

**THE THEORY OF IMPLEMENTATION OF SOCIALLY  
OPTIMAL DECISIONS IN ECONOMICS**

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# The Theory of Implementation of Socially Optimal Decisions in Economics

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To my wife, Maria del Mar

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# Introduction

A major purpose of social choice theory is to study the tradeoffs between different desiderata. This book focuses its attention on a well-defined subset of social choice theory, namely the implementation of socially optimal decisions in economics.

Implementation problems arise when the social planner (sometimes a real person, sometimes a surrogate of society) cannot distinguish between things that are indeed different. This may be due to the fact that certain characteristics are unobservable to the planner or that even if the planner has this information she cannot use it because of legal requirements. For example, public monopolies are sometimes bound by laws not to discriminate among consumers.

The quintessential implementation problem is that of the 'free rider', that is, the agent who hopes to improve her luck by not telling the truth about her own unobservable characteristic. (Imagine that you are asked how much you would pay for the construction of a park near your home and that in the case where the park is actually built this amount is the one you would pay. Would you give a truthful answer?) A closely related question was spotted by the ancient Romans and summarized by the question 'who controls the controllers?' In the words of Roger Myerson: 'An organization must give its members the correct incentives to share information and act appropriately. An individual cannot be relied upon to testify against himself or to exert efforts for which he will not be rewarded' (in L. Hurwicz, D. Schmeidler and H. Sonnenschein (eds) 1985, *Social Goals and Social Organization*, (Cambridge University Press), chapter 8). But if these incentives must be the right ones they bind the choice of the planner as much as the scarcity of resources does: 'The basic insight of mechanism theory is that *incentive constraints* should be considered coequally with *resource constraints* in the formulation of the economic problem. In situations where individuals' private information and actions are difficult to monitor, the need to give people an incentive to share information and exert efforts may impose constraints on the economic system just as much as the limited availability of raw materials. The theory of mechanism design is the fundamental mathematical methodology for analysing these constraints' (Roger Myerson (1989), entry on 'Mechanism Design' in the J. Eatwell, M. Milgate and P. Newman (eds), *New Palgrave* (London:

Macmillan). Chapter 1 of this book presents the 'classical' theory of resource allocation in which incentive problems are entirely disregarded. Chapter 2 presents the main ingredients of the approach that will be followed in the rest of the book where incentives of agents are carefully modeled.

Some writers have argued that the kind of opportunistic behavior implied by the 'free rider' problem accounts only for a part of the story (see, for instance, D. North, *Structure and Change in Economic History*, New York, Norton, 1981, chapter 5). Indeed one of the most fundamental contributions of the Theory of Implementation has been to show that the 'free rider' problem may or may not occur, depending on the kind of game that agents play and on the (game-theoretical) solution concept. In fact the story of implementation theory is that of a liberation from constraints. The first major development was in the work of Gibbard–Hurwicz–Satterthwaite in the early 1970s: when information is private, and thus the appropriate equilibrium concept is dominant strategies, incentives bite a lot. These incentives adopt the form of incentive compatibility constraints where for each agent to tell the truth about her characteristic, must be a dominant strategy. This and related topics are discussed in Chapter 3. The second major development came from Maskin in the late 1970s: when the information about the characteristics of the agents is shared by them (but not by the planner), and thus the relevant equilibrium concept is Nash equilibrium, incentive compatibility does not matter. Thus, for instance, by putting agents in a circle, if each agent is able to monitor the characteristics of her neighbors the free rider problem dissolves. What it bites here is a, generally much weaker, monotonicity condition that can be explained as follows. Suppose that an allocation, say  $a$ , is optimal for some preferences. Now preferences change in such a way that  $a$  goes up in all individual rankings about allocations. Then  $a$  must also be socially optimal for the new preferences. Chapter 4 is devoted to explaining the theory of Nash implementation. The third big push came in the mid-1980s from Moore and Repullo.<sup>1</sup> They and their followers exploited the knowledge gathered on Nash equilibrium refinements in the late 1970s and early 1980s that followed the lead of Reinhard Selten. By discarding some Nash equilibria (because they are not subgame perfect or they are weakly dominated, etc), they were able to show that neither incentive compatibility, nor monotonicity bite (see Chapter 6 below). Thus, in this approach, incentives do not have any cutting power.

We should mention here two more highlights of implementation theory.

One is the theory of implementation in Bayesian equilibrium (a concept developed by John Harsanyi) where agents act under incomplete information. Constraints implied by Bayesian implementation are both a form of incentive compatibility and a certain monotonicity restriction. Another important topic is the construction of 'nice' mechanisms (including those inspired in the market) implementing specific social goals. These issues are discussed in chapters 7 and 5 respectively of this book. My guess about the direction of future research is that it will move towards the consideration of dynamic models and a less mechanical formalization of the rules of the game (on this matter the paper by Hurwicz in the first issue of *Economic Design* is warmly recommended).

This book concentrates on the study of two questions. First of all, we will study which kind of social decisions can be implemented by non-cooperative games by means of various types of equilibrium (dominant strategies, Nash or some of its refinements, Bayesian equilibria, etc.). Secondly, we will study some concrete mechanisms which implement 'good' social decision rules. The emphasis of the applications will lie on the implementation of the Lindahl correspondence in economies with public goods. Pure exchange economies are also considered. Each chapter includes a collection of problems. These problems are meant to supplement the exposition of the main text and to test the understanding of the reader about certain questions. They also provide additional references of important work.

This book is based on my lecture notes for a course in the PhD program of the University of Alicante. It is a pleasure to acknowledge the challenging intellectual atmosphere of my department led by people like Carmen Herrero, Fernando Vega-Redondo, Ignacio Ortuño-Ortín, Paco Marhuenda, Subir Chattopadhyay and Antonio Villar. Thanks to a Fulbright scholarship I could visit the Department of Economics of the University of Rochester. There I learnt Implementation Theory from William Thomson. My debt to William is immense not only because of his guidance and care but also because my contact with his students proved to be very fruitful for my future research. Among them, Simon Wilkie, Baskhar Chakravorty and Tömas Sjöström became co-authors. Parts of our joint research have been used freely in this book. The group also included J.P. Conley, D. Diamantaras, T. Shinotsuka and T. Yamato among others. I also owe to Ignacio Ortuño-Ortín countless illuminating discussions on the role of Implementation Theory. A part of them came out as a joint paper reviewed in Chapter 7. Another

joint paper with my former student Carmen Beviá forms the basis for a section in Chapter 3. John P. Conley corrected some serious misunderstandings of mine in Chapters 1 and 2. Chapter 3 benefited from the insightful comments of Salvador Barberá. I am also indebted to my fellows Jose Alcalde, Iñigo Iturbe-Ormaetxe, Diego Moreno, Javier López-Cuñat and to several generations of students of the PhD program of the University of Alicante, especially to Pablo Amorós, Bernardo Moreno and Socorro Puy for correcting my mistakes. J. Alcalde and B. Moreno are co-authors of a paper that became the backbone of Appendix I to Chapter 6 and S. Baliga is co-author of a paper reviewed in Chapter 5. I am specially indebted to Jörg Naëve for his careful reading. He amended several inadequacies in propositions 1 (Chapter 3), 2 (Chapter 4) and 2 and 3 in Chapter 7. The proof of these results presented in this book are virtually his. My secretary, Mercedes Mateo, did many of the drawings and with her efficiency contributed to my dedication to the book. The latter also apply to Vera Emmen. It is only fair that I thank warmly all these persons for the various kind of help that they have given me. This book could not have been written without them, but I alone am responsible for any error.

LUIS C. CORCHÓN