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Andrzej Granas James Dugundji

Fixed Point Theory

With 14 Illustrations

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Andrzej Granas
Département de Mathématiques et Statistique
Université de Montréal
CP 6128
Montréal, QC H3C 3J7
Canada
granasa@dms.umontreal.ca
and
Department of Mathematics and Computer Science
University of Warmia and Mazury
Olsztyn
Poland

James Dugundji
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For

Monique,

Stanisław, Janusz, Jean-Jacques, Andrzej

&

To the Memory of

Jim Dugundji

Preface

The aim of this monograph is to give a unified account of the classical topics in fixed point theory that lie on the border-line of topology and non-linear functional analysis, emphasizing developments related to the Leray–Schauder theory. Using for the most part geometric methods, our study centers around formulating those general principles of the theory that provide the foundation for many of the modern results in diverse areas of mathematics.

The main text is self-contained for readers with a modest knowledge of topology and functional analysis; the necessary background material is collected in an appendix, or developed as needed. Only the last chapter presupposes some familiarity with more advanced parts of algebraic topology.

The "Miscellaneous Results and Examples", given in the form of exercises, form an integral part of the book and describe further applications and extensions of the theory. Most of these additional results can be established by the methods developed in the book, and no proof in the main text relies on any of them; more demanding problems are marked by an asterisk. The "Notes and Comments" at the end of paragraphs contain references to the literature and give some further information about the results in the text.

This monograph evolved from *Fixed Point Theory*, *Vol. I*, published in the Monografie Matematyczne series in 1982. An outline of the entire treatise was conceived by the authors in 1978; in spite of its appearance many years later than expected, the content follows the original plan.

The following is a brief note about the life and work of my close friend Jim Dugundji (1919–1985). Jim Dugundji received his B.A. degree from New York University in 1940 and, for the next two years, studied at the University of North Carolina. After serving in the US Air Force from 1942 to 1946 he enrolled at the Massachussetts Institute of Technology, where he earned his Ph.D. in 1948 under Witold Hurewicz. Since 1948, Jim Dugundji taught at the University of Southern California in Los Angeles, where he became a full professor in 1958. For many years he served as one of the editors of the *Pacific Journal of Mathematics* and of *Topology and its Applications.*

While Dugundji's mathematical work lay mainly in the field of topology, he also contributed to dynamical systems and functional analysis, and to problems in applied mathematics (electrical engineering, geology, and theoretical chemistry). Among his books are *Topology* (Allyn and Bacon, 1965) and *Perspectives in Theoretical Stereochemistry* (Springer, 1984), the latter written with I. Ugi, R. Kopp and D. Marquarding.

Jim Dugundji's mathematical publications are marked by their lucidity and frequently by the decisiveness of his results. His work was, in fact, in many ways an expression of his character. Although he was self-effacing and lacking in any wish for self-advancement, he was totally independent and would not tolerate anything which he considered second best. He spent his life for science's sake, aware of the sacrifice and dedication this requires, and what he asked from himself—which was quite a lot—he expected from others. Man of high integrity and moral strength, he had a great sensitivity, and all who were close to him could testify to his caring concern.

I wish to express my gratitude to our many friends and collaborators who so generously assisted us during the long years of preparation of this book. Most important, without the help and encouragement of Merope Dugundji and my wife Monique, it would not have been possible for us to conceive this project nor for me to bring it to its completion. First and foremost my thanks go to Cezary Bowszyc, who read and commented upon the entire manuscript; his detailed and constructive criticism has led to many improvements and has been of a very great help. Alberto Abbondandolo, Robert Burckel, Haïm Brézis, Ed Fadell, Marlène Frigon, Kazimierz Gęba, Tadeusz Iwaniec, Marc Lassonde, Isaac Namioka, and Gencho Skordev offered valuable suggestions in various stages of the writing, all of which are sincerely appreciated. Special thanks go to Jerzy Trzeciak for his excellent editorial job; to Anna Rudnik for her considerable help with the typesetting; and to the staff of Springer-Verlag for their most efficient handling of publication matters.

I thank also the Killam Foundation and the National Research Council of Canada for providing support for my research projects on various topics, which are now summarized in this book.

Finally, I would like to express my gratitude to Albrecht Dold, Ky Fan and Louis Nirenberg for their encouragement and inspiration over the years.

Montreal and Olsztyn, September 2002 *Andrzej Granas*

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