LEARNING DISCOURSE

Discursive approaches to research in mathematics education*

Edited by

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While looking at the papers collected in this volume one feels that, in spite of their diverse themes, these seven studies have quite a lot in common and, as a collection, seem to be signaling the existence of a distinct, relatively new type of research in mathematics education. A comparison with, say, a fifteen-year-old issue of *Educational Studies in Mathematics* or of *Journal for Research in Mathematics Education* would reveal a long series of differences. To begin with, the present articles simply look different from their older counterparts: They are longer and have a highly variable format, often not even remotely reminiscent of the classical background-method-sample-findings-discussion structure that reigns in the former research reports. Long segments of conversation transcripts take the place of the once ubiquitous graphs and tables. As we start reading, we discover substantial differences in vocabulary. The language of mental schemes, misconceptions, and cognitive conflicts seems to be giving way to a discourse on activities, patterns of interaction, and communication failures. While the older texts speak of learning in terms of personal acquisition, the newer ones portray it as the process of becoming a participant in a collective doing. And last but not least, the classroom scenes that we see as we go on reading have very little in common with what we got used to in the older papers. To be sure, finding a detailed description of a learning activity in a research paper was a rare occurrence until recently, and in the majority of cases we had to rely on our own experience while trying to imagine the life of the class in which the authors conducted their study. In spite of this limitation, much can be said also about the differences in the ways of learning investigated in the two types of research: The traditional mathematical classroom featuring one blackboard, one outspoken teacher and twenty to forty silent students seems to belong to history. It has been replaced by small teams of learners talking to each other, by groups of students voicing their opinions in whole class discussions, and by children and grownups grappling with mathematical problems in real-life situations.
All these innovative features, when taken together, seem to make a real difference and to define a distinctive research framework that, because of its obvious emphasis on the issues of language and communication, can be called discursive or communicational. To be sure, this special framework, although quite widespread and increasingly popular these days, is still under construction. And yet, considering the progress that has already been made, the time seems ripe for an intermediary summary and reflection. The aim of this special issue is to put discursive research in the limelight and to spur some thinking about the reasons for its appearance, about its nature, and about its possible advantages and pitfalls. Let us now address these issues briefly, one by one.

The first question to ask concerns the reasons for the advent of the discursive approach. To give a proper answer, one has to take a broader look at the history, and not just of mathematics education, but of research on human thinking in general. This latter research is, and always has been, torn between two complementary, but not necessarily compatible, goals. On the one hand, the intention of the researcher is to fathom the phenomenon of human thinking in all its uniqueness and with all its ramifications. On the other hand, the method employed must be rigorous enough to put this research on a par with any other scientific endeavor with respect to cogency, trustworthiness, and, above all, usefulness. These two goals create an essential tension that fuels the incessant change. While the request of scientficity (whatever this term means at a given moment) pushes toward simplicity and feeds the belief in cross-contextual invariants, the wish for an all-encompassing, true-to-life picture of human cognitive activities implies that the formidable complexity of the phenomenon should never disappear from the researcher’s sight. No wonder then, that the relatively brief history of cognitive studies is stormy and replete with dramatic turnabouts.

On the face of it, the main question that needs to be answered before the dilemma of the conflicting goals can be solved is that of the proper method of inquiry. And yet, not in many fields of research is the way of conceptualizing the object of investigation more sensitive to methodological issues than in the study of the human mind. Judging from history, the uncompromising insistence on methodological rigor, especially if gauged according to criteria borrowed from the ‘exact’ sciences, forces researchers to bend, and eventually forget, the original focus of their endeavor. This is what happened when behaviorists decided to purge psychological discourse of any reference to mental non-observables, and this is what happened again not long after the advent of computer science, when technology brought back the hope of a truly scientific insight into the workings
of the human mind. Jerome Bruner, one of the founding fathers of the ‘cognitive revolution’ of the late 1950s admits that his and his colleagues’ ‘all-out effort to establish meaning as the central concept of psychology…’ (Bruner, 1990, p. 2), grounded in the computer metaphor of mind, did not achieve its goal and, in consequence, failed to deliver on its promise of groundbreaking insights into the specificity of the human intellect. As the recent proliferation of critical publications makes clear, also Piaget’s impressive attempt to meet the challenge of the conflicting goals by modeling the development of human thinking on Darwinian theory of evolution proved unsatisfactory in many respects (see e.g. Bruner, 1985).

The insufficiency of all these approaches expressed itself, among others, in their inability to bring about a lasting betterment of the human condition, which is the ultimate goal of any scientific endeavor. Thus, for example, none of the theories produced by the different frameworks could account in a satisfactory way for such phenomena as the persistent failure of many students in mathematics or the stubborn irreproducibility of educational success.

The first notions about possible reasons for this pervasive difficulty came following a wave of cognition-oriented cross-cultural studies that began in the early 1920s. At that time, psychologists and educators from diverse scientific traditions began arriving in cultures far removed from their own, convinced that “[i]n the realm of culture, outsideness is a most powerful factor in understanding” (Bakhtin, 1986, p.7) and keen to observe what came to be known as higher psychological processes cast against the background of foreign traditions (for an historical survey see Cole, 1996). Mathematical thinking, considered as a paradigmatic example of such a process, and as one that is particularly liable to rigorous investigation, became the preferred object of study. The guiding assumption of the early studies was that this uniquely human form of cognitive activity may be found in pre-industrial cultures in their nascent, underdeveloped form. By watching the incipient editions of these processes, psychologists hoped to learn about the cultural invariants of human cognition. And yet, as it soon became clear, venturing into unfamiliar cultural settings to look for phenomena defined according to one’s own cultural heritage is an inherently problematic, ultimately misguided, endeavor. Initially, doubts were raised about the methods of study. The traditional forms of experimental design became questioned when the experimenters realized that school mathematical problems, imported directly from the researchers’ own culture, would only too often turn out to be completely foreign to the respondent. This clearly created the possibility of major misinterpretations, with the invest-
igators conferring on their findings meanings dictated by their own cultural background (Cole, 1996).

Increasingly suspicious about the experimental method, some of the researchers began supplementing their investigations with descriptive studies in which the focus shifted from laboratory problem solving to spontaneous everyday activities. A long series of research projects devoted to what came to be known as everyday, street, workplace or supermarket mathematics followed. The main merit of all these studies was that they obviated the need for the researcher’s regulatory intervention, at least in the initial phase of the investigation that was usually carried out as an ethnographical observation. An experimental study would then often be devised so as to make it possible for the subjects to communicate with the researcher on their own terms. The change of approach proved itself when the non-interventional studies began producing results dramatically different from those one would expect on the grounds of the subjects’ former performance on school tasks purported to involve ‘the same’ cognitive functions. It soon became clear that the superior everyday mathematical performance of people who tended to fail on school tasks is not an accidental, isolated phenomenon. What was found among Kpelle rice sellers in the mid-1960s (Cole, 1996) was observed over and over again among Vai tailors (Reed and Lave, 1979), Brazilian street vendors (Saxe, 1991; Nunes et al., 1993), dairy warehouse workers (Scribner, 1983/1997), and American weight-watchers and shoppers (Lave, 1988).

At this point the methodological doubt turned epistemological. Psychologists started questioning what until now had been taken for granted even without being explicitly spelled out. A common denominator of all the traditional approaches to thinking was the vision of mind as a ‘mirror of nature’ (Rorty, 1979) – a container to be filled with reflections of, or structures residing in, the external world. Whether simply received or individually constructed, it was believed that these structures – known as knowledge, concepts, or mental schemes – were regulated by universal external factors, and should thus be more or less the same for all human beings. Once acquired, each such structure should lead to similar behaviors in all the situations in which this structure could be identified. Similarly, the cognitive processes that produced and used these entities were expected to be cross-contextually invariant, that is, governed by universal rules that remain basically the same across different social, cultural, historical and situational settings. Those who were taking a closer look at cognition across cultural and situational boundaries could not help wondering about the soundness of this assumption, or at least about its testability. Sooner or later, this essential doubt would force them to question the concep-
tual foundations of the traditional framework. This is how the acquisition metaphor, upon which the time-honored cognitivist approach was resting, became the primary suspect.

To this very day, the acquisitionist framework, its impressive history notwithstanding, is a target of criticism coming from the somewhat eclectic group of thinkers who are often called sociocultural. In fact, many different names have been given to this rich and diverse cluster of approaches. What sets these approaches apart as a distinct group is the fact that most of them are associated with the Vygotskian school of thought, and that they all promote the vision of human thinking as essentially social in its origins and as inextricably dependent on historical, cultural, and situational factors. It is important to stress that our historical account by no means exhausts the list of approaches that can be called sociocultural, nor does it cover all the events that led to the advent of this variegated trend. In our selective and, of necessity, very brief survey we have focused on those developments that had a direct bearing on cognitive research in general, and on research in mathematical education, in particular.

The discursive approach announced in the title of this special issue can be viewed as one of many possible implementations of the sociocultural call for research that acknowledges the inherently social nature of human thought. Not all the contributors to this volume are using the name ‘discursive’ and some of them may eschew any explicit descriptions of the epistemological and ontological underpinnings of their research. Nevertheless, a number of theoretical assumptions can be identified that seem to be guiding all the authors. These overarching foundational motifs are what defines the discursive framework. The reader will come across the common theoretical threads while reading the papers. The articles by van Oers, by Lerman, and by Sfard, which all deal with the conceptual infrastructure of the discursive research explicitly, will help in revealing these common threads. At this point, suffice it to say that within the discursive framework, thinking is conceptualized as a special case of the activity of communication and learning mathematics means becoming fluent in a discourse that would be recognized as mathematical by expert interlocutors. As will be explained by the contributors themselves, these deceptively simple definitions turn out to have quite far reaching theoretical and practical entailments. In the remainder of this introduction, let us limit ourselves to the question of how the discursive approach helps in resolving the dilemmas that have been challenging our research and fueling its incessant change ever since its earliest beginnings.

Let us start with the question of whether the discursive approach stands a good chance of capturing what is unique in human thinking. The first
thing to note is that while the more traditional frameworks conceptualize learning as intellectual acquisition, and thus as a change in the individual learner, the discursive approach focuses on the change in one’s ways of communicating with others. This complicates the picture and makes it much richer. While the place of the individual is not denied, it is conceptualized in a whole new way. No longer is the individual learner viewed as the only object of change; furthermore, the change itself is no longer regarded as stand-alone and independent of that which affects the community of learners as a whole. Indeed, the vision of learning as becoming a participant in a practice must lead to the conclusion that in this process, the practice itself is bound to undergo modifications. Thus, the inclusion of the community in the picture of learning affects the scope of things that must be considered when the change in the activities of an individual learner is studied.

When regarded not as an isolated entity but as a part of a larger whole, the learner becomes but an inextricable element of a new, much broader unit of analysis, many ingredients of which must be brought into the account even if the ultimate focus of study is change in the individual learner’s activities. More specifically, when learning mathematics is conceptualized as developing a discourse, probably the most natural units of analysis can be found in the discourse itself (as opposed to such formerly favored units as concepts, mental schemes, or student’s knowledge).

Indeed, the focus of the studies reported in this volume is on the discourse generated by students grappling with mathematical problems. Thus, it is interesting to see how the classroom conversation develops on both the collective and individual level when the group of children in O’Connor’s study responds to the teacher’s challenging question “Can any fraction be turned into a decimal?” O’Connor examines the fit between the mathematical content (rational numbers and their representational forms) and a whole class position-driven discussion in an upper elementary school classroom. Position-driven discussions occur when a teacher orchestrates an argument among a group of students of one conceptually challenging central question with a limited number of options. Like O’Connor, Forman and Ansell investigate how a teacher orchestrates the discourse in her elementary school classroom. Unlike O’Connor, however, they argue that voices from the past, present, and future, and from outside as well as inside the classroom walls, animate the discussion of students’ strategies for solving multi-digit word problems. These voices come from the students’ families and the teacher’s educational experiences; they represent the memories, attitudes, emotions, and expectations about traditional and reform educational practices in mathematics.
While the conversations in both of these articles involve the whole class and are orchestrated by the teacher, the study in the Zack and Graves article looks at the discourse of groups of students in problem-solving situations. The particular focus is the way in which the differences among the positions of the participants function and how they enable the learners to jointly construct new knowledge. Similarly, the studies in Kieran’s and Sfard’s articles offer a glimpse into dyadic peer interactions. Kieran explores the emergence of collective mathematical thinking and the ways in which the mathematical discourse of some individuals changes as a result of the group experience. Sfard, on the other hand, tries to fathom the nature and the reasons for the evident ineffectiveness of an interaction between two students who try to solve a mathematical problem.

It must be immediately stressed that discourse is not the only possible source of units of analysis for sociocultural research, nor even the only one considered by the contributors to this special issue. Among the most widely known alternatives are *activity*, the unit proposed by Vygotskian scholars who call themselves activity theorists (Leont’ev, 1978; Engestrom, 1987), *culture*, as preferred by at least some of cultural psychologists who view learning as *enculturation* (Tomasello, 1999); and *practice*, introduced by those among sociocultural thinkers who are most strongly oriented toward sociological issues (Lave, 1988; Lave and Wenger, 1991; Wenger, 1998).

A review of these other possibilities and the explanation of their relative advantages can be found in the article by van Oers, who organizes his exposition around the fundamental question “What is really mathematical?” He provides an historical overview of research on mathematics learning in classroom settings before articulating the discursive approach. Building upon the theories of Vygotsky and Bakhtin, van Oers outlines an emerging framework for future research in which notions of activity, practice, and discourse play prominent roles. Lerman also surveys a variety of theories that have influenced mathematics education and provides his own version of cultural, discursive psychology. In this survey, he discusses discursive psychology, cultural psychology, and sociocultural research, in order to work towards a synthesis. In contrast, Sfard makes a clear choice and argues for the advantages of the framework that takes discourse and communication as its pivotal concepts.

Whether one speaks about learning in terms of discourse, activity, culture, or practice, the focus is on the change generated by interpersonal interactions, and this, as has already been mentioned, results in a picture which is more complex and closer to life than in the traditional cognitivist studies. The question that now begs to be asked is whether all this rich-
ness does not come at the expense of the scientific elegance, cogency, and trustworthiness of the research. While looking at recent publications that can count as sociocultural, one may say that, indeed, we are still paying the methodological cost of the decision to put a premium on the goal of capturing the intricacies of learning in all their specificity and uniqueness. We must realize that when it comes to tools and techniques that would match this endeavor, we have yet a long way to go. Unlike in the former, positivist era, we now have to craft our ways of analyzing data each time anew, appropriately to the questions we are asking, and in accord with the data we were able to collect.

This does not mean, however, that it is not possible to build a basic reservoir of sound methodological tools. With its well-defined, directly accessible object of study, the discursive approach seems to be on its way to becoming a fully-fledged research framework, complete with a set of reliable methods of data analysis. In the last years, many impressive methodological advances have been made within this area. In addition to the general-purpose techniques, such as those gathered under the names conversation analysis and discourse analysis, numerous new tools specially crafted to fit the particular needs of the research in mathematics education are appearing these days with an increasing frequency.

An assortment of such methods may be found in this special issue. O’Connor, who comes to mathematics education from the field of applied linguistics, has always made extensive use of the methods of discourse analysis in her studies on interaction patterns in mathematical classrooms. In the paper included in this volume, she builds her own techniques of looking while trying to capture the ways in which mathematical content evolves as a result of interaction. In her detailed account, O’Connor shows us how the whole-class discussion unfolds, helping us understand the conceptual, pedagogical, and interpersonal dilemmas that emerge during discussions of challenging mathematical content. She uses units that parse the argument into claims and counterclaims with supporting evidence. She also identifies units that illustrate the teacher’s skill at managing conceptual and communicative confusion. Forman and Ansell employ a hierarchy of units in their analysis from the molar (such as a lesson) to the molecular (sequences of talk about a particular topic). Furthermore, they examine critical junctures or changes in the structure of events, which may allow one to make inferences about participants’ interpretations of those events. Zack and Graves structure the extract that they present into four parts, which emerge as a function of participants’ differing roles and stances during the mathematical interaction. This structuring device affords an analysis of the process whereby individual and group developmental trajectories are constructed,
as well as an exploration of the relationship between discourse and knowing. Kieran uses an interactivity flowchart, adapted from an earlier study for which it was created (cf., Sfard and Kieran, 2001), to segment the discourse of participants into personal and interpersonal channels of talk. With a focus directed toward those object-level utterances that move the mathematical dimensions of the discourse forward, Kieran hypothesizes why there might be discrepancies between partners in their subsequent individual work. In her attempt to understand the nature of and reasons for the observed communication failure, Sfard applies the interactivity flowchart along with another type of analysis developed in her former study with Kieran: She follows the course of the mathematical conversation with the help of focal analysis – a method that aims at ‘mapping the trajectory’ of the object of conversation.

In this special issue, the complexity of the phenomena under study is reflected in the multi-level analyses of the discourse. All the authors are discussing the development of mathematical communication, and while doing so, they are alternating between the analysis of students’ single turns and the examination of patterns to be found in sequences of thematically connected utterances. This may be compared to the study of the mechanics of water where, at some points, the researchers may be watching regularities in the movement of individual particles, and at other times may choose to investigate the geometry and periodic recurrence of waves and whirls. The macro- and micro-level pictures obtained in these ways do not resemble each other, and yet, both are needed by those who try to understand the complex phenomenon under study. In the same vein, whatever the particular focus or level of analysis in the studies presented in this volume, the phenomenon under study remains the same: All the authors are looking at classroom communication that evolves so as to become genuinely mathematical and to allow for solving problems that were intractable within other discourses.

A message similar to the one conveyed by the above comparison can be found in the ‘zoom of lens’ metaphor invoked in this volume by Lerman to explain the relation between the individual and social research perspectives. The much debated split between these two perspectives is referred to in the title of this special issue, ‘Bridging the Individual and the Social’. This split has been worrying researchers for some time now. The seemingly incompatible perspectives are producing two incomplete types of studies, each of which is ‘telling only half of the good story’ (Cobb, 1996). The call for bridging the two halves follows. We turned this call into the title for the special issue, but not necessarily because we believe that bridging is what needs to be done. Rather, we used the slogan because it points to
the dilemma that seems to be still at the center of researchers’ attention. Our solution to this dilemma is to deconstruct the dichotomy, and not to unify the halves. Indeed, as the water-study metaphor makes clear, by defining thinking as communicating we are sidestepping the split rather than bridging a gap. The problematic dichotomy between the individual and social research perspectives is no longer an issue when one realizes that the cognitivist (‘individualistic’) and interactionist (‘social’) approaches are but two ways of looking at what is basically one and the same phenomenon: the phenomenon of communication, one that originates between people and does not exist without the collective even if it may temporarily involve only one interlocutor. The social nature of the individual is the principal message of this special issue.

NOTES

1. Even if this is not true for many mathematics classrooms around us, it certainly is true for those in which researchers nowadays choose to conduct their studies.

2. The reasons for the particular appropriateness of mathematics for uncovering factors contributing to human cognitive development are eloquently spelled out by Reed and Lave (1979) in the paper with the telling title *Arithmetic as a tool for investigating relations between culture and cognition*.

3. Several different terms have been used to characterize the school of thought that began with Vygotsky: sociocultural, cultural-historical activity theory, cultural psychology, neo-Vygotskian. Vygotsky himself used the term cultural-historical (Cole, 1995; van Oers, 1998).

4. Among the thinkers whose work had a decisive influence on this development one should mention, above all, the Austrian-British philosopher Ludwig Wittgenstein whose seminal work on language brought the issue of communication to the center of psychological research; and of the American and German social philosophers George Herbert Mead and Alfred Schutz, who stressed, each one of them in his own way, the tight relations between human thought and social interactions. (See Valsiner and van de Veer, 2000, and Cole, 1996, for historical overviews of the relevant theories.)


REFERENCES


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We, the guest editors, are pleased to have had the opportunity to produce this Special Issue, one of the series emanating from the International Group for the Psychology of Mathematics Education (PME). However, we hope that this Special Issue will be of interest not only to PME researchers, but also to the broader scientific community concerned with issues related to mathematical discourse and communication.

The Special Issue would not have been possible without the collaboration and cooperation of several individuals. We especially wish to thank the authors of the seven main papers who share their research with readers of this volume. Their theoretical discussions and analyses touch upon crucial aspects of discursive interactions in the mathematics classroom. We are also grateful to Celia Hoyles and Falk Seeger for their contributions in the form of commentary papers. Among the issues raised for consideration, Celia emphasizes in particular the importance of tool mediation and the design of mathematical activities, while Falk argues for more long-term studies on the formation of proficient discursive classrooms.

We want to acknowledge, as well, the work done by the reviewers of the research papers. Their timely, thorough, and insightful comments were greatly appreciated by the authors. We express our gratitude to the Kluwer editorial staff for their patient and expert handling of the various stages of the development of this Special Issue. Last, but not least, we owe special thanks to Heinz Steinbring who, as shadow editor, shepherded this volume to its completion and helped us to deal with several matters along the way. A warm thank-you to all!