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Hans Kurzweil Bernd Stellmacher

The Theory of Finite Groups

An Introduction

Springer

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Preface

Since its early beginnings in the nineteenth century the theory of finite groups has grown to be an extensive and diverse part of algebra. In the beginning of the 1980s, this development culminated in the classification of the finite simple groups, an impressive and convincing demonstration of the strength of its methods and results.

In our book we want to introduce the reader—as far as an introduction can do this—to some of the developments in this area that contributed to this success or may open new perspectives for the future.

The first eight chapters are intended to give a fast and direct approach to those methods and results that everybody should know who is interested in finite groups. Some parts, like nilpotent groups and solvable groups, are only treated as far as they are necessary to understand and investigate finite groups in general.

The notion of *action*, in all its facets, like action on sets and groups, coprime action, and quadratic action, is at the center of our exposition.

In the last chapters we focus on the correspondence between the local and global structure of finite groups. Our particular goal is to investigate non-solvable groups all of whose 2-local subgroups are solvable. The reader will realize that nearly all of the methods and results of this book are used in this investigation.

At least two things have been excluded from this book: the representation theory of finite groups and—with a few exceptions—the description of the finite simple groups. In both cases we felt unable to treat these two themes in an adequate way within the framework of this book.

For the more important results proved or mentioned in this book we tried to give the original papers as references, and in a few cases also some with alternative proofs. In the Appendix we state the classification theorem of the finite simple groups and also some of the fundamental theorems that are related to the subject of the last chapters.

The first eight chapters are accompanied by exercises. Usually they are not ordered by increasing difficulty and some of them demand serious thinking and persistence. They should allow the reader to get engaged with group theory and to find out about his or her own abilities.

The reader may want to postpone and revisit later some of the apparently more difficult exercises using the greater experience and insight gained from following chapters.

It should be pointed out here that—with the exception of the first chapter—all groups under consideration are meant to be finite.

Our special thanks go to our colleague H. Bender. Without him this book would not have been written, and without his encouraging support it would have taken a different shape.

We would like to thank J. Hall for reading the entire manuscript and A. Chermak for reading parts of it. We are also grateful to B. Baumann, D. Bundy, S. Heiss, and P. Flavell for their helpful comments and suggestions.

A German version of this book has been published in 1998 as a Springer-Lehrbuch.

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Kiel, Germany
February 2003

Hans Kurzweil
Bernd Stellmacher

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