Interfacial Instability

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With 61 Figures

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To Ruth and Vasudha

Preface: A Guide for the Reader

This book presents a set of essays on interfacial instability, drawing attention to the surface tension acting at an interface between two phases. The stability of a liquid jet is the model problem. In this problem, surface tension works on the pressures of the fluids, and thence on their velocities. In other problems, it works on gradients of temperature and concentration. It affects the freezing point in solidification, it affects the solubility in precipitation and it affects the rate of electron transfer in electrodeposition.

The essays are nine in number and there are three appendices. A set of derivations is collected at the end of many of the essays to serve as a basis for discussion. These are called discussion notes to set them apart from endnotes. The endnotes ordinarily explain background information which the reader may need in order to enjoy an essay, however, on one or two occasions an endnote presents the derivation of a formula needed in the essay.

The subject matter is tied together by two or three common themes.

Each essay presents a problem where the domain on which the problem is defined is not fixed in advance. Determining the domain is then part of solving the problem. This is one theme and it calls for some explanation. This is given in the second essay, which presents a way in which a domain perturbation can be carried out. As our plan is to project the nonlinear equations onto a reference domain, it is necessary to explain how this can be done. The equations that hold at a deflecting surface present the main problem and, although intuition might lead to the correct perturbation equations in simple cases, in other cases a definite method might be welcome. Appendix A presents several short, but interesting, illustrations of the ideas worked out in the second essay.

Most problems start with a base state upon which a perturbation is imposed. The base state will be a solution to the nonlinear equations and it will depend on the values to which its input or control variables are set. The base state may be stable or it may be unstable to the perturbation. Determining the critical values of the input variables at which the base state loses stability is a second theme.

This brings us to the aims of the essays and to another important theme. The essays were written to fulfill two needs at one and the same time. They can be used by teachers who would like to add lectures on interfacial instabilities to their courses and they can be used by graduate students trying to bring themselves to the point of doing their research on interfacial problems. To achieve both aims, we have selected problems that can be solved by hand, using only pencil and paper, or chalk and blackboard. This is the third main theme. Little numerical work is required and it follows at once that no nonlinear calculations are going to be presented. A lot of physics can be learned by doing the linear problem first, physics that can help direct a nonlinear calculation.

The simpler the base solution, the further we can go using only a pencil and a piece of paper. This guides the selection of the topics presented in this book. But, by limiting ourselves to pencil-and-paper work, we are not limited to the same problems that have been presented to students for many years. The fifth, seventh, eighth and ninth essays are largely unfamiliar and so, of course, is the second essay. The first, third, fourth and sixth essays may be familiar, but that is not to say that no new results can be found therein. It is surprising just how much turns up in the course of reworking the classical problems.

Now, let us indicate the way in which the essays fit together.

The first four essays draw attention to the breakup of a cylindrical thread of liquid, called a jet. This is addressed first by way of Rayleigh's work principle and then by way of a perturbation calculation. Rayleigh's work principle is a useful rule of thumb, and we try to use it as much as possible throughout the essays. But forces cause instabilities, and forces are what the perturbation equations are all about. As a deflecting surface calls into question the domain on which the fluid lies, writing the perturbation equations draws on the material presented in the second essay.

The first and third essays explain the problem in the case of an infinitely long jet. Then, in the fourth essay, the jet is made of finite length in order to introduce a control variable. This brings in the various ways in which a jet can meet its end walls and introduces their influence on the question of its stability. The finite jet calculation retains the flavor of the infinite jet in only one case. Save this, the fundamental modes of displacement do not have discernible wavelengths.

Two end conditions, free and fixed, are of interest. By free conditions, contact at right angles is intended. The other case is called fixed. This end condition, which can be realized quite easily in an experiment, precludes symmetric eigenfunctions. At the same time it eliminates contact line motion, which presents something of a technical difficulty if the no-slip condition must be taken into account.

At first, the fluids are take to be inviscid. This simplifies our calculations and it determines the critical point, the point of zero growth, entirely correctly. This is just as one might guess. Critical points are determined by a balance of forces. If viscosity does not enter into this balance, then critical points can be determined as though the fluids were inviscid. Still, the viscosity of the fluids is not entirely neglected. Its role is explained, first by way of a picture, and then by way of an energy calculation.

Now, something a little odd turns up in the three essays on jets and bridges and asserts itself as a theme well into the sixth essay. It is this: Maintaining the volume of a jet fixed, as its surface is displaced, turns out to be crucial in using Rayleigh's work principle, however, on carrying out the perturbation calculation, it is, at first, of little consequence. It is not until the jet is made finite that holding its volume fixed leads to an important perturbation equation, but even then, it is not important for all end conditions.

In the fifth essay, our attention turns to the effect of rotation in an effort to see if Rayleigh's criterion for detecting an adverse stratification of angular momentum carries the same force, when a liquid cylinder is confined by its own surface tension, as it does when it is confined by a rigid wall.

The sixth essay, then, brings to a close the sequence of essays wherein surface tension plays its role through pressure differences across surfaces and the effect of pressure on fluid motions. This essay deals with the stability of a heavy fluid lying over a light fluid, and it adds the effect of gravity to the effect of surface tension. Due to this, the base solution turns out to be a little more interesting, yet the problem is very much like the jet problem, and all the questions raised and answered there are again raised and answered in this essay.

In the seventh essay, the stability of freezing and melting fronts is taken up; in the eighth essay, attention is directed to the stability of the front dividing a solid and the solution into which the solid is dissolving or out of which it is precipitating.

These two problems are very much alike, but one interesting difference is discovered and it is tied up in the fact that in both cases surface tension, although it is ordinarily stabilizing, can make a destabilizing contribution to the shape of the front. These two essays continue the theme of displaced surfaces, and the role of surface tension in determining their stability. They lead into the ninth essay, where two such surfaces must be taken into account in order to determine the stability of the surface of a metal being plated out of an electrolytic solution. Surface tension continues to play its stabilizing role, but now by way of its influence on the electron transfer reaction that must take place at both surfaces.

Some of the background information for this essay, mainly information on the rates of electron transfer reactions, may not be widely known, and so it is presented, for it is essential to a physical explanation of the instability.

It is in this essay that our explanations for the causes of the instabilities by way of pictures achieve their greatest precision. It is here that the signature of the base solution on its stability can be most confidently decoded.

As a guide to the reader, we list references to books and papers that got us started on these essays or that helped us over some of the rough points:

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