Appendix A

The multi-phase signal is developed in the following manner.

If $y(t) > 0$, the following rules are applied

\[
\begin{align*}
    u(t) &= d, \quad 0 \leq t - t_{on,k} < x(t)P_{on,k-1} \\
    u(t) &= 0, \quad x(1)P_{on,k-1} \leq t - t_{on,k} < x(2)P_{on,k-1} \\
    u(t) &= d, \quad x(2)P_{on,k-1} \leq t - t_{on,k} < x(3)P_{on,k-1} \\
    u(t) &= 0, \quad x(3)P_{on,k-1} \leq t - t_{on,k} < x(4)P_{on,k-1} \\
    u(t) &= d, \quad x(4)P_{on,k-1} \leq t - t_{on,k} < (1 - x(4))P_{on,k-1} \\
\end{align*}
\]

(A.1)

If $y(t) < 0$, the following rules are applied

\[
\begin{align*}
    u(t) &= -d, \quad 0 \leq t - t_{off,k} < x(t)P_{off,k-1} \\
    u(t) &= 0, \quad x(1)P_{off,k-1} \leq t - t_{off,k} < x(2)P_{off,k-1} \\
    u(t) &= -d, \quad x(2)P_{off,k-1} \leq t - t_{off,k} < x(3)P_{off,k-1} \\
    u(t) &= 0, \quad x(3)P_{off,k-1} \leq t - t_{off,k} < x(4)P_{off,k-1} \\
    u(t) &= -d, \quad x(4)P_{off,k-1} \leq t - t_{off,k} < (1 - x(4))P_{off,k-1} \\
    u(t) &= 0, \quad (1 - x(4))P_{off,k-1} \leq t - t_{off,k} < (1 - x(3))P_{off,k-1} \\
    u(t) &= -d, \quad (1 - x(3))P_{off,k-1} \leq t - t_{off,k} < (1 - x(2))P_{off,k-1} \\
    u(t) &= 0, \quad (1 - x(2))P_{off,k-1} \leq t - t_{off,k} < (1 - x(1))P_{off,k-1} \\
    u(t) &= -d, \quad (1 - x(1))P_{off,k-1} \leq t - t_{off,k} \\
\end{align*}
\]

(A.2)
where $P_{\text{on},k-1}$ and $P_{\text{off},k-1}$ denote the time length of half cycle corresponding to on status and off status of $(k-1)$th cycle in relay feedback and $t_{\text{on},k}$ and $t_{\text{off},k}$ represent starting time of the on status and off status of the $k$th cycle in relay feedback.

The values of $x$ are obtained by solving the constrained non-linear optimization problem given by Eqs. A.1 and A.2. The six step replay proposed in Sect. 2.4.1 is shown in Fig. A.1.

Fig. A.1  a Proposed relay feedback signal; b its cyclic steady-state part when the period is 1
Appendix B

Improved Continuous Cycling Method for Stable FOPTD System

**Case study 1** Consider the first-order stable system as

\[ G_p(s) = \frac{e^{-0.5s}}{s+1} \]  \hspace{1cm} (B.1)

Formulating the amplitude and the phase angle criteria, we get

\[-\pi = -0.5\omega_c - \arctan(\omega_c) \text{ and } \frac{1}{K_{c,\text{max}}} = \frac{1}{\sqrt{\omega_c^2 + 1}}\]  \hspace{1cm} (B.2)

Solving Eq. (B.2) at the crossover frequency (\(\omega_c\)), the values of \(\omega_c\) and \(K_{c,\text{max}}\) are obtained as 3.673 and 3.8069. Using the ZN tuning rule \((K_{c,\text{des}} = 0.6K_{c,\text{max}}; \tau_I = \frac{t_a}{2}; \tau_D = \frac{t_a}{8})\), the PID controller parameters are calculated as \(K_{c,\text{des}} = 2.2841, \tau_I = 0.8553\) and \(\tau_D = 0.2138\).

In order to determine the updated value of the controller gain, once the integral and derivative actions come into effect, an additional element \((G_c = 1 + \frac{1}{\tau_I s} + \tau_D s)\) is formulated and added along with the controller. The new proposed system is given as

\[ G_p'(s) = \left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \left(\frac{e^{-0.5s}}{s+1}\right) \]  \hspace{1cm} (B.3)

where,

\[ \tau_D = \frac{\tau' \tau_D'}{\tau'_I + \tau'_D}; \quad \tau_I = \tau'_I + \tau'_D; \]  \hspace{1cm} (B.4)
\[
\tau' = \frac{5}{\omega_u} ; \quad \tau'_D = \frac{0.8}{\omega_u} ; \quad (B.5)
\]

For the new proposed system, the phase angle criterion is written as

\[
-\pi = a \tan \left( \tau_D \omega_c - \frac{1}{\tau_1 \omega_c} \right) - 0.5 \omega_c - a \tan(\omega_c) \quad (B.6)
\]

Solving Eq. (B.6), we get \( \omega_c \) as 4.8784. On substituting the value of \( \omega_c \) in the amplitude criteria to calculate the value of updated controller gain \( K_{c,max} \).

\[
GM = \sqrt{\left( \tau_D \omega_c - \frac{1}{\tau_1 \omega_c} \right)^2 + 1 \over \omega_c^2 + 1} ; \quad K_{c,max} = {1 \over GM} \quad (B.7)
\]

The updated PID controller settings obtained using the proposed method are \( K_c,\text{des} = 2.3490 \), \( \tau_1 = 1.5790 \) and \( \tau_D = 0.1878 \). Figure B.1 represents the servo and the regulatory response of the process for a unit step change in set point and in load. It can be seen that the proposed method gives improved performance for servo response, whereas the performance is slightly deteriorated in case of regulatory response.

To select a method for comparing the performance of the present method, four methods, based on ultimate values, are considered. Figure B.2 represents the performance of all the methods. It can be seen that Tan et al. (1999) method is better than the other methods for servo problem and ZN method is better for the regulatory problem. Among the other three methods, Smith (2003) method gives better

**Fig. B.1** Servo and regulatory response of the process to unit step change in input and load for system given by Eq. B.1. Solid line: proposed method; dashed line: ZN method
response for regulatory case. For the present work, Tan et al. (1999) method is considered for comparing the proposed method. Figure B.3 compares the servo response and the regulatory response of both the methods. It is seen that the present method gives improved servo and regulatory performances than the Tan et al. method.

The methods are also compared based on the time-domain integral performance. Table B.1 suggests an improvement in time integral values for the servo response for the present method over ZN method and the Tan et. al. method. For regulatory response, the present improved method is better than Tan et al. method. However, ZN method is always best for the regulatory response.
Case study 2 Consider the first-order stable system with slightly large \( L/\tau \) ratio as given below

\[
G_p(s) = \frac{e^{-0.7s}}{s+1}
\]  \hspace{1cm} (B.8)

Similar procedure as given in case study 1 is followed for the system. The settings obtained for both the conventional and proposed methods are

ZN method: \( K_{c,\text{des}} = 1.7519 \), \( \tau_1 = 1.1452 \) and \( \tau_D = 0.2863 \)  \hspace{1cm} (B.9)

Proposed method: \( K_{c,\text{des}} = 1.7660 \), \( \tau_1 = 2.1142 \) and \( \tau_D = 0.2541 \)  \hspace{1cm} (B.10)

The servo and regulatory response of the process to the step change in set point and the load are given in Fig. B.4. Figure B.4 shows the significant improvement in servo response, whereas for the regulatory response, the ZN method is better. The time-domain integral performances given in Table B.2 also suggest the better performance of the present method.

![Graph showing servo and regulatory response](image_url)

**Fig. B.4** Servo and regulatory response of the process to unit step change in input and load for system given by Eq. B.8. Legends same as Fig. B.1

<table>
<thead>
<tr>
<th>Table B.1</th>
<th>Time-domain integral performance of the systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servo response</td>
<td>Regulatory response</td>
</tr>
<tr>
<td>ZN</td>
<td>Tan et al. (1999) method</td>
</tr>
<tr>
<td>ISE</td>
<td>0.7254</td>
</tr>
<tr>
<td>ITAE</td>
<td>0.870</td>
</tr>
<tr>
<td>IAE</td>
<td>1.068</td>
</tr>
</tbody>
</table>
Generalizing the above method for stable system with varying time delay to process time constant ratio ($\varepsilon = L/\tau$).

$$G_p(s) = \frac{e^{-\varepsilon s}}{s + 1}$$  \hfill (B.11)

The information obtained using the stability criteria, regarding the controller gain and frequency, is shown in Table B.3. In addition, the controller parameters obtained using the proposed method are also given in Table B.3.

Using the data (Table B.3), the following correlations are proposed, based on the ultimate frequency and controller gain, to directly tune the PID controller. The proposed correlations are

<table>
<thead>
<tr>
<th></th>
<th>Servo response</th>
<th>Regulatory response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZN</td>
<td>IM</td>
</tr>
<tr>
<td>ISE</td>
<td>0.9567</td>
<td>0.928</td>
</tr>
<tr>
<td>ITAE</td>
<td>1.317</td>
<td>1.541</td>
</tr>
<tr>
<td>IAE</td>
<td>1.336</td>
<td>1.295</td>
</tr>
</tbody>
</table>

The values of $K_{c, \text{max}}$ and $\omega_u$ can be determined using the relay auto-tuning method, incorporating higher-order harmonics. If the transfer function is known, then the following relations are proposed (by fitting data given in Table B.3) to design the controller.

The values of $K_{c, \text{max}}$ and $\omega_u$ can be determined using the relay auto-tuning method, incorporating higher-order harmonics. If the transfer function is known, then the following relations are proposed (by fitting data given in Table B.3) to design the controller.

$$K_{c, \text{des}} = 0.655K_{c, \text{max}} - 0.145$$  \hfill (B.12)

$$\tau_1 = \frac{5.798}{\omega_u}$$  \hfill (B.13)

$$\tau_D = \frac{0.707}{\omega_{u1.01}}$$  \hfill (B.14)

$$K_{c, \text{des}}K_p = 1.248e^{-0.94}$$  \hfill (B.15)

$$\frac{\tau_1}{\tau} = -0.596\varepsilon^2 + 3.43\varepsilon + 0.014$$  \hfill (B.16)

$$\frac{\tau_D}{\tau} = -0.073\varepsilon^2 + 0.412\varepsilon + 0.001$$  \hfill (B.17)
To check the applicability of the correlations proposed, for the higher-order system, the method is applied to a second order stable system with real poles. One way is to get an equivalent FOPTD model by reaction curve method using SK method. However, the objective of the present work is to check the modified continuous ZN method works, and the ultimate parameters are calculated by writing the gain and phase angle criterion. Consider the example

\[ G_p(s) = \frac{e^{-0.5s}}{(2s + 1)(5s + 1)} \quad (B.18) \]

The values of \( K_{c,\text{max}} \) (14.8134) and \( \omega_c \) (1.1576) are calculated using the stability criteria.

\[ -\pi = -0.5\omega_c - \tan^{-1}(2\omega_c) - \tan^{-1}(5\omega_c) \quad (B.19) \]

\[ \frac{1}{K_{c,\text{max}}} = \frac{1}{\sqrt{(4\omega_c^2 + 1)(25\omega_c^2 + 1)}} \quad (B.20) \]

The controller settings are determined using the tuning rules proposed in Eqs. B.12–B.14. The servo response and the regulatory response are shown in Fig. B.5. It can be seen that the method is improved for servo response along with the less overshoot and settling time. In addition, the time-domain integral values (Table B.5) show the significant reduction of 24, 46 and 26% approximately in ISE, ITAE and IAE values, respectively, for the servo response. For regulatory problem, ZN method is slightly better than the present method (Table B.4).
In the appendix, the improved continuous cycling method is proposed (Eqs. B.12–B.14) and the corresponding setting is given by model parameters of a FOPTD system (Eqs. B.15–B.17). The present method is best among the available method for the servo response. For regulatory problem, the method is better than Tan et al. (1999) method. However, the ZN method is best for the regulatory response.

**Fig. B.5** Servo and regulatory response of the process to unit step change in input and load for system given by Eq. B.16. Legends same as Fig. B.1

**Table B.4** Controller settings for stable SOPTD system

<table>
<thead>
<tr>
<th></th>
<th>$K_{c,des}$</th>
<th>$\tau_I$</th>
<th>$\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM</td>
<td>9.5578</td>
<td>5.0086</td>
<td>0.6098</td>
</tr>
<tr>
<td>ZN method</td>
<td>8.888</td>
<td>2.7138</td>
<td>0.6784</td>
</tr>
</tbody>
</table>

**Table B.5** Time-domain integral performance of the stable SOPTD system

<table>
<thead>
<tr>
<th></th>
<th>Servo response</th>
<th>Regulatory response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZN</td>
<td>IM</td>
</tr>
<tr>
<td>ISE</td>
<td>2.263</td>
<td>1.717</td>
</tr>
<tr>
<td>ITAE</td>
<td>15.64</td>
<td>8.291</td>
</tr>
<tr>
<td>IAE</td>
<td>3.968</td>
<td>2.902</td>
</tr>
</tbody>
</table>
Appendix C

CODE 1: MATLAB Code for Determining the Minimum and Maximum Value of Controller Gain
%FUNCTION FILE

function f = w_1(w,epi)
    f = - epi*w + atan(w);
end

% COMMAND

epi = [ 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2];
%w_guess = 50; % for maximum value of critical frequency
%w_guess = 0.01 % for minimum value of critical frequency
for i= 1:length(epi)
    w(1,i) = fsolve(@(w) w_1(w, epi(1,i)), w_guess); %% solve for w_c
    amtf(1,i) = 1/sqrt(1*w(1,i)^2 +1); % amplitude of TF
    kc(1,i) = 1/amtf(1,i); % solve for k_c
    pu(1,i) = 2*pi/w(1,i); % period of oscillation
    tau_i(1,i) = pu(1,i)/2; % ZN tuning rule
    tau_d(1,i) = pu(1,i)/8; % ZN tuning rule
end
CODE II: MATLAB Code for Determining the Updated Minimum and Maximum Value of Controller Gain

% FUNCTION FILE

function f = w_2(W,tau_i,tau_d, epi)
f = atan((tau_d*W)-(1/(tau_i*W))) - epi*W + atan(1*W);
end

% COMMAND

global w
epi= [ 0.01 0.02 0.05 0.08 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1];

for i=1:5
  tau_i_s(1,i) = 5/w(1,i); %series form of integral time
  tau_d_s(1,i) = 0.8/w(1,i); %series form of derivative time
  tau_i(1,i) = tau_i_s(1,i) + tau_d_s(1,i); %parallel form of integral time
  tau_d(1,i) = (tau_i_s(1,i).* tau_d_s(1,i))./(tau_i_s(1,i) + tau_d_s(1,i)); %parallel form of derivative time
  W(1,i) = fsolve(@(W) w_2(W, tau_i(1,i),tau_d(1,i),epi(1,i)),100);
  a = sqrt(1*W(1,i)^2 +1);
  b = sqrt(1 +((tau_d(1,i).*W(1,i)) - (1./(tau_i(1,i).*W(1,i))))^2);
  amtf_pid(1,i) = b/a; %amplitude of the updated system
  kcup(1,i)= 1/amtf_pid(1,i); %updated controller gain
end
Appendix D

Simulink Diagrams

Linearized bioreactor with relay feedback

Non linear bioreactor model with relay feedback
Bioreactor subsystem
Two input two output system with relay feedback
Index

A
Amplitude criterion, 61, 127–129, 131, 132, 135, 144
Asymmetric relay, 62, 95

B
Bioreactor, 53, 68, 70, 72, 73, 135, 155, 164
BLT, 44

C
Centralised control, 26, 113
Characteristic equation, 35, 44, 46, 94, 96
Characteristic loci, 35, 36, 38, 114
Cohen—coxon method, 7
Conventional method, 20, 24, 62, 63, 66–68, 70–72, 75, 79–81, 85, 87, 88, 90, 91, 97–100, 102, 103, 106, 107, 118, 124, 145, 168, 180
Critical gains, 37–40 points, 9, 23, 34, 36, 44, 60, 77, 78, 94
CSTR, 10, 141, 143, 144, 149

D
Davison method, 45
DCP, 9, 36–38
Decentralised control, 26, 28, 50, 75, 94
Decentralised relay, 40, 41, 76, 94
Decoupling control, 26
Derivative control, 2
Derivative kick, 3
Derivative time, 3, 8, 27, 114, 118, 128, 129, 131, 136, 139, 142, 145, 152, 169, 170, 181
Describing function analysis, 15, 16, 19, 40
Detuning factor, 27, 31, 33, 101
Distillation column, 78, 81, 89

G
Gain margin, 39, 42, 43, 59, 60, 64, 66, 67, 73, 128, 133, 141, 153, 154, 157

H
Identification, 6
Independent tuning, 26, 29
Integral control, 2
Internal stability, 47, 50
ISP reactor, 84

L

M
Maciejowski method, 45
Manual loop tuning, 4
Maximum sensitivity, 59, 73, 134, 141, 154
Maximum singular value, 46, 50, 75, 107, 110, 116, 119, 120, 124, 125, 169, 174, 180
Model uncertainty, 46
Modified Fourier series, 21
Modified Ziegler-Nichols, 167
Multivariable system, 5, 25, 26, 29, 34–36, 39–43, 45, 49, 50, 77, 95, 113

N
Niederlinski index, 29
Noise, 20, 64, 72, 73, 82, 83, 117

P
Padé approximation, 61, 96
Phase angle, 54, 61, 127, 128, 130, 131, 135, 136, 138, 139, 141, 142, 144, 152, 153, 157, 158, 185, 186, 190
Phase margin, 39, 42, 43, 59, 60, 64, 66, 67, 128, 134, 141, 153
Preload relay, 22, 23
Principle harmonics, 56
Process activation method, 23
Process reaction curve method, 6
Proportional controller, 2
Proportional-derivative-integral control, 2

Q
Quarter amplitude damping, 7

R
Refined Ziegler Nichols methods, 5
Regulatory, 63, 65, 67, 73, 79, 88, 98, 102, 110, 118, 123, 125, 130–133, 136, 138, 140, 144, 146, 149, 153, 154, 158, 161, 165, 178, 180, 186, 188, 190, 191
Relative gain array, 28
Relay, 8, 11, 13, 18–20, 33, 50, 53, 62–65, 69, 72, 78, 84, 87, 89, 96, 102, 106, 144–147, 165, 169, 170, 177, 189
Relay equations, 14
Relay feedback method, 9
Reset time, 127–129, 131, 136, 139, 142, 145, 152, 169, 170
Reset windup, 3
Robustness, 46, 124, 165
Routh Hurwitz method, 61, 94, 96

S
Scalar system, 2, 18
Sensitivity function, 59
Sequential loop, 26, 28
Sequential relay, 39
Simultaneous loop tuning, 26, 28
Single test relay, 167, 172, 173, 180, 181
Six-step relay, 20
SOPTD systems, 54
Spectral radius, 47
Stability analysis, 44, 50
Stability criterion, 29, 32, 35, 38, 43, 44, 46, 50
Stable, 7, 9, 14, 22, 29, 32, 35, 36, 39, 43, 44, 46, 47, 50, 53, 54, 59, 65, 75–78, 81, 91, 94, 102, 113, 115, 127, 128, 133, 141, 151, 157, 180, 185, 188–190
Steady-state gain matrix, 28, 37, 40, 41, 43, 45, 105, 110, 116, 117, 122
Steady-state gains, 36–38, 125, 168
Structured Singular Value, 49
Synthesis method, 8

T
Tanttu and Lieslehto tuning method, 45
Time-domain performance, 65, 67, 72, 75, 79, 87, 90, 93, 102, 107
Total variation, 60, 72, 107, 149, 158, 172, 178
Tuning, 3, 9, 60, 144, 145
Two Tests Relay (TTR), 167, 171, 173, 180, 181

U
Ultimate gain, 4, 17, 19–24, 29, 30, 32, 33, 38, 53, 57, 60, 64, 71, 73, 76–80, 84, 85, 88, 90–93, 95, 99, 101, 103, 107, 110, 113, 115, 127, 144–146, 159

W
Waveform, 13, 14, 17, 19, 35, 40, 53, 65, 76–78, 84, 87, 89, 96, 97, 115, 145, 146, 177

Z
Ziegler-Nichols, 4, 5, 7, 10, 11, 44, 90, 140, 158