

# Missile Guidance and Control Systems

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# Missile Guidance and Control Systems



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## Preface

*In every department of physical science there is only so much science, properly so-called, as there is mathematics.*

*Immanuel Kant*

Most air defense systems in use or under development today, employ homing guidance to effect intercept of the target. By virtue of the use of onboard data gathering, the homing guidance system provides continually improving quality of target information right up to the intercept point. More than any single device, the guided missile has shaped the aerospace forces of the world today. Combat aircraft, for example, are fitted with airborne weapons that can be launched against enemy aircraft, ground forces, or strategic targets deep inside enemy territory. Also, the guided missile can be employed as a diversionary weapon to confuse ground and air forces. Ground-based missile systems have various range capabilities from a few miles to several thousand miles. These ground-based missiles are ballistic or nonballistic types, depending on their mission requirements. The design of a guided weapon (i.e., a missile) is a large undertaking, requiring the team effort of many engineers having expertise in the areas of aerodynamics, flight controls, structures, and propulsion, among others. The different design groups must work together to produce the most efficient weapon in terms of high accuracy and low cost.

The intent of this book is to present the fundamental concepts of guided missiles, both tactical, and strategic and the guidance, control, and instrumentation needed to acquire a target. In essence, this book is about the mathematics of guided flight. This book differs from similar books on the subject in that it presents a detailed account of missile aerodynamic forces and moments, the missile mathematical model, weapon delivery, *GPS* (global positioning system) and *TERCOM* (terrain contour matching) guidance, cruise missile mechanization equations, and a detailed analysis of ballistic guidance laws. Moreover, an attempt has been made to give each subject proper emphasis, while at the same time special effort has been put forth to obtain simplicity, both from the logical and pedagogical standpoint. Typical examples are provided, where necessary, to illustrate the principles involved. Numerous figures give the maximum value of visual aids by showing important relations at a glance and motivating the various topics. Finally, this book will be

of benefit to engineers engaged in the design and development of guided missiles and to aeronautical engineering students, as well as serving as a convenient reference for researchers in weapon system design.

The aerospace engineering field and its disciplines are undergoing a revolutionary change, albeit one that is difficult to secure great perspective on at the time of this writing. The author has done his best to present the state of the art in weapons systems. To this end, all criticism and suggestions for future improvement of the book are welcomed.

The book consists of seven chapters and several appendices. Chapter 1 presents a historical background of past and present guided missile systems and the evolution of modern weapons. Chapter 2 discusses the generalized missile equations of motion. Among the topics discussed are generalized coordinate systems, rigid body equations of motion, D'Alembert's principle, and Lagrange's equations for rotating coordinate systems. Chapter 3 covers aerodynamic forces and coefficients. Of interest here is the extensive treatment of aerodynamic forces and moments, the various types of missile seekers and their function in the guidance loop, autopilots, and control surface actuators. Chapter 4 treats the important subject of the various types of tactical guidance laws and/or techniques. The types of guidance laws discussed in some detail are homing guidance, command guidance, proportional navigation, augmented proportional navigation, and guidance laws using modern control and estimation theory. Chapter 5 deals with weapon delivery systems and techniques. Here the reader will find many topics not found in similar books. Among the numerous topics treated are weapon delivery requirements, the navigation/weapon delivery system, the fire control computer, accuracies in weapon delivery, and modern topics such as situational awareness/situation assessment. Chapter 6 is devoted to strategic missiles, including the classical two-body problem and Lambert's theorem, the spherical Earth hit equation, explicit and implicit guidance techniques, atmospheric reentry, and ballistic missile intercept. Chapter 7 focuses on cruise missile theory and design. Much of the material in this chapter centers on the concepts of cruise missile navigation, the terrain contour matching concept, and the global positioning system. Each chapter contains references for further research and study. Several appendices provide added useful information for the reader. Appendix A lists several fundamental constants, Appendix B presents a glossary of terms found in technical publications and books, Appendix C gives a list of acronyms, Appendix D discusses the standard atmosphere, Appendix E presents the missile classification, Appendix F lists past and present missile systems, Appendix G summarizes the properties of conics that are useful in understanding the material of Chapter 6, Appendix H is a list of radar frequencies, and Appendix I presents a list of the most commonly needed conversion factors.

Such is the process of learning that it is never possible for anyone to say exactly how he acquired any given body of knowledge. My own knowledge was acquired from many people from academia, industry, and the government. Specifically, my knowledge in guided weapons and control systems was acquired and nurtured during my many years of association with the Department of the Air Force's Aeronautical Systems Center, Wright-Patterson AFB, Ohio, while participating in the theory,

design, operation, and testing (i.e., from concept to fly-out) the air-launched cruise missile (*ALCM*), *SRAM II*, *Minuteman III*, the *AIM-9 Sidewinder*, and other programs too numerous to list.

Obviously, as anyone who has attempted it knows, writing a book is hardly a solitary activity. In writing this book, I owe thanks and acknowledgment to various people. For obvious reasons, I cannot acknowledge my indebtedness to all these people, and so I must necessarily limit my thanks to those who helped me directly in the preparation and checking of the material in this book. Therefore, I would like to acknowledge the advice and encouragement that I received from my good friend Dr. Guanrong Chen, formerly Professor of Electrical and Computer Engineering, University of Houston, Houston, Texas, and currently Chair Professor, Department of Electronic Engineering, City University of Hong Kong. In particular, I am thankful to Professor Chen for suggesting this book to Springer-Verlag New York and working hard to see that it received equitable consideration. Also, I would like to thank my good friend Dr. Victor A. Skormin, Professor, Department of Electrical Engineering, Thomas J. Watson School of Engineering and Applied Science, Binghamton University (SUNY), Binghamton, New York, for his encouragement in this effort. To Dr. Pravas R. Mahapatra, Professor, Department of Aerospace Engineering, Indian Institute of Science, Bangalore, India, I express my sincere thanks for his commitment and painstaking effort in reviewing Chapters 2–4. His criticism and suggestions have been of great service to me. Much care has been devoted to the writing and proof-reading of the book, but for any errors that remain I assume responsibility, and I will be grateful to hear of these.

The author would like to express his appreciation to the editorial and production staff of Springer-Verlag New York, for their courteous cooperation in the production of this book and for the high standards of publishing, which they have set and maintained.

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Dayton, Ohio  
November, 2003

George M. Siouris



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