PART III: APPLICATIONS

Part I of this book addressed methods for reaching a feasible solution quickly in optimization models. Part II addressed the analysis of infeasible models. As you might imagine, there are countless direct applications for algorithms in these classes: reducing the time to reach a feasible solution reduces the overall solution time; good tools for analysis of infeasible instances reduces the overall time for the complete modeling and solution cycle. However, there are a surprising number of applications beyond these straightforward ones. These "spin-off" applications are the subject of Part III.

Closest to home is the application of methods for the analysis of infeasibility to the analysis of other model forms. Unboundedness in primal linear programs is directly related to infeasibility in the dual (Sec. 9.1). Network models can be plagued by the inability to carry flow in some of the arcs, a condition known as *nonviability* that can be analyzed by a simple transformation to an infeasibility problem (Sec. 9.2). The interaction of the objectives in multiple objective linear programs can also be transformed into an infeasibility problem and analyzed with the assistance of the tools developed in Part II (Sec. 9.3).

Other important and seemingly unrelated problems can also be addressed by simple transformations to infeasibility analysis problems. Many of these are well addressed by methods for the solution of the maximum feasibility problem (see Chap. 7). A standard problem in classification and data mining is the placement of hyperplanes to separate data points of one type from data points of other types in the training set. This is easily transformed into a MAX FS problem, and hence solutions are returned that tend to minimize the number of misclassified points, whereas more traditional approaches may have minimized other measures such as the sum of the squared misclassification distances (Sec. 10.1). This is the same as providing the initial training for a neural network. A related problem is determining the *data depth* of a particular point in a multidimensional cloud of data points, defined as the minimum number of data points on one side of a hyperplane through the point in question (Sec 10.2). This again is easily transformed into a MAX FS problem and addressed via the methods of Chap. 7. Massive data sets, such as census data, are routinely screened for errors using linear relationships. The data validation rules can be analyzed for internal inconsistencies using the methods of Chaps. 6 and 7 (Sec. 10.3).

Several specific applications are well addressed as instances of the MAX FS problem. Radiation treatment planning results in a large set of linear inequalities that express the fact that diseased tissue must receive more than some minimum amount of radiation while nearby healthy tissue and important organs should receive less than some maximum dose. Since these requirements often conflict, this can be addressed as an instance of a MAX FS problem (Sec. 11.1). The problem in protein folding is to find the natural folding shape that minimizes the energy, which will be smaller than for other folded shapes. Given the energy inequalities associated with similar "decoy" shapes, information about the natural folded shape is gleaned by solving a MAX FS problem that is typically extremely large (Sec. 11.2). The digital video broadcasting problem is another MAX FS problem of very large scale (Sec. 11.3). Here the broadcast coverage area is subdivided into small regions, each of which should receive a signal with a minimum amount of power from a set of transmitters. This is again a MAX FS problem.

The best approximation methods of Chap. 8 are needed in automated test assembly (Sec. 11.4) in which the idea is to meet the requirements imposed by the test assembly rules as well as possible. IIS isolation is used to analyze problems of buffer overrun in computer programs (Sec. 11.5) when linear constraints describing the growth in the size of the buffer generate infeasibilities. User preferences used to rank the value of internet pages can be expressed as linear inequalities (Sec. 11.6), but these can result in infeasibilities, which are analyzed using either a best correction approach (Chap. 8) or a MAX FS strategy (Chap. 7).

IIS analysis is a common feature in tree-structured search, such as in branch and bound solution of MIPs or modern constraint programming systems, and is used to direct the backtracking process more efficiently (Sec. 11.7). Piecewise linear models are often used to approximate various physical phenomena such as signals. Estimation of such models (Sec. 11.8) can be represented as an instance of the MIN PFS problem of Sec. 7.9. Finding sparse solutions for systems of linear equations amounts to a MAX FS problem (Sec. 11.9). Various NP-hard problems can also be converted to the MAX FS problem (Sec 11.10), though some of these are in binary rather than continuous variables, for which we do not as yet have good solution heuristics.