

DERIVATIVES AND INTERNAL MODELS

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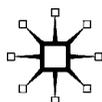


Derivatives and Internal Models

Fourth Edition

Dr Hans-Peter Deutsch

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Preface

The philosophy of this book is to provide an introduction to the valuation and risk management of modern financial instruments formulated in precise (and mathematically correct) expressions, covering all pertinent topics with a consistent and exact notation and with a depth of detail sufficient to give the reader a truly sound understanding of the material – an understanding which even places the reader in a position to independently *develop* pricing and risk management algorithms (including actually writing computer programs), should this be necessary. Such tasks will greatly be facilitated by the CD-ROM accompanying the book. This CD-ROM contains Microsoft Excel™ workbooks presenting concrete realizations of the concepts discussed in the book in the form of executable algorithms. Of course, the reader has full access to all source codes of the Visual Basic™ modules as well as to all calculations done in the spread sheet cells. The CD-ROM thus contains a collection of literally thousands of examples providing the reader with valuable assistance in understanding the complex material and serving as the potential basis for the further development of the reader's own particular pricing and risk management procedures.

The book should equip the reader with a wide array of tools needed for all essential topics in the field of modern market risk management and derivatives pricing. The reader is not expected to have previous knowledge of finance, but rather a sound mathematical and analytical background typical of scientists, mathematicians, computer scientists, engineers, etc. The novice is not even required to be familiar with ideas as fundamental as compounding interest. The book, however, is certainly also of interest to the experienced risk manager or financial engineer, since the concepts introduced are widely elaborated upon and analyzed down to the very foundations, making a comprehension of the material possible which goes significantly beyond the level held to be “common knowledge” in this field.

Since the beauty of a room behind a closed door is of little use if the door itself cannot be found, emphasis has been placed on providing an easy *entry* into the analysis of each of the various topics. As the author does not wish to

lose the reader at the outset, or expect the reader to first engage in the study of quoted literature before proceeding, the book is practically self-contained. An explanation of almost every expression or notion needed can be found in the book itself, ranging from compounding interest to term structure models, from expectation to value at risk, from time series analysis to GARCH models, from arbitrage to differential equations and exotic options, from the normal distribution to martingales, and so on.

The selection of the topics and the nature of their presentation result to a great extent from my personal experience as a consultant in the world of financial services; first with the Financial Risk Consulting division of Arthur Andersen in Germany, which it has been my pleasure to establish and direct for many years, and later with the consulting firm *d-fine*, which is in fact this former Financial Risk Consulting division, now operating as a company on its own. In these functions, I have been in a position to observe and identify exactly what knowledge and methods are required in the financial world as well as to see what tools are indispensable for a newcomer to this world.

I would like to take this opportunity to thank many of the (in part former) members of the Financial Risk Consulting team and of *d-fine* for their valuable input and many fruitful discussions not only concerning this book, but also in our day to day consulting work.

HANS-PETER DEUTSCH
Frankfurt, October 2008