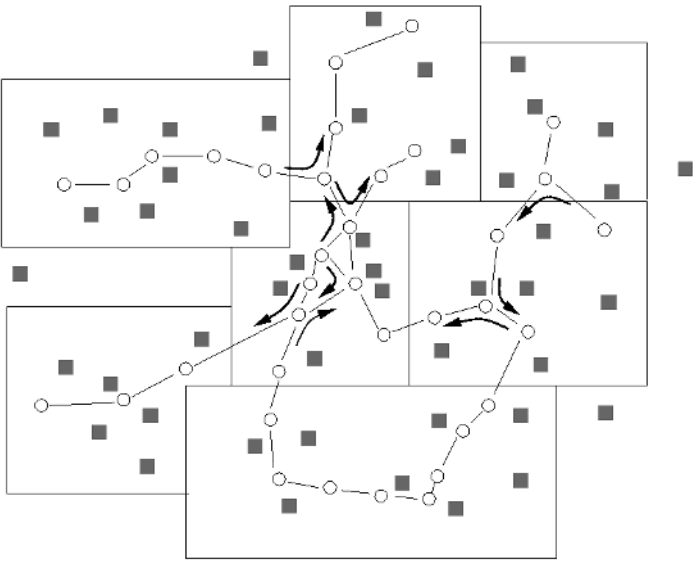


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# OPTIMIZATION IN PUBLIC TRANSPORTATION

Stop Location, Delay Management and Tariff Zone Design in a Public Transportation Network



# Optimization and Its Applications

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VOLUME 3

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*Aims and Scope*

Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.

The series *Springer Optimization and Its Applications* publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multi-objective programming, description of software packages, approximation techniques and heuristic approaches.

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Stop Location, Delay Management and Tariff Zone Design  
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By

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To my parents  
Helga and Volker Schumacher

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# Preface

*Der Kunde ist König.*

(German saying)

Public transportation plays an important role in most populated areas. Especially in metropolitan regions public transportation systems are widely used. But unfortunately, public transportation is often a subject of complaints. Customers are annoyed about “unfair prices”, about “bad service” and in particular get upset in case of delays. Such complaints are understandable, but for the public transportation companies it is often impossible to provide a better service without increasing the costs. The reason for these difficulties is the complexity and the size of the planning problems arising.

The theory of optimization provides a sound methodology for finding good solutions, if a mathematical model of the respective problem is known. Moreover, due to the availability of fast computers many problems that seemed to be intractable some years ago can nowadays be solved.

This work provides suitable models for planning public transportation systems from a customer-oriented point of view, but taking into account the limited budget public transportation companies have to respect. In particular, we develop and analyze optimization models for the following three problems:

Part I: Stop location. Here we deal with the location of stops along bus routes, or of stations along railway tracks. As objective functions we consider the number of customers living close to a station and the additional travel time arising by the stopping activities of the trains or buses. In particular, we discuss how to find the minimal number of stops to cover

a given set of demand points or demand regions, how to cover as many customers as possible with a given budget and both problems together in a bicriteria setting.

Part II: Delay management. If a vehicle arrives at a station with a delay, passengers who wish to change into another vehicle, say a bus, may miss their connection, if this bus departs on time. Such wait-depart decisions and their impact on the whole transportation system are investigated from the customers' perspective. As objective functions we hence discuss the sum of all delays over all passengers, the number of missed connections, and the sum of all delays over all vehicles. The latter two objectives are treated as a bicriteria optimization problem.

Part III: Zone planning. In order to design a zone tariff system, the complete transportation area has to be partitioned into zones, and prices for traveling through 1,2,3,... zones have to be defined in such a way that the current income of the public transportation company does not decrease too much. As objective function we consider the deviations between the new prices and some given reference prices. These deviations can be interpreted as the fairness of the new tariff system or as the changes to the current ticket prices.

All three problems were brought to my attention within real-world projects, and some of the obtained results have already been implemented and applied in practice. Nevertheless, the main focus of this work is to develop a consistent mathematical theory and to present basic results within all three fields.

- The stop location problem is treated using the concept of gauges and ideas of continuous location theory. A finite dominating set of possible new stops can be derived. This allows us to formulate the stop location problem as a set covering problem. By using the special structure of the covering matrix which is due to the geometrical properties of the stop location problem, efficient solution methods for this type of set covering problems are developed.
- For the delay management problem three different, but equivalent mixed integer programming formulations are presented. By combining these models many structural results for the delay management problem are obtained. In particular, it is possible to identify cases in which the problem is solvable efficiently. Furthermore, methods of project planning are applied to determine Pareto solutions.
- Finally, the design of zone tariff systems in public transportation is modeled by methods of graph theory. The obtained theoretical results together with ideas of clustering theory are utilized for deriving solution approaches.

The theory presented in this text and the obtained results open a wide field for further developments and implementations of the suggested approaches. The algorithms that have already been tested on our real-world data confirm the practical usefulness of the models and show their potential for future applications.

Before concluding the preface I wish to add several acknowledgments. First of all, I thank Horst W. Hamacher for his support, for the pleasant and constructive work together with him, and for his helpful advice in any question I had.

For many valuable suggestions I am indebted to Kathrin Klamroth and Dagmar Tenfelde-Podehl. I also appreciate the comments of Teresa Melo, Martin C. Müller, and Michael Schröder.

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I also want to thank Robert Saley of Springer for his kind assistance during the publication of the manuscript.

Last, but not least, my special thanks go to my husband Georg and to my children Svenja and Malte for their encouragement, patience, and understanding which made it possible for me to write this text.

Göttingen,  
December 2005

*Anita Schöbel*



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