THE THEORY OF SEARCH GAMES AND RENDEZVOUS

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by

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Preface

Search Theory is one of the original disciplines within the field of Operations Research. It deals with the problem faced by a Searcher who wishes to minimize the time required to find a hidden object, or "target." The Searcher chooses a path in the "search space" and finds the target when he is sufficiently close to it. Traditionally, the target is assumed to have no motives of its own regarding when it is found; it is simply stationary and hidden according to a known distribution (e.g., oil), or its motion is determined stochastically by known rules (e.g., a fox in a forest).

The problems dealt with in this book assume, on the contrary, that the "target" is an independent player of equal status to the Searcher, who cares about when he is found. We consider two possible motives of the target, and divide the book accordingly. Book I considers the zero-sum game that results when the target (here called the Hider) does not want to be found. Such problems have been called Search Games (with the "zero-sum" qualifier understood). Book II considers the opposite motive of the target, namely, that he wants to be found. In this case the Searcher and the Hider can be thought of as a team of agents (simply called Player I and Player II) with identical aims, and the coordination problem they jointly face is called the Rendezvous Search Problem. This division of the book according to Player II's motives can be summarized by saying that in a Search Game the second player (Hider) wishes to maximize the capture time T, while in a Rendezvous Problem the second player (Rendezvouser) wishes to minimize T. (In both cases, the first player wishes to minimize T.)

Of the two problems dealt with in the book, the area of Search Games (Book I) is the older. These games stem in part from the "The Princess and the Monster" games proposed by Rufus Isaacs (1965) in his well known book on Differential Games. Beginning with the first search game with mobile hider to be solved (that on the circle, by Alpern (1974), Foreman (1974), and Zelinkin (1972)), and the subsequent solutions of search games on networks and regions in space by Gal (1979), the early work on such games culminated in the classic book of Gal (1980). This work has stimulated much subsequent research in the field including applications in computer science, economics, and biology. Much of this research is covered in Book I which contains many new results on Search Games as well as the classical results, presented with simpler exposition and proofs. However there are many open questions, even some of a fairly elementary nature, which are also covered here. For an extensive introduction to the area of Search Games, see Chapter 1.

The Rendezvous Search Problem (Book II) is a more recent area of interest. It asks how quickly two (or maybe more) players can meet together at a single location, when placed in a known search region, without a common labelling of locations. Although posed informally by Alpern as early as 1976, a rigorous formulation for the continuous time version did not appear until Alpern (1995). Beginning with the early subsequent papers of Alpern and Gal (1995) and Anderson and Essegaier (1995) on rendezvous on the line, the interest in this problem has expanded to encompass many variations, including multiple player rendezvous and different forms studied by V. Baston, A. Beck, S. Fekete, S. Gal, J. V. Howard, W. S. Lim, L. Thomas, and others. Particular interest has been paid to some discrete time rendezvous models, which have a separate history going back to the original papers of Crawford and Haller (1990) on coordination games in the economics literature, and Anderson and Weber (1990) in a search theory context. Much of this work is surveyed in the paper of Alpern (2002a). An extensive introduction to the field of Rendezvous Search can be found in Chapter 10.

Although both authors have worked in the two fields of Search Games and Rendezvous Search Theory, the division of this book into two parts reflects the emphasis of their work. As such, Book I (Search Games) was mainly written by Shmuel Gal, and results in this part which are not otherwise ascribed are due to him. Similarly, Book II (Rendezvous Search) was mainly written by Steve Alpern, with unascribed results there due to him. Of course both authors take joint responsibility for this book as a whole.

We would like to put the work of this book into its historical context with respect to earlier survey articles and books on search. Search Theory is usually considered to have begun with the work of Koopman and his colleagues on "Search and Screening" (1946). (An updated edition of his book appeared in 1980.) The problem of finding the optimal distribution of effort spent in search is the main subject of the classic work of Stone (1989, 2nd ed.), "Theory of Optimal Search", which was awarded the 1975 Lanchester Prize by the Operations Research Society of America. Much of the early work on search theory surveyed by Dobbie (1968) was concerned with aspects other than optimal search trajectories, and as such is very different from our approach. The later survey of Benkoski, Monticino, and Weisinger (1991) shows how the determination of such trajectories has come to be studied more extensively. Recent books on Search Theory include those of Ahlswede and Wegener (1987), Haley and Stone (1980), Iida (1992), and Chudnovsky and Chudnovsky (1989). The first book to introduce game theoretic aspects of search problems was of course Gal (1980), but these are also considered in Ruckle (1983a) and form the basis of the recent stimulating book of Garnaev (2000). This volume is the first to cover the new field of rendezvous search theory.

Frequently Used Notations

Search Games

μ	Lebesgue measure of the search space
$\bar{\mu}$	Minimal length of a tour that covers the search space
ρ	Rate of discovery of the searcher
$C(S, H)$	Cost function (the payoff to the hider)
$c(s, h)$	Expected cost
$\hat{C}(S, H) = C(S, H)/\|H\|$	Normalized cost function
$\hat{c}(s, h)$	Expected normalized cost
$d(Z_1, Z_2)$	Distance between Z_1 and Z_2
D	Diameter of the search space
E	Expectation
H	A pure hiding strategy
$\mathcal{H}$	The set of all pure hiding strategies
h	A mixed hiding strategy
h_μ	The uniform hiding strategy
O	Origin (usually, the starting point of the searcher)
Pr	(A) Probability of an event A
Q	Search space
$r, r(Z)$	Discovery (or detection) radius
S	A pure search strategy (a search trajectory)
$\mathcal{S}$	The set of all admissible search trajectories
s	A mixed search strategy
T	Capture time
t	Time parameter
$v(H), v(h)$	Value of the hiding strategy ($v(h) = \inf_s c(s, h)$)
$v(S), v(s)$	Value of the search strategy ($v(s) = \sup_h c(s, h)$)
$\bar{V}$	Minimal value obtained by a pure search strategy (the "pure value" $\bar{V} = \inf_S v(S)$)
v	Value of the search game ($v = \inf_s v(s) = \sup_h v(h)$)
w	Maximal velocity of the hider
Z	A point in the search space
$\lfloor . \rfloor$	Integer part

Rendezvous

$\hat{T}$	Expected rendezvous time
R^a, R^s	Player-asymmetric, player-symmetric, rendezvous values
G	A given group of symmetries of Q
$t_{f,g}$	The maximum rendezvous time for f, g
$H(Q, p, q)$	The H-network based on Q
$\mathcal{N}_e, \mathcal{N}_o$	The sets of even and odd nodes of an H-network
$(var\ s)(r)$	Total variation of function s up to time r
$Var(s)$	Total variation of s over its domain

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