A POSTERIORI ERROR ANALYSIS VIA DUALITY THEORY

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A POSTERIORI ERROR ANALYSIS VIA DUALITY THEORY

With Applications in Modeling and Numerical Approximations

by

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Dedicated to

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Preface

This work provides a posteriori error analysis for mathematical idealizations in modeling boundary value problems, especially those arising in mechanical applications, and for numerical approximations of numerous nonlinear variational problems. An error estimate is called a posteriori if the computed solution is used in assessing its accuracy. A posteriori error estimation is central to measuring, controlling and minimizing errors in modeling and numerical approximations. In this book, the main mathematical tool for the developments of a posteriori error estimates is the duality theory of convex analysis, documented in the well-known book by Ekeland and Temam ([49]). The duality theory has been found useful in mathematical programming, mechanics, numerical analysis, etc.

The book is divided into six chapters. The first chapter reviews some basic notions and results from functional analysis, boundary value problems, elliptic variational inequalities, and finite element approximations. The most relevant part of the duality theory and convex analysis is briefly reviewed in Chapter 2. This brief review is sufficient for the applications of the duality theory in all the following chapters. In mathematical modeling of differential equation problems, usually assumptions are made on various data. Qualitatively, for many problems, it is known that the solution depends continuously on the problem data. Frequently though, it is desirable also to estimate or bound quantitatively the effect on the solutions of the problems caused by the adoption of the assumptions on the data. In Chapter 3, a posteriori error estimates are derived for the effect on the solutions of mathematical idealizations on the data of elliptic linear boundary value problems. In Chapter 4, a posteriori error estimates are given for linearization in a number of nonlinear boundary value problems. The last two chapters are devoted to a posteriori error analysis of numerical solutions. In Chapter 5, the regularization method and the Kačanov method are considered, both being useful in handling certain types of nonlinearity. In Chapter 6, a posteriori error estimates are derived and studied for finite element solutions of some elliptic variational inequalities.

This book is intended for researchers and graduate students in Applied and Computational Mathematics, and Engineering. Mathematical prerequisites include calculus, linear algebra, some exposures of differential equations, and concepts of normed spaces, Banach spaces and Hilbert spaces. In the theoretical development, some basic notions and results in functional analysis, duality thoery, weak formulations of boundary value problems, variational inequalities, and the finite element method are used. Brief reviews of these notions and results in the first two chapters provide background materials for a reader who lacks knowledge in these areas.

This work avoids giving the results in the most general, abstract form so that it is easier for the reader to understand more clearly the essential ideas involved. Many examples are included to show the usefulness of the derived error estimates.

In preparing this book, I have benefited from many individuals. I am grateful to Professor Ivo Babuška for introducing me the research topic and for providing valuable advice. Several of my collaborators (teachers, friends, and students) made contributions to various parts of the book, and I especially thank Dr. Viorel Bostan, Dr. Jiuhua Chen, Professor Hongci Huang, late Professor Søren Jensen, Professor B.D. Reddy. I express my gratitude to Professor Kendall Atkinson and Professor Mircea Sofonea for their constant support. I thank Professor D.Y. Gao and Professor R.W. Ogden for inviting me to make the contribution in their Kluwer book series on Advances in Mechanics and Mathematics (AMMA).

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