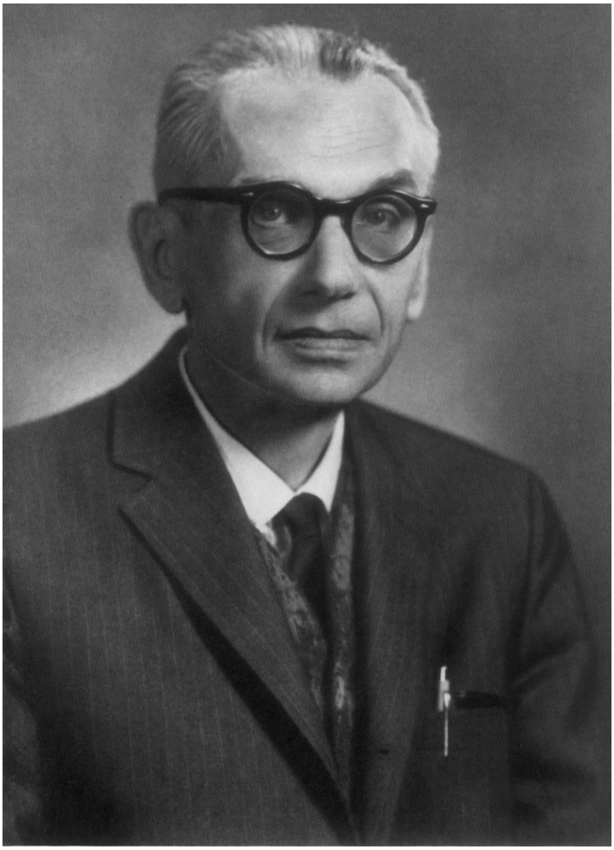

Kurt Gödel (1906–1978)

S.M. Srivastava

A Course on Mathematical Logic

Springer

S.M. Srivastava
Indian Statistical Institute
Kolkata
India
e-mail: smohan@isical.ac.in

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Preface

This book is written on the occassion of the birth centenary year of Kurt Gödel (1906–1978), the most exciting logician of all times, whose discoveries shook the foundations of mathematics. His beautiful technique to examine the whole edifice of mathematics within mathematics itself has been likened, not only figuratively but also in precise technical terms, to the music of Bach and drawings of Escher [4]. It had a deep impact on philosophers and linguists. In a way, it ushered in the era of computers. His idea of arithmetization of formal systems led to the discovery of a universal computer program that simulates all programs. Based on his incompleteness theorems, physicists have propounded theories concerning artifical intelligence and the mind–body problem [10].

The main goal of this book is to state and prove Gödel's completeness and incompleteness theorems in precise mathematical terms. This has enabled us to present a short, distinctive, modern, and motivated introduction to mathematical logic for graduate and advanced undergraduate students of logic, set theory, recursion theory, and computer science. Any mathematician who is interested in knowing what mathematical logic is concerned with and who would like to learn the famous completeness and incompleteness theorems of Gödel should also find this book particularly convenient. The treatment is thoroughly mathematical, and the entire subject has been approached like any other branch of mathematics. Serious efforts have been made to make the book suitable for both instructional and self-reading purposes. The book does not strive to be a comprehensive encyclopedia of logic, nor does it broaden its audience to linguists and philosophers. Still, it gives essentially all the basic concepts and results in mathematical logic.

The main prerequisite for this book is the willingness to work at a reasonable level of mathematical rigor and generality. However, a working knowledge of elementary mathematics, particularly naive set theory and algebra, is required. We suggest [12, pp. 1–15] for the necessary prerequisites in set theory. A good source for the algebra needed to understand some examples and applications would be [7].

Students who want to specialize in foundational subjects should read the entire book, preferably in the order in which it is presented, and work out all the problems. Sometimes we have only sketched the proof and left out the routine arguments for readers to complete. Students of computer science may leave out sections on model theory and arithmetical sets. Mathematicians working in other areas and who want to know about the completeness and incompleteness theorems alone may also omit these sections. However, sections on model theory give applications of logic to mathematics. Chapters 1 to 4, except for Section 2.4 and Sections 5.1 and 5.4, should make a satisfactory course in mathematical logic for undergraduate students.

The book prepares students to branch out in several areas of mathematics related to foundations and computability such as logic, model theory, axiomatic set theory, definability, recursion theory, and computability. Hinman's recent book [3] is the most comprehensive one, with representation in all these areas. Shoenfield's [11] is still a very satisfactory book on logic. For axiomatic set theory, we particularly recommend Kunen [6] and Jech [5]. For model theory, the readers should also see Chang and Keisler [2] and Marker [8]. For recursion theory we suggest [9].

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