

Markov Chains: Models, Algorithms and Applications

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To Anna, Cecilia, Mandy and our Parents

Contents

1	Introduction	1
1.1	Markov Chains	1
1.1.1	Examples of Markov Chains	2
1.1.2	The n th-Step Transition Matrix	5
1.1.3	Irreducible Markov Chain and Classifications of States	7
1.1.4	An Analysis of the Random Walk	8
1.1.5	Simulation of Markov Chains with EXCEL	10
1.1.6	Building a Markov Chain Model	11
1.1.7	Stationary Distribution of a Finite Markov Chain	14
1.1.8	Applications of the Stationary Distribution	16
1.2	Continuous Time Markov Chain Process	16
1.2.1	A Continuous Two-state Markov Chain	18
1.3	Iterative Methods for Solving Linear Systems	19
1.3.1	Some Results on Matrix Theory	20
1.3.2	Splitting of a Matrix	21
1.3.3	Classical Iterative Methods	22
1.3.4	Spectral Radius	24
1.3.5	Successive Over-Relaxation (SOR) Method	26
1.3.6	Conjugate Gradient Method	26
1.3.7	Toeplitz Matrices	30
1.4	Hidden Markov Models	32
1.5	Markov Decision Process	33
1.5.1	Stationary Policy	35
2	Queueing Systems and the Web	37
2.1	Markovian Queueing Systems	37
2.1.1	An M/M/1/ $n - 2$ Queueing System	37
2.1.2	An M/M/ $s/n - s - 1$ Queueing System	39
2.1.3	The Two-Queue Free System	41
2.1.4	The Two-Queue Overflow System	42
2.1.5	The Preconditioning of Complex Queueing Systems	43

2.2	Search Engines	47
2.2.1	The PageRank Algorithm	49
2.2.2	The Power Method	50
2.2.3	An Example	51
2.2.4	The SOR/JOR Method and the Hybrid Method	52
2.2.5	Convergence Analysis	54
2.3	Summary	58
3	Re-manufacturing Systems	61
3.1	Introduction	61
3.2	An Inventory Model for Returns	62
3.3	The Lateral Transshipment Model	66
3.4	The Hybrid Re-manufacturing Systems	68
3.4.1	The Hybrid System	69
3.4.2	The Generator Matrix of the System	69
3.4.3	The Direct Method	71
3.4.4	The Computational Cost	74
3.4.5	Some Special Cases Analysis	74
3.5	Summary	75
4	Hidden Markov Model for Customers Classification	77
4.1	Introduction	77
4.1.1	A Simple Example	77
4.2	Parameter Estimation	78
4.3	Extension of the Method	79
4.4	Special Case Analysis	80
4.5	Application to Classification of Customers	82
4.6	Summary	85
5	Markov Decision Process for Customer Lifetime Value	87
5.1	Introduction	87
5.2	Markov Chain Models for Customers' Behavior	89
5.2.1	Estimation of the Transition Probabilities	90
5.2.2	Retention Probability and CLV	91
5.3	Stochastic Dynamic Programming Models	92
5.3.1	Infinite Horizon without Constraints	93
5.3.2	Finite Horizon with Hard Constraints	95
5.3.3	Infinite Horizon with Constraints	96
5.4	Higher-order Markov decision process	102
5.4.1	Stationary policy	103
5.4.2	Application to the calculation of CLV	105
5.5	Summary	106

- 6 Higher-order Markov Chains** 111
 - 6.1 Introduction 111
 - 6.2 Higher-order Markov Chains 112
 - 6.2.1 The New Model 113
 - 6.2.2 Parameters Estimation 116
 - 6.2.3 An Example 119
 - 6.3 Some Applications 121
 - 6.3.1 The DNA Sequence 122
 - 6.3.2 The Sales Demand Data 124
 - 6.3.3 Webpages Prediction 126
 - 6.4 Extension of the Model 129
 - 6.5 Newboy’s Problems 134
 - 6.5.1 A Markov Chain Model for the Newsboy’s Problem ... 135
 - 6.5.2 A Numerical Example 138
 - 6.6 Summary 139

- 7 Multivariate Markov Chains** 141
 - 7.1 Introduction 141
 - 7.2 Construction of Multivariate Markov Chain Models 141
 - 7.2.1 Estimations of Model Parameters 144
 - 7.2.2 An Example 146
 - 7.3 Applications to Multi-product Demand Estimation 148
 - 7.4 Applications to Credit Rating 150
 - 7.4.1 The Credit Transition Matrix 151
 - 7.5 Applications to DNA Sequences Modeling 153
 - 7.6 Applications to Genetic Networks 156
 - 7.6.1 An Example 161
 - 7.6.2 Fitness of the Model 163
 - 7.7 Extension to Higher-order Multivariate Markov Chain 167
 - 7.8 Summary 169

- 8 Hidden Markov Chains** 171
 - 8.1 Introduction 171
 - 8.2 Higher-order HMMs 171
 - 8.2.1 Problem 1 173
 - 8.2.2 Problem 2 175
 - 8.2.3 Problem 3 176
 - 8.2.4 The EM Algorithm 178
 - 8.2.5 Heuristic Method for Higher-order HMMs 179
 - 8.2.6 Experimental Results 182
 - 8.3 The Interactive Hidden Markov Model 183
 - 8.3.1 An Example 183
 - 8.3.2 Estimation of Parameters 184
 - 8.3.3 Extension to the General Case 186
 - 8.4 The Double Higher-order Hidden Markov Model 187

X Contents

8.5 Summary	189
References	191
Index	203

List of Figures

Fig. 1.1. The random walk.	4
Fig. 1.2. The gambler's problem.	4
Fig. 1.3. The $(n + 1)$ -step transition probability.	6
Fig. 1.4. Simulation of a Markov chain.	12
Fig. 1.5. Building a Markov chain.	13
Fig. 2.1. The Markov chain for the one-queue system.	38
Fig. 2.2. The Markov chain for the one-queue system.	40
Fig. 2.3. The two-queue overflow system.	42
Fig. 2.4. An example of three webpages.	48
Fig. 3.1. The single-item inventory model.	63
Fig. 3.2. The Markov chain	64
Fig. 3.3. The hybrid system	70
Fig. 4.1. The graphical interpretation of Proposition 4.2.	82
Fig. 5.1. EXCEL for solving infinite horizon problem without constraint.	94
Fig. 5.2. EXCEL for solving finite horizon problem without constraint.	97
Fig. 5.3. EXCEL for solving infinite horizon problem with constraints.	99
Fig. 6.1. The states of four products A,B,C and D.	125
Fig. 6.2. The first (a), second (b), third (c) step transition matrices.	128

List of Tables

Table 2.1. Number of iterations for convergence ($\alpha = 1 - 1/N$).	58
Table 2.2. Number of iterations for convergence ($\alpha = 0.85$).	59
Table 4.1. Probability distributions of dice A and dice B.	77
Table 4.2. Two-third of the data are used to build the HMM.	84
Table 4.3. The average expenditure of Group A and B.	84
Table 4.4. The remaining one-third of the data for the validation of HMM.	85
Table 5.1. The four classes of customers.	90
Table 5.2. The average revenue of the four classes of customers.	92
Table 5.3. Optimal stationary policies and their CLVs.	95
Table 5.4. Optimal promotion strategies and their CLVs.	98
Table 5.5. Optimal promotion strategies and their CLVs.	100
Table 5.6. Optimal promotion strategies and their CLVs.	101
Table 5.7. The second-order transition probabilities.	105
Table 5.8. Optimal strategies when the first-order MDP is used.	107
Table 5.9. Optimal strategies when the second-order MDP is used.	108
Table 5.10. Optimal strategies when the second-order MDP is used.	109
Table 6.1. Prediction accuracy in the DNA sequence.	123
Table 6.2. Prediction accuracy in the sales demand data.	125
Table 6.3. Prediction accuracy and χ^2 value.	133
Table 6.4. Prediction accuracy and χ^2 value.	133
Table 6.5. The optimal costs of the three different models.	139
Table 7.1. Prediction accuracy in the sales demand data.	150
Table 7.2. Results of the multivariate Markov chain models.	156
Table 7.3. The first sequence results.	162
Table 7.4. The second sequence results.	163
Table 7.5. Results of our multivariate Markov chain model.	165
Table 7.6. Prediction results	166
Table 8.1. $\log P[O A]$.	183
Table 8.2. Computational times in seconds.	183

Preface

The aim of this book is to outline the recent development of Markov chain models for modeling queueing systems, Internet, re-manufacturing systems, inventory systems, DNA sequences, genetic networks and many other practical systems.

This book consists of eight chapters. In Chapter 1, we give a brief introduction to the classical theory on both discrete and continuous time Markov chains. The relationship between Markov chains of finite states and matrix theory will also be discussed. Some classical iterative methods for solving linear systems will also be introduced. We then give the basic theory and algorithms for standard hidden Markov model (HMM) and Markov decision process (MDP).

Chapter 2 discusses the applications of continuous time Markov chains to model queueing systems and discrete time Markov chain for computing the PageRank, the ranking of website in the Internet. Chapter 3 studies re-manufacturing systems. We present Markovian models for re-manufacturing, closed form solutions and fast numerical algorithms are presented for solving the systems. In Chapter 4, Hidden Markov models are applied to classify customers. We proposed a simple hidden Markov model with fast numerical algorithms for solving the model parameters. An application of the model to customer classification is discussed. Chapter 5 discusses Markov decision process for customer lifetime values. Customer Lifetime Values (CLV) is an important concept and quantity in marketing management. We present an approach based on Markov decision process to the calculation of CLV with practical data.

In Chapter 6, we discuss higher-order Markov chain models. We propose a class of higher-order Markov chain models with lower order of model parameters. Efficient numerical methods based on linear programming for solving the model parameters are presented. Applications to demand predictions, inventory control, data mining and DNA sequence analysis are discussed. In Chapter 7, multivariate Markov models are discussed. We present a class of multivariate Markov chain model with lower order of model parameters. Effi-

cient numerical methods based on linear programming for solving the model parameters are presented. Applications to demand predictions and gene expression sequences are discussed. In Chapter 8, higher-order hidden Markov models are studied. We proposed a class of higher-order hidden Markov models with efficient algorithm for solving the model parameters.

This book is aimed at students, professionals, practitioners, and researchers in applied mathematics, scientific computing, and operational research, who are interested in the formulation and computation of queueing and manufacturing systems. Readers are expected to have some basic knowledge of probability theory Markov processes and matrix theory.

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