# FEASIBILITY AND INFEASIBILITY IN OPTIMIZATION:
## Algorithms and Computational Methods

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Library of Congress Control Number: 2007935595

ISBN-13: 978-0-387-74931-0 (HB) ISBN-13: 978-0-387-74932-7 (e-book)

Printed on acid-free paper.



9 8 7 6 5 4 3 2 1

springer.com

For my parents, Mac and Shirley Chinneck, who fed that spark of curiosity with science books, telescopes and chemistry sets, even while I was busily dismantling useful household items, and for Linda and Annie, who make it all worthwhile.

# Preface

Most applied optimization problems involve constraints: What is the maximum profit that a manufacturer can make given a limited number of machines and a limited labour force? What is the minimum amount of fuel that a fleet of trucks can consume while making a specified set of deliveries? What is the smallest amount of silicon needed to etch an electronic circuit while respecting limits on signal propagation time, inter-wire distance, etc.? Applications of constrained optimization are everywhere in industry, business, and government.

Of course, the solution returned by an optimization algorithm must also be feasible: we want the best possible value of the objective function that satisfies all constraints and variable bounds. Some optimization algorithms are not even able to proceed towards optimality until a feasible solution is available. In addition, the optimization question can be converted to a feasibility question, and vice versa. And what happens when an algorithm is unable to find a feasible solution? How do we know what went wrong? How do we repair the model? Questions of optimization, feasibility, and infeasibility are inextricably linked.

There has been a surge of important developments related to feasibility and infeasibility in optimization in the last two decades, a trend that continues to accelerate even today. New and more efficient methods for seeking feasibility in difficult optimization forms such as mixed-integer programs and nonlinear programs are emerging. The first effective algorithms for analyzing infeasible models have been discovered and implemented in commercial software. A community of researchers in constraint programming has begun to integrate their knowledge and approaches with the optimization community. Unanticipated spin-off applications of the new algorithms are being found. It's an exciting time.

The goal of this book is to summarize the state of the art in recent work at the interface of optimization and feasibility. It should serve as a useful reference for researchers, graduate students, and software developers working on optimization, feasibility, infeasibility, and related topics. Readers having a reasonable grounding in optimization (linear and nonlinear programming, mixed-integer programming, etc.) should have no difficulty following the material.

Lightweight coverage of topics in constraint programming, with an emphasis on constraint satisfaction problems, is included to illustrate the extensive overlap and convergence in the two literatures. An ideal version of the book would cover topics in constraint programming in the same depth as topics in optimization, but this is beyond the scope of this project: collecting and organizing the wealth of new developments relating to feasibility and infeasibility in optimization. I hope the resulting book is useful to both optimizers and constraint programmers, and that it helps accelerate the ongoing merger of the two communities merge into a stronger hybrid.

## Acknowledgements

My graduate work was conducted during the late 1970s and early 1980s. Inspired by the energy crises of those times, I constructed network optimization models to minimize the use of energy in large industrial plants. Later I found the optimization modeling more interesting than the energy aspects of this work. I had noticed that some of the processing network models that I was using in the energy work suffered from an inability to carry flow in some of the arcs, a pathology later labeled *nonviability* (see Sec. 9.2). I developed algorithms to automatically identify and analyze this problem.

Enter Harvey Greenberg. At that time he was involved in a project to develop an *Intelligent Mathematical Programming System* (IMPS) (see e.g. Greenberg (1996b)), and consequently had an interest in algorithms for analyzing modeling errors of various types, such as nonviability. Harvey organized an extraordinary series of meetings on the IMPS topic for an eclectic group of researchers from academia and industry. Harvey invited me to one of these meetings and, as they say, the rest is history. Sitting in the bar one night after the IMPS meeting we had a discussion about whether or not you could isolate the cause of infeasibility in a linear program to an irreducible subset of the constraints defining the model. At the time, Harvey didn't think it could be done, but I did, so I bet him a beer that I could find a way to do so. As you will see in Part II of the book, I won that bet.

But there is a postscript to this story. I have now known Harvey for around twenty years, and we have gone on to make numerous one-beer bets on other issues in optimization. I have not won a single one of those subsequent bets, so I am currently several hundred beers in debt to him. But I have an even bigger debt than that. Harvey became my unofficial mentor, always ready to provide advice and suggestions and listen to my ideas. His influence on my work has been profound.

Harvey and Pascal Van Hentenryck both took the time to read an early draft of the book and provide advice and suggestions that greatly improved it. Both pointed out topics that should be greatly expanded upon, especially the material on constraint programming, but time is unfortunately limited, so a full treatment of that topic remains another project. And as clever as those two fellows are, I'm sure I've managed to hide a few errors in the manuscript that they did not find: those are mine alone.

Last but not least are the two incredible ladies in my life, my wife Linda and daughter Annie, who can finally look at this book and see what kept me glued to the computer for such long hours over the past year. Thanks for being there.

John W. Chinneck

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# Introduction

*To be, or not to be: that is the question…*
From *Hamlet* by William Shakespeare

Shakespeare certainly hit the nail on the head: the most basic question of all is whether or not something exists: an object, a person or a solution that satisfies a given set of constraints. For Shakespeare, human existence was a fundamental question of life; for this book, existence of a feasible solution is a fundamental question of optimization.

Why such interest in feasibility and infeasibility in the context of optimization? Surely it is most important to find the best (i.e. optimum) solution, rather than just any feasible solution? The questions of feasibility and optimality are in fact two sides of the same coin. First, the existence of a feasible solution is a very fundamental question: before you can determine which solution is the best, you must first determine whether or not it is even possible for a feasible solution to exist at all. Second, it is easy to convert an optimization question to a feasibility question (and vice-versa), so the two questions are fundamentally the same. For example, you can pose the feasibility question as to whether or not a solution exists with an objective function value that is at least as good as a certain stated aspiration value. Over a series of iterations this aspiration value can be adjusted until we can definitely answer that no solution exists beyond a certain value. That last feasible solution is the optimum value of the objective function, found by answering a series of feasibility questions.

Looking at this the opposite way, it is common practice to pose feasibility questions as optimization problems. This is the basic idea of any phase 1 technique: create an objective function that measures the degree of violation of the constraints at any given point, and then minimize this function. If a value of zero can be found for this phase 1 optimization problem, then a feasible point exists, otherwise the model may not be feasible.

Third, there are unique and interesting questions associated with feasibility and infeasibility in optimization. For example, given a set of constraints that a solver determines to be infeasible, provide a diagnosis of why this is so. This question has grown in importance in recent years as optimization models have grown larger and more complex in step with the phenomenal increases in inexpensive computing power. One approach to this question is to isolate an *irreducible infeasible subset (IIS)* of the constraints, i.e. a (small) subset of constraints that is itself infeasible, but becomes feasible if one or more constraints is removed. This helps focus the diagnosis and model repair efforts and is especially helpful in very large models. This approach is well summarized by Greenberg's aphorism: "diagnosis = isolation + explanation" (Greenberg 1993). A related diagnostic question is this: given an infeasible model, what is the smallest number of constraints to remove such that the remaining constraints constitute a feasible set? Another is: what is the best way to repair the infeasible system (e.g. what is the smallest set of changes that can be made to the constraint right hand sides such that the set of constraints becomes feasible)?

Many of the algorithms used in answering these diagnostic questions depend on assessing the feasibility of numerous subsets of the original set of constraints. Hence those algorithms operate much more efficiently if the feasibility status of an arbitrary set of constraints can be determined quickly (which is of course a fundamental feasibility question itself). This is not difficult for sets of linear constraints, but it can be extremely difficult and time-consuming to determine feasibility status at all when there are nonlinear constraints or integer restrictions on some or all of the variables. Hence one focus of this book is algorithms for improving the speed with which the first feasible solution can be found (if one exists) for the more difficult cases in optimization.

A fourth major reason for interest in feasibility-related algorithms is the many applications that have been found for them. Some of these applications are surprising: data classification, training of neural networks, radiation treatment planning, analysis of protein folding, automatic test assembly, applications in statistics, etc. Some of these are briefly reviewed in Part III.

Finally, the question of feasibility or infeasibility is a major overlap between the field of optimization and the field of constraint programming. Constraint programming, arising from computer science, has special strength in seeking a yes/no answer to the question of whether a solution exists for a stated set of constraints; this is identical to the feasibility question in optimization. However, because of their different roots and traditions, constraint programming researchers approach the question in a different way and with different techniques. The two fields have begun to merge in recent years, resulting in stronger hybrid techniques. Constraint programming techniques and their links with optimization are addressed at an elementary level.

The emphasis in this volume is on algorithms and computational methods, specifically practical algorithms for solving the feasibility/infeasibility related problems that are the main subject. The book summarizes the main developments over the last twenty years or so, a very active period for the field, spurred by improvements in computing power and an increase in the size and complexity of optimization models. It should prove useful for academics teaching and conducting research in the field and their graduate students, as well as practitioners.

As opposed to a mathematical treatment, we take the involvement of a computer as a given: modern optimization problems are normally of such scale and complexity that they simply cannot be solved without using a computer. The essential element in solving a feasibility or optimization problem via computer is an efficient and effective algorithm. The computer implementation of these algorithms introduces a number of practical issues and complications, such as tolerances. These are also dealt with as they arise.

***A Note on Theorems***: There is a significant amount of mathematical development underlying the algorithms and computational methods that are the main topic of this book. To keep the focus on algorithms, proofs are generally included where a theorem relates to whether an algorithm functions as intended. However, where theorems relate to mathematical underpinnings, the proof is generally omitted in favour of a simple reference to the original publication containing the proof.