PRACTICAL OPTIMIZATION
Algorithms and Engineering Applications
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Practical Optimization: Algorithms and Engineering Applications
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To Lynne
and
Chi-Tang Catherine
with our love
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Dedication vi
Biographies of the authors vii
Preface xv
Abbreviations xix

1. THE OPTIMIZATION PROBLEM 1
   1.1 Introduction 1
   1.2 The Basic Optimization Problem 4
   1.3 General Structure of Optimization Algorithms 8
   1.4 Constraints 10
   1.5 The Feasible Region 17
   1.6 Branches of Mathematical Programming 22
       References 24
       Problems 25

2. BASIC PRINCIPLES 27
   2.1 Introduction 27
   2.2 Gradient Information 27
   2.3 The Taylor Series 28
   2.4 Types of Extrema 31
   2.5 Necessary and Sufficient Conditions for Local Minima and Maxima 33
   2.6 Classification of Stationary Points 40
   2.7 Convex and Concave Functions 51
   2.8 Optimization of Convex Functions 58
       References 60
       Problems 60

3. GENERAL PROPERTIES OF ALGORITHMS 65
   3.1 Introduction 65
   3.2 An Algorithm as a Point-to-Point Mapping 65
   3.3 An Algorithm as a Point-to-Set Mapping 67
   3.4 Closed Algorithms 68
   3.5 Descent Functions 71
   3.6 Global Convergence 72
10.3 Classification of Constrained Optimization Problems 273
10.4 Simple Transformation Methods 277
10.5 Lagrange Multipliers 285
10.6 First-Order Necessary Conditions 294
10.7 Second-Order Conditions 302
10.8 Convexity 308
10.9 Duality
   References 312
   Problems 313

11. LINEAR PROGRAMMING PART I: THE SIMPLEX METHOD 321
   11.1 Introduction 321
   11.2 General Properties 322
   11.3 Simplex Method 344
      References 368
      Problems 368

12. LINEAR PROGRAMMING PART II: INTERIOR-POINT METHODS 373
   12.1 Introduction 373
   12.2 Primal-Dual Solutions and Central Path 374
   12.3 Primal Affine-Scaling Method 379
   12.4 Primal Newton Barrier Method 383
   12.5 Primal-Dual Interior-Point Methods 388
      References 402
      Problems 402

13. QUADRATIC AND CONVEX PROGRAMMING 407
   13.1 Introduction 407
   13.2 Convex QP Problems with Equality Constraints 408
   13.3 Active-Set Methods for Strictly Convex QP Problems 411
   13.4 Interior-Point Methods for Convex QP Problems 417
   13.5 Cutting-Plane Methods for CP Problems 428
   13.6 Ellipsoid Methods 437
      References 443
14. SEMIDEFINITE AND SECOND-ORDER CONE PROGRAMMING 449
  14.1 Introduction 449
  14.2 Primal and Dual SDP Problems 450
  14.3 Basic Properties of SDP Problems 455
  14.4 Primal-Dual Path-Following Method 458
  14.5 Predictor-Corrector Method 465
  14.6 Projective Method of Nemirovski and Gahinet 470
  14.7 Second-Order Cone Programming 484
  14.8 A Primal-Dual Method for SOCP Problems 491
    References 496
    Problems 497

15. GENERAL NONLINEAR OPTIMIZATION PROBLEMS 501
  15.1 Introduction 501
  15.2 Sequential Quadratic Programming Methods 501
  15.3 Modified SQP Algorithms 509
  15.4 Interior-Point Methods 518
    References 528
    Problems 529

16. APPLICATIONS OF CONSTRAINED OPTIMIZATION 533
  16.1 Introduction 533
  16.2 Design of Digital Filters 534
  16.3 Model Predictive Control of Dynamic Systems 547
  16.4 Optimal Force Distribution for Robotic Systems with Closed
      Kinematic Loops 558
  16.5 Multiuser Detection in Wireless Communication Channels 570
    References 586
    Problems 588

Appendices 591
A  Basics of Linear Algebra 591
  A.1 Introduction 591
A.2 Linear Independence and Basis of a Span 592
A.3 Range, Null Space, and Rank 593
A.4 Sherman-Morrison Formula 595
A.5 Eigenvalues and Eigenvectors 596
A.6 Symmetric Matrices 598
A.7 Trace 602
A.8 Vector Norms and Matrix Norms 602
A.9 Singular-Value Decomposition 606
A.10 Orthogonal Projections 609
A.11 Householder Transformations and Givens Rotations 610
A.12 QR Decomposition 616
A.13 Cholesky Decomposition 619
A.14 Kronecker Product 621
A.15 Vector Spaces of Symmetric Matrices 623
A.16 Polygon, Polyhedron, Polytope, and Convex Hull 626
References 627
B Basics of Digital Filters 629
B.1 Introduction 629
B.2 Characterization 629
B.3 Time-Domain Response 631
B.4 Stability Property 632
B.5 Transfer Function 633
B.6 Time-Domain Response Using the Z Transform 635
B.7 Z-Domain Condition for Stability 635
B.8 Frequency, Amplitude, and Phase Responses 636
B.9 Design 639
Reference 644
Index 645
The rapid advancements in the efficiency of digital computers and the evolution of reliable software for numerical computation during the past three decades have led to an astonishing growth in the theory, methods, and algorithms of numerical optimization. This body of knowledge has, in turn, motivated widespread applications of optimization methods in many disciplines, e.g., engineering, business, and science, and led to problem solutions that were considered intractable not too long ago.

Although excellent books are available that treat the subject of optimization with great mathematical rigor and precision, there appears to be a need for a book that provides a practical treatment of the subject aimed at a broader audience ranging from college students to scientists and industry professionals. This book has been written to address this need. It treats unconstrained and constrained optimization in a unified manner and places special attention on the algorithmic aspects of optimization to enable readers to apply the various algorithms and methods to specific problems of interest. To facilitate this process, the book provides many solved examples that illustrate the principles involved, and includes, in addition, two chapters that deal exclusively with applications of unconstrained and constrained optimization methods to problems in the areas of pattern recognition, control systems, robotics, communication systems, and the design of digital filters. For each application, enough background information is provided to promote the understanding of the optimization algorithms used to obtain the desired solutions.

Chapter 1 gives a brief introduction to optimization and the general structure of optimization algorithms. Chapters 2 to 9 are concerned with unconstrained optimization methods. The basic principles of interest are introduced in Chapter 2. These include the first-order and second-order necessary conditions for a point to be a local minimizer, the second-order sufficient conditions, and the optimization of convex functions. Chapter 3 deals with general properties of algorithms such as the concepts of descent function, global convergence, and
rate of convergence. Chapter 4 presents several methods for one-dimensional optimization, which are commonly referred to as line searches. The chapter also deals with inexact line-search methods that have been found to increase the efficiency in many optimization algorithms. Chapter 5 presents several basic gradient methods that include the steepest descent, Newton, and Gauss-Newton methods. Chapter 6 presents a class of methods based on the concept of conjugate directions such as the conjugate-gradient, Fletcher-Reeves, Powell, and Partan methods. An important class of unconstrained optimization methods known as quasi-Newton methods is presented in Chapter 7. Representative methods of this class such as the Davidon-Fletcher-Powell and Broydon-Fletcher-Goldfarb-Shanno methods and their properties are investigated. The chapter also includes a practical, efficient, and reliable quasi-Newton algorithm that eliminates some problems associated with the basic quasi-Newton method. Chapter 8 presents minimax methods that are used in many applications including the design of digital filters. Chapter 9 presents three case studies in which several of the unconstrained optimization methods described in Chapters 4 to 8 are applied to point pattern matching, inverse kinematics for robotic manipulators, and the design of digital filters.

Chapters 10 to 16 are concerned with constrained optimization methods. Chapter 10 introduces the fundamentals of constrained optimization. The concept of Lagrange multipliers, the first-order necessary conditions known as Karush-Kuhn-Tucker conditions, and the duality principle of convex programming are addressed in detail and are illustrated by many examples. Chapters 11 and 12 are concerned with linear programming (LP) problems. The general properties of LP and the simplex method for standard LP problems are addressed in Chapter 11. Several interior-point methods including the primal affine-scaling, primal Newton-barrier, and primal dual-path following methods are presented in Chapter 12. Chapter 13 deals with quadratic and general convex programming. The so-called active-set methods and several interior-point methods for convex quadratic programming are investigated. The chapter also includes the so-called cutting plane and ellipsoid algorithms for general convex programming problems. Chapter 14 presents two special classes of convex programming known as semidefinite and second-order cone programming, which have found interesting applications in a variety of disciplines. Chapter 15 treats general constrained optimization problems that do not belong to the class of convex programming; special emphasis is placed on several sequential quadratic programming methods that are enhanced through the use of efficient line searches and approximations of the Hessian matrix involved. Chapter 16, which concludes the book, examines several applications of constrained optimization for the design of digital filters, for the control of dynamic systems, for evaluating the force distribution in robotic systems, and in multiuser detection for wireless communication systems.
The book also includes two appendices, A and B, which provide additional support material. Appendix A deals in some detail with the relevant parts of linear algebra to consolidate the understanding of the underlying mathematical principles involved whereas Appendix B provides a concise treatment of the basics of digital filters to enhance the understanding of the design algorithms included in Chaps. 8, 9, and 16.

The book can be used as a text for a sequence of two one-semester courses on optimization. The first course comprising Chaps. 1 to 7, 9, and part of Chap. 10 may be offered to senior undergraduate or first-year graduate students. The prerequisite knowledge is an undergraduate mathematics background of calculus and linear algebra. The material in Chaps. 8 and 10 to 16 may be used as a text for an advanced graduate course on minimax and constrained optimization. The prerequisite knowledge for this course is the contents of the first optimization course.

The book is supported by online solutions of the end-of-chapter problems under password as well as by a collection of MATLAB programs for free access by the readers of the book, which can be used to solve a variety of optimization problems. These materials can be downloaded from the book's website: http://www.ece.uvic.ca/~optimization/.

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ABBREVIATIONS

AWGN additive white Gaussian noise
BER bit-error rate
BFGS Broyden-Fletcher-Goldfarb-Shanno
CDMA code-division multiple access
CMBER constrained minimum BER
CP convex programming
DFP Davidon-Fletcher-Powell
D-H Denavit-Hartenberg
DNB dual Newton barrier
DS direct sequence
FDMA frequency-division multiple access
FIR finite-duration impulse response
FR Fletcher-Reeves
GCO general constrained optimization
GN Gauss-Newton
IIR infinite-duration impulse response
IP integer programming
KKT Karush-Kuhn-Tucker
LCP linear complementarity problem
LMI linear matrix inequality
LP linear programming
LSQI least-squares minimization with quadratic inequality
LU lower-upper
MAI multiple access interference
ML maximum likelihood
MPC model predictive control
PAS primal affine-scaling
PCM predictor-corrector method
PNB primal Newton barrier
QP quadratic programming
SD steepest descent
SDP semidefinite programming
SDPR-D SDP relaxation-dual
SDPR-P SDP relaxation-primal
SNR signal-to-noise ratio
SOCP second-order cone programming
SQP sequential quadratic programming
SVD singular-value decomposition
TDMA time-division multiple access