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(continued after index)

Frank C. Hoppensteadt Charles S. Peskin

Modeling and Simulation in Medicine and the Life Sciences

Second Edition

With 93 Illustrations

Springer

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This book is dedicated to the memory of
ANNELI LAX,
who helped and inspired us.

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics* (*TAM*).

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of the textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences* (*AMS*) series, which will focus on advanced textbooks and research level monographs.

Pasadena, California	J.E. Marsden
Providence, Rhode Island	L. Sirovich
Houston, Texas	M. Golubitsky
College Park, Maryland	S.S. Antman

Preface

Important knowledge in medicine and the life sciences has been created using mathematical models that help predict and interpret what we observe. This book describes several major contributions that have been made to physiology, engineering, and population biology by such theoretical work.

We have tried to keep the presentation brief, and tried to ensure that the topics are presented at a level that is accessible to a wide audience. Each topic could serve as a launching point for more advanced study, and suitable references are suggested to help with this. If the underlying mathematics is understood for these basic examples, then mathematical aspects of more advanced life science problems will be within reach.

The material in each chapter is presented in general order of increasing mathematical difficulty, which ranges up to calculus, matrix theory, and elementary differential equations. Some exercises deal with material in preceding sections, while others are projects that extend preceding material. Some topics in matrices, MATLAB® and probability are described in appendices.

Our purpose in this book is not the systematic presentation of mathematical material, although there are important threads that run through several chapters. Instead, we hope to illustrate how mathematics can be used. In particular, our goal is to make available to students having at least one term of calculus topics in the life sciences and medicine that have benefited from mathematical modeling and analysis. In addition to exposing students to current ideas, the material is intended to reinforce their mathematics education by presenting familiar mathematical topics from novel points of view. Finally, enabling students to think in terms of models early in their academic experience should motivate them to develop and apply modeling skills further.

While hoping this cross-disciplinary book will be useful to a wide variety of individuals, we believe that it can have special significance for the premedical student, who will find a mathematical introduction to a host of phenomena that are central to the practice of medicine. These include genetics and epidemics as well as the functions of the heart, lungs, and kidneys. It is our hope that the mathematical study of these topics will give the student a depth of understanding and insight that could not have been achieved through traditional, descriptive education in the medical sciences.

The mix of topics, taken largely from physiology and from population biology, includes important phenomena that are within reach of the students described above. The population part of the book draws its material from the areas of demographics, genetics, epidemics, and molecular biology, while the physiological part surveys cardiovascular, pulmonary, renal, and muscle physiology. One chapter (Chapter 6) is intended to introduce students to models of nerve cells and some neural circuits as a basis for studying how the brain works from the point of view of a neural engineer. We are on the rise of a wave of understanding brain function, and mathematical modeling can be useful in understanding this complex organ.

Anneli Lax maintained an interest in the course that led to this book and provided helpful discussions during the preparation of the lecture notes on which this book is based. We are grateful to her. Besides the authors, the course has been taught by W. Stephen Childress, H. Michael Lacker, Daniel Tranchina, and Tatyana Izhikevich, and we are indebted to them for their comments and advice.

Arizona State University
Courant Institute
New York University
May 2001

Frank C. Hoppensteadt
Charles S. Peskin

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