Surveys and Tutorials in the Applied Mathematical Sciences

Volume 4

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Editors
S.S. Antman, J.E. Marsden, L. Sirovich

Mathematics is becoming increasingly interdisciplinary and developing stronger interactions with fields such as biology, the physical sciences, and engineering. The rapid pace and development of the research frontiers has raised the need for new kinds of publications: short, up-to-date, readable tutorials and surveys on topics covering the breadth of the applied mathematical sciences. The volumes in this series are written in a style accessible to researchers, professionals, and graduate students in the sciences and engineering. They can serve as introductions to recent and emerging subject areas and as advanced teaching aids at universities. In particular, this series provides an outlet for material less formally presented and more anticipatory of needs than finished texts or monographs, yet of immediate interest because of the novelty of their treatments of applications, or of the mathematics being developed in the context of exciting applications. The series will often serve as an intermediate stage of publication of materials which, through exposure here, will be further developed and refined to appear later in one of Springer's more formal series in applied mathematics.

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Multiscale Finite Element Methods

Theory and Applications

Springer

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Dedicated to my parents, Rafik and Ziba,
my wife, Denise, and my son, William
Yalchin Efendiev

Dedicated to my parents, Sum-Hing and Sau-Ying,
my wife, Yu-Chung, and my children, George and Anthony
Thomas Y. Hou

Preface

The aim of this monograph is to describe the main concepts and recent advances in multiscale finite element methods. This monograph is intended for the broader audience including engineers, applied scientists, and for those who are interested in multiscale simulations. The book is intended for graduate students in applied mathematics and those interested in multiscale computations. It combines a practical introduction, numerical results, and analysis of multiscale finite element methods. Due to the page limitation, the material has been condensed.

Each chapter of the book starts with an introduction and description of the proposed methods and motivating examples. Some new techniques are introduced using formal arguments that are justified later in the last chapter. Numerical examples demonstrating the significance of the proposed methods are presented in each chapter following the description of the methods. In the last chapter, we analyze a few representative cases with the objective of demonstrating the main error sources and the convergence of the proposed methods.

A brief outline of the book is as follows. The first chapter gives a general introduction to multiscale methods and an outline of each chapter. The second chapter discusses the main idea of the multiscale finite element method and its extensions. This chapter also gives an overview of multiscale finite element methods and other related methods. The third chapter discusses the extension of multiscale finite element methods to nonlinear problems. The fourth chapter focuses on multiscale methods that use limited global information. This is motivated by porous media applications where some type of nonlocal information is needed in upscaling as well as multiscale simulations. The fifth chapter of the book is devoted to applications of these methods. Finally, in the last chapter, we present analyses of some representative multiscale methods from Chapters 2, 3, and 4.

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