## Annexure-1

### Details of the Dielectric Materials and Their Suppliers

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<th>Supplier or manufacturer</th>
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<td>MgO–SiO&lt;sub&gt;2&lt;/sub&gt; (CD-6)</td>
<td>6.3</td>
<td><strong>Countis Laboratories</strong>&lt;br&gt;12295 Charles Dr, Grass Valley, CA 95945, United States&lt;br&gt;+1 530-272-8334&lt;br&gt;<a href="mailto:tcountis@countis.com">tcountis@countis.com</a></td>
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<td>Unit 2507-8, 25/F, Office Tower, Langham</td>
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<tr>
<td></td>
<td></td>
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<td>Place, 8 Argyle Street, Mongkok, Mongkok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Kowloon, Hong Kong Tel: +852-2923 0600 Call:</td>
</tr>
<tr>
<td></td>
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<td>+852-2923 0605 Email: <a href="mailto:sales@hk.eccosorb.com">sales@hk.eccosorb.com</a></td>
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<td>Ti Zr Nb Zn oxide (E6000)</td>
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<td><a href="http://www.smcel.com">http://www.smcel.com</a></td>
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<td>TE-36</td>
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Annexure-2

Two-Dimensional Mathematical Model of Resonant Modes in Cavity Resonator

See Figs. A2.1, A2.2 and A2.3.

Probe inserted d/
Characteristic equation of RDRA is given below:

\[ k_x^2 + k_y^2 + k_z^2 = \varepsilon_r k_{mn}^2 \quad \text{(A2.1)} \]

The field \( E_z \) can be expressed as follows:

\[ (\nabla^2 + h^2)(H_z \text{ or } E_z) = 0; \quad \text{Helmholtz equation} \]

\[ E_z = \sum C_{mn} \cdot \sin(m\pi x/a) \cdot \sin(n\pi y/b) \cdot e^{-i\gamma_{mn}} e^{i\omega t} \]

In the above equation, \( C_{mn} \) are the amplitude coefficients and wave is propagating in z-direction

\[ \gamma_{mn} \text{(Propagation constant)} = \sqrt{h_{mn}^2 - k^2} = \sqrt{h_{mn}^2 - \omega^2 \mu \varepsilon} \]

where \( h_{mn} = k_c = n\pi/b \); are possible eigenvalues.

Hence, computation of field \( E_z \) when all the four sides of resonator are transparent and magnetic walls (PMC walls) and top and bottom walls are PEC (Electrical walls). We are well versed that \( H_z = 0 \) at magnetic walls and \( E_z = 0 \) at electric walls.

The feed probe is inserted into rectangular DRA at point \((a/2, b/2)\) in z-direction. \( I(t) \) Current can be expressed in terms of magnetic vector potential \( A_z \).
Fig. A2.1  RDRA without ground plane

Fig. A2.2  Ground plane of RDRA

Fig. A2.3  RF feed

\[ \mathbf{A}_z = \frac{\hat{z}}{4\pi r} \mu I dl e^{-jkr}, \quad r \text{ is far field point.} \]

\[ \text{div} \ A = \frac{j k (\hat{r} \cdot \hat{z}) \mu I dl - e^{-jkr}}{4\pi r} \]

\[ \text{div} \ A = \frac{j k \cos \theta \mu I dl - e^{-jkr}}{4\pi r}; \]

\[ = -\frac{j \omega \beta}{c^2} \]
Let,
\[ \mathcal{J} = \sqrt{a^2 + b^2} \frac{k c}{\omega} \frac{\mu d l}{4 \pi r} e^{-jk r} \]
\[ = \cos \theta \frac{\mu d l}{4 \pi r} e^{-jk r} \]

Now,
\[ E = -\nabla \varphi - j \omega A; \] Lorentz’s gauge condition
\[ E_r = 0 + O\left(\frac{1}{r^2}\right) \]
\[ E_\theta = j \omega A_\theta = j \omega A_z, \quad \text{at } z = 0 \]
\[ E_\phi = -j \omega A_\phi = 0 \quad \text{at } z = 0 \]

Hence,
\[ E_\theta = E_z = j \omega A_z \]
\[ = j \omega \frac{\mu d l}{4 \pi r} e^{-jk r} \frac{2}{\sqrt{ab}} \sin\left(\frac{m \pi x}{a}\right) \sin\left(\frac{n \pi y}{b}\right) \]

where \( r = \sqrt{x^2 + y^2} \)
\delta m, n', \delta n, n' = \langle u_{mn}, u_{m'n'} \rangle; \text{ where } z = 0; \text{ (property of orthogonality as product of basis function becomes zero)}

\[ E_z = \sum_{mn} C_{mn} u_{mn}(x, y) e^{-j \omega \mu l} \]
\[ h_{mn} \leq \omega \]

At \( z = 0 \)
\[ E_z \Rightarrow \sum_{mn} C_{mn} u_{mn}(x, y) \frac{j \omega \mu d l}{4 \pi \sqrt{x^2 + y^2}} e^{-jk \sqrt{x^2 + y^2}}; \]

Hence, amplitude coefficient
\[ C_{mn} = \frac{j \omega \mu d l}{4 \pi} \int_{0}^{a} \int_{0}^{b} \frac{u_{mn}(x, y)}{\sqrt{x^2 + y^2}} e^{-jk \sqrt{(x - a/2)^2 + (y - b/2)^2}} \text{dxdy}; \]
\[ C_{mn} = \frac{j\omega \mu Idl}{4\pi} \int_0^a \int_0^b \frac{u_{mn}(x,y)}{\sqrt{(x-a/2)^2 + (y-b/2)^2}} e^{-jk\sqrt{(x-a/2)^2 + (y-b/2)^2}} \, dx \, dy \]

\[ (A2.2) \]

Hence \( \pi^2 \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right] \leq \omega^2 \)

if \( a > b \) and \( m = 1, 2, 3, \ldots \) \( n = 1, 2, 3, \ldots \)

\[ \pi^2 \left( \frac{1}{a^2} + \frac{1}{b^2} \right) \leq \omega^2 < \pi^2 \left( \frac{2}{a^2} + \frac{1}{b^2} \right) \]

\[ \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} \leq \frac{\omega}{\pi} < \sqrt{\frac{2}{a^2} + \frac{1}{b^2}} \]

\[ \gamma_{mn} = \frac{j\pi p}{d} \]

\[ k^2 + \gamma_{mn}^2 = h_{mn}^2 \]

hence, \( k^2 = h_{mn}^2 + \frac{\pi^2 p^2}{d^2} \)

\( C_{mn} \) Fourier coefficients of modes;
\( u_{mn} \) depends on input excitation;
\( h_{mn} \) resonant mode (cut off frequency); and
\( k \)-propagation constant.

Generation of modes or characteristics frequencies \( \omega(mnp) \) e.m. of electromagnetic fields oscillations inside the cavity resonator has been discussed. The basic Maxwell’s theory can be applied with boundary conditions to express resonator fields as superposition of these characteristics frequencies.

The fields

\[ E_x(x, y, z, t) = \sum_{mnp} \text{Re} \int C_{mnp} e^{j\omega(mnp)t} u_{mnp}(x, y, z) \]

or

\[ \sum_{mnp} |C_{mnp}| u_{mnp}(x, y, z) \cos(\omega(mnp) + \Phi(mnp)) ; \]
where \( u_{mn}(x, y) = \frac{2}{\sqrt{ab}} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \); \( \omega(mnp) \) is the characteristic frequency and \( \omega(mnp) \) is the phase of current applied. The rectangular cavity resonator is excited at the centre with an antenna probe carrying current \( I(t) \) of some known frequency \( \omega(mnp) \). This generates the field \( E_z \) inside the cavity of the form given below:

\[
E_z(x, y, \delta, t) = \int G(x, y) \frac{j\omega \mu \text{Idl}(x^2 + y^2)}{4\pi(x^2 + y^2 + \delta^2)^{3/2}} e^{j(\omega - \frac{\pi}{C_0/C_1} \sqrt{x^2 + y^2 + \delta^2})} I(\omega) e^{j\omega t} d\omega
\]

where \( G(x, y) \) are the constant terms associated with current.

Equating resonator field with the antenna current fields at \( z = \delta \) plane;

Antenna or resonator radiation current or fields

\[
= \sum |C_{mnp}| \sqrt{\frac{2}{d}} \sin\left(\frac{p\pi}{d}\right) \cos(\omega(mnp)t + \phi(mnp))
\]

Antenna probe current

\[
= \int G(x, y) \frac{j\omega \mu \text{Idl}(x^2 + y^2)}{4\pi(x^2 + y^2 + \delta^2)^{3/2}} I(\omega) e^{j\omega t} d\omega \left( e^{j(\omega - \frac{\pi}{C_0/C_1} \sqrt{x^2 + y^2 + \delta^2 + \psi_{mnp}})} u_{mn}(x, y) dx dy; \right)
\]

Multiply both sides by \( e^{-j\omega(mnp)t} \) and then taking time averaging (KAM) gives us the following

\[
|C_{mnp}| \sqrt{\frac{2}{d}} \sin\left(\frac{p\pi}{d}\right) e^{j\omega(mnp)} = \lim_{T \to \infty} \frac{1}{2\pi} \int_{-T}^{T} G(x, y) \frac{j\omega \mu \text{Idl}(x^2 + y^2)}{4\pi(x^2 + y^2 + \delta^2)^{3/2}} e^{-j\omega(mnp)t} I(\omega) e^{j\omega t} d\omega \left( e^{j(\omega - \frac{\pi}{C_0/C_1} \sqrt{x^2 + y^2 + \delta^2 + \psi_{mnp}})} u_{mn}(x, y) dx dy; \right)
\]

It is clear that for these two expressions to be equal, the probe current can be defined as

\[
I(\omega) = \frac{1}{2} \sum_{mnp} |I(mnp)| \left[ \delta(\omega - \omega(mnp)) e^{j\omega(mnp)} + e^{j\omega(mnp)} \delta(\omega - \omega(mnp)) \right]
\]

The antenna probe current must contain only the resonator characteristics frequencies \( \omega(mnp) \), then
\[
\sum_p |C_{mnp}| \sqrt{\frac{2\pi}{d}} \sin \left( \frac{p \pi \delta}{d} \right) \cos(\omega(mnp)t + \phi(mnp)) = \int G(x, y) \frac{j \omega \mu I_{dl}(x^2 + y^2)}{4\pi(x^2 + y^2 + \delta^2)^{3/2}} I(\omega) \quad (A2.3)
\]

\[e^{j\omega t} \int \epsilon(x) \left( \frac{\omega}{c} \right) \sqrt{x^2 + y^2 + \delta^2 + \psi_{mnp}} \right) u_{mnp}(x, y) dx dy \]

Antenna probe current = Resonator radiated current or magnetic fields, as per the law of conservation of energy. The modes’ diagrams are given below (Figs. A2.4, A2.5, A2.6, A2.7, A2.8, A2.9, A2.10, A2.11, A2.12, A2.13, A2.14 and A2.15):

Fig. A2.4  Mode diagram

Fig. A2.5  TE_{112}

Fig. A2.6  TE_{113}

Fig. A2.7  TE_{114}
Fig. A2.8  $\text{TE}_{111}$

Fig. A2.9  $\text{TE}_{112}$

Fig. A2.10  $\text{TE}_{113}$

Fig. A2.11  $\text{TE}_{115}$

Fig. A2.12  $\text{TE}_{116}$
Mode sketch

Fig. A2.13  TE_{114}

Fig. A2.14  TE_{118}

Fig. A2.15  TE_{114}

Rectangular design

\[ \omega_{mn} = \frac{\pi}{\sqrt{a^2 + b^2}} \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^{\frac{1}{2}}, \]  

in two-dimensional case

\[ \frac{\partial^2 X}{\partial x^2} + k_x^2 \cdot x = 0; \quad \frac{\partial^2 Y}{\partial y^2} + k_y^2 \cdot y = 0; \quad \frac{\partial^2 Z}{\partial z^2} + k_z^2 \cdot z = 0 \]

\[ \varepsilon_r k_0^2 = k_x^2 + k_y^2 + k_z^2; \quad \text{where } k \text{ is wave number} \]

\( h(k, x) \) is the harmonic function and can be written as follows: \( \sin(k, x) \) or \( \cos(k, x) \).

These are solutions of wave function and if boundary conditions are applied, then eigenvalues can be defined as follows:

\[ k_0 = \frac{2\pi f_0}{c}, \quad \tan(k_y d/2) = \sqrt{(\varepsilon_r - 1)k_0^2 - k_y^2} \]

\[ k_x^2 + k_y^2 + k_z^2 = \varepsilon_r k_0^2, \quad \text{Resonant frequency } \quad f_0 = \frac{c}{2\pi \sqrt{\varepsilon_r}} \sqrt{k_x^2 + k_y^2 + k_z^2} \]

where \( k_x = m \frac{\pi}{a}, k_y = n \frac{\pi}{b}, \) and \( k_z \tan \left( \frac{k_z d}{2} \right) = \sqrt{(\varepsilon_r - 1)k_0^2 - k_z^2}. \)
The resonance frequency of this antenna can be estimated using the approximate analytical expressions for the resonance frequency of TE_{111} mode in a rectangular resonator (three dimensional) given by

\[ f_{111} = \frac{c}{2\pi \sqrt{\varepsilon_r}} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{2b}\right)^2 + \left(\frac{\pi}{d}\right)^2}, \]

Three-dimensional case

Propagation constant, \( \gamma^2 = k^2 - k_c^2 \)

### A2.1 Fourier Series

\[ f(x) = f_0 + \sum_{n=1}^{\infty} \left[ a_n \cos \left(\frac{2n\pi}{a} x\right) + b_n \sin \left(\frac{2n\pi}{a} x\right) \right] \]

\[ a_n = \frac{2}{a} \int_{0}^{a} f(x) \cos \left(\frac{2n\pi}{a} x\right) dx \]

\[ b_n = \frac{2}{a} \int_{0}^{a} f(x) \sin \left(\frac{2n\pi}{a} x\right) dx \]

Half-wave Fourier analysis will have odd or even terms, i.e., sine–sine or cosine–cosine.

If \( f(x) = f(-x) \), then even harmonics will take place and only cosine terms will occur, i.e.,

\[ f(x) = \sum_{n=1}^{\infty} C_n \cos \left(\frac{n\pi x}{a}\right) \]

where \( C_n = \frac{2}{a} \int_{0}^{a} f(x) \cos \left(\frac{n\pi}{a} x\right) dx \)

Similarly for odd terms, \( f(x) \neq f(-x) \),

\[ f(x) = \sum_{n=1}^{\infty} B_n \sin \left(\frac{n\pi x}{a}\right) \]

where \( B_n = \frac{2}{a} \int_{0}^{a} f(x) \sin \left(\frac{n\pi}{a} x\right) dx \).
A2.2 Spectral Resolution of EM Waves

Every wave can be subjected to the process of spectral resolution, i.e., can be represented as a superposition of monochromatic waves of various frequencies. The character of this expansion varies according to the character of the time dependence of the fields.

One category consists of those cases where the expansion contains frequencies forming a discrete sequence of values. The simplest case of this type arises in the resolution of a purely periodic field. This is the usual expansion in Fourier series. It contains the frequencies which are integral multiples of the “fundamental” frequency \( \omega_0 = \frac{2\pi}{T} \), where \( T \) is the period of the field. We therefore write it in the form as follows:

\[
|f| = \sum_{n=-\infty}^{\infty} f_n e^{-j\omega_0 n t}
\]

where \( f \) is any of the quantities describing the field. The quantities \( f_n \) are defined in terms of the function \( f \) by the integrals

\[
f_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{j\omega_0 t} dt.
\]

Because \( f(t) \) must be real

\[
f_n = f_n^*.
\]

in more complicated cases, the expansion may contain integral multiples of several different incommensurable fundamental frequencies. When the sum is squared and averaged over the time, the product of terms with different frequencies is given zero because they contain oscillating factors.

Only terms of the form \( f_nf_{-n} = |f_n|^2 \) remain. Thus, the average of the square of the field, i.e., the average intensity of the wave, is the sum of the intensities of its monochromatic components.

\[
\bar{f}^2 = \sum_{n} = -\infty^{\infty} |f_n|^2 = 2 \sum_{n=1}^{\infty} |f_n|^2,
\]

where it is assumed that the average of the function \( f \) over a period is zero. Another category consists of fields which are expandable in a Fourier integral containing a continuous distribution of different frequencies. For this to be possible, the function \( f(t) \) must satisfy certain definite conditions; usually we consider functions which vanish for \( t \rightarrow \pm \infty \).

Similarly, \( f_{-\omega} = f_{\omega}^* \); let us express the total intensity of the wave, i.e., the integrals of \( f^2 \) over all time, in terms of the intensity of the Fourier components. Now, we have

\[
\int f^2 dt = 2 \sum_{n=1}^{\infty} |f_n|^2.
\]
\[ \int_{-\infty}^{\infty} f^2 dt = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} f_{\omega} e^{-j\omega t} \frac{d\omega}{2\pi} \right\} dt = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} f_{\omega} e^{-j\omega t} \right\} \frac{d\omega}{2\pi} \]

or

\[ \int_{-\infty}^{\infty} f^2 dt = \int_{-\infty}^{\infty} |f_{\omega}|^2 \frac{d\omega}{2\pi} = 2 \int_{0}^{\infty} |f_{\omega}|^2 \frac{d\omega}{2\pi}. \]

\[ f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_{\omega} e^{-j\omega t} d\omega, \] where the Fourier components are given in terms of the function \( f(t) \) by the integrals,

\[ f_{\omega} = \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt. \]

### A2.3 Coordinate System and Their Transformations

Rectangular \((x, y, z)\), cylindrical \((\rho, \phi, z)\), and spherical \((r, \theta, \phi)\) coordinates can be expressed as follows:

\[
\begin{align*}
    x &= \rho \cos \phi = r \sin \theta \cos \phi, \\
    y &= \rho \sin \phi = r \sin \theta \sin \phi, \\
    z &= r \cos \theta. \\
    \rho &= \sqrt{x^2 + y^2} = r \sin \theta, \\
    \theta &= \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right) = \tan^{-1} \frac{\rho}{z}.
\end{align*}
\]

Transformations of the coordinate components of a vector among the three coordinate systems are given by

\[
\begin{align*}
    A_z &= A_{\rho} \cos \phi - A_{\phi} \sin \phi, \\
    &= A_r \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_{\phi} \sin \phi, \\
    A_y &= A_{\rho} \sin \phi - A_{\phi} \cos \phi, \\
    &= A_r \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi - A_{\phi} \cos \phi, \\
    A_z &= A_r \cos \theta - A_\theta \sin \theta, \\
    A_\rho &= A_z \cos \phi + A_y \sin \phi = A_r \sin \theta + A_\theta \cos \theta, \\
    A_{\phi} &= -A_z \sin \phi + A_y \cos \phi.
\end{align*}
\]
\[ \begin{align*}
A_r &= A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta \\
&= A_\rho \sin \theta + A_z \cos \theta \\
A_\theta &= A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta \\
&= A_\rho \cos \theta - A_z \sin \theta
\end{align*} \]

unit vector in the three systems are denoted by \((u_x, u_y, u_z), (u_\rho, u_\phi, u_z), \) and \((u_r, u_\theta, u_\phi)\)

\[ \begin{align*}
dr &= dx dy dz = \rho d\rho d\phi dz = r^2 \sin \theta dr d\theta d\phi \\
\text{Area}(D_3) &= u_x dy dz + u_y dx dz + u_z dx dy \\
&= u_\rho \rho d\phi dz + u_\phi \rho d\rho dz + u_z \rho d\rho d\phi \\
&= u_r r^2 \sin \theta d\theta d\phi + u_\theta r \sin \theta dr d\phi + u_\phi r dr d\theta \\
\text{Length}(L) &= u_x dx + u_y dy + u_z dz \\
&= u_\rho d\rho + u_\phi \rho d\phi + u_z dz \\
&= u_r dr + u_\theta r d\theta + u_\phi r \sin \theta d\phi
\end{align*} \]

Scalar multiplication is defined by

\[ A \cdot B = A_1 B_1 + A_2 B_2 + A_3 B_3 \]

\[ \nabla \cdot \nabla v = \nabla^2 v \]

\[ \nabla \times \nabla \times A = \nabla (\nabla \cdot A) - \nabla^2 A \]

\[ \text{Re}(re^{i\alpha}) = r \cos(\omega t + \theta) \]

\[ \text{Im}(re^{i\alpha}) = r \sin(\omega t + \theta) \]

Kronecker Tensor \( \otimes \)

\[ f = \frac{1}{2\pi\sigma^2} e^{-\frac{(a - u)^2}{2\sigma^2}} \] where \( a \) is the mean and \( \sigma \) is the variance and vector multiplication can be defined as:

\[ A \times B = \begin{vmatrix}
    u_1 & u_2 & u_3 \\
    A_1 & A_2 & A_3 \\
    B_1 & B_2 & B_3
\end{vmatrix} \]

The differential operators are the gradient \((\nabla \omega)\),
Divergence ($\nabla \cdot A$),

$\text{curl } (\nabla \times A)$

Laplacian operator ($\nabla^2_{\omega}$)

In rectangular coordinates, we can think of $\text{del } (\nabla)$ as the vector operator

$$\nabla = u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}$$

$$\nabla \omega = u_x \frac{\partial \omega}{\partial x} + u_y \frac{\partial \omega}{\partial y} + u_z \frac{\partial \omega}{\partial z}$$

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times A = \begin{vmatrix} u_x & u_y & u_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 \omega = \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2}$$

In cylindrical coordinates, we have

$$\nabla \omega = \frac{u_\rho}{\rho} \frac{\partial \omega}{\partial \rho} + u_\phi \frac{1}{\rho} \frac{\partial \omega}{\partial \phi} + u_z \frac{\partial \omega}{\partial z}$$

$$\nabla \cdot A = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho A_\rho \right) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times A = u_\rho \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + u_\phi \left( \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + u_z \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho A_\phi \right) - \frac{1}{\rho} \frac{\partial A_\rho}{\partial \phi} \right]$$

$$\nabla^2 \omega = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \omega}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \omega}{\partial \phi^2} + \frac{\partial^2 \omega}{\partial z^2}$$

In spherical coordinates, we have
\[ \nabla_\omega = u_r \frac{\partial \omega}{\partial r} + u_\phi \frac{1}{r} \frac{\partial \omega}{\partial \phi} + u_\theta \frac{1}{r \sin \theta} \frac{\partial \omega}{\partial \theta} \]

\[ \nabla \cdot A = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 A_r \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( A_\theta \sin \theta \right) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \]

\[ \nabla \times A = u_r \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} \left( A_\theta \sin \theta \right) - \frac{\partial A_\phi}{\partial \phi} \right] + u_\theta \frac{1}{r} \left( \frac{\partial}{\partial r} \left( r A_r \right) - \frac{\partial A_\phi}{\partial \phi} \right) + u_\phi \frac{1}{r} \left( \frac{\partial}{\partial \theta} \left( r A_\phi \right) - \frac{\partial A_\theta}{\partial \phi} \right) \]

\[ \nabla^2 \omega = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \omega}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \omega}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \omega}{\partial \phi^2} \]

\[ R = u_r x + u_\theta y + u_\phi z \]

And the “source coordinates” by

\[ r' = u_r x' + u_\theta y' + u_\phi z' \]

\[ |r - r'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \]

\[ A = \frac{Il e^{-jk|r-r'|}}{4\pi|r - r'|} \]

To emphasize that \( A \) is evaluated at the field point \((x, y, z)\) and \( Il \) is situated at the source point \((x', y', z')\) (Table A2.1),

\[ A(r) = A = \frac{Il(r') e^{-jk|r-r'|}}{4\pi|r - r'|} \]

**Table A2.1**: Frequency in \( H_z \)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Symbol</th>
<th>Frequency in ( H_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tera</td>
<td>T</td>
<td>( 10^{12} )</td>
</tr>
<tr>
<td>Giga</td>
<td>G</td>
<td>( 10^{9} )</td>
</tr>
<tr>
<td>Mega</td>
<td>M</td>
<td>( 10^{6} )</td>
</tr>
<tr>
<td>Kilo</td>
<td>K</td>
<td>( 10^{3} )</td>
</tr>
<tr>
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<td>H</td>
<td>( 10^{2} )</td>
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<tr>
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<td>mm</td>
<td>( 10^{-3} )</td>
</tr>
<tr>
<td>Micro</td>
<td>( \mu )</td>
<td>( 10^{-6} )</td>
</tr>
<tr>
<td>Nano</td>
<td>n</td>
<td>( 10^{-9} )</td>
</tr>
<tr>
<td>Pico</td>
<td>p</td>
<td>( 10^{-12} )</td>
</tr>
<tr>
<td>Femto</td>
<td>f</td>
<td>( 10^{-15} )</td>
</tr>
</tbody>
</table>
Annexure-3

Design Steps of RDRA Using ADS Software

Steps →

1. Export the model from HFSS and save in G drive or any file (without Path).
2. Now right click the ADS icon and click run as administration.
3. Then, click the yes button.
4. Then, click the cancel and go create new project.
5. Now on schematic will open, then go to layout button, then go to create update layout.
6. Then go to file button and then go to import button, the layout model is complete.
7. Now click on line which is connected to the patch on layout model then delete it.
8. Then go to the view button then go to layer view then go to by name. Then go to conductor 2 button, now then drag the feed or patch and date it.
9. Now go to each capacitor then click double and give it value according to the formula.

\[
C_{(v)} = 26f \quad \text{at} \quad v = 0 \quad C_{i} = 1.298 \mu f \\
C_{f} = 0f \quad \text{and add each capacitor by line by clicking on line icon.}
\]

10. Now go to S-parameter then click on termination which also given in Fig. A3.1.
11. Now go to the S.P (S-parameter) button and put on schematic window then click the S-parameter which is on the schematic window and put frequency 1 to 3 by stepping 1 MHz frequency then ok.
12. Now go to simulate button and simulate it then after completing the button.
13. Now then go to EDS model, then go to substrate and create update then go to open button put substrate (RT Duroid-5880) then put the thickness of the substrate (1.524 mm) loss tangent (0.001) then go to apply and then go to ok.
14. Now again go to EDS model then go to component. Now go to create update then put start frequency and stop frequency 3 GHz.
15. Now put the port on the patch by single clicking on the patch from port Ze on.
17. Now go to schematic window and then go to library file and click on anywhere on schematic window.
18. Now go to lumped element and select on capacitor and put three by pressing control button.

PCB manufacturing from HFSS model

1. Save HFSS model bottom as view .dxf file after going to modeler and exporting it
2. open .dfx in AutoCAD to generate .pdf or image as .jpg format.
3. use butter paper to place this design on to PCB
4. now connect SMA connectors and it is ready for testing antenna parameters.

II. HFSS design steps:

**APPLY MAGNETIC AND ELECTRIC BIAS TO MHD ANTENNA MAGNETIC BIASING STEPS WITH HFSS:**

1. MAKE THREE SLOTS
2. SLOTS SHOULD BE ENCLOSING MICRO-STRIP FEED LINE
3. THE UPPER EDGE OF ALL THE SLOTS SHOULD TOUCH EACH OTHER
4. THE SUBS AND SLOTS SHOULD NOT INTERSECT
5. UNITE ALL THE SLOTS
6. SELECT MATERIAL
7. (A) FERRITE
8. (B) MAGNETIC SATURATION EG 500 TESLA
9. GO TO BOX → ASSIGN EXCITATION → MAGNETIC BIAS—
10. NEXT
11. PERMEABILITY
12. X, Y, Z VALUE-DESIRED
13. FINISH
14. CHECK FOR VALIDATION
15. **RELOCATE SLOT IF REQUIRED**
16. **SIMULATE**

**ELECTRIC BIASING STEPS WITH HFSS:**

1. **INSERT TWO BOXES OF COPPER INSIDE THE DRA OVER THE SLOT**

2. **NOW APPLY VOLTAGE BIAS BY RIGHT CLICK AND APPLY +15 V**
3. WHEN CLICK ON VOLTAGE, THIS WINDOW COME WHERE WE ENTER VOLTAGE AND E FIELD DIRECTION

4. IN THE SAME WAY, WE APPLY ELECTRIC BIAS TO SECOND ELECTRODE
Annexure-4

Resonating Modes in Rectangular Resonators

See Fig. A4.1.

Rectangular waveguide solution:

Helmholtz equation

\[ \nabla^2 \psi + k^2 \psi = 0 \text{ (source less medium)} \]
\[ \nabla^2 \psi + k^2 \psi = -\mu j \text{ (medium with source)} \]

Maxwell’s equations

\[ \nabla \times E = -\mu \frac{\partial H}{\partial t} \]
\[ \nabla \times H = j + \frac{\partial E}{\partial t} \]

Solving LHS of both sides first

\[ \nabla \times E = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = i \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) - j \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) + k \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \]

\[ \nabla \times H = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = i \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) - j \left( \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right) + k \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \]

Comparing with RHS in both equations and getting value of \( H_x, H_y, H_z \) and \( E_x, E_y, E_z \), we get
Substituting: $-\frac{\partial}{\partial z} = \gamma$; 

$H_x = \frac{j\omega \mu}{\gamma^2 + \omega^2 \mu \epsilon} \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right)$  
(A4.1)

$H_y = \frac{j\omega \mu}{\gamma^2 + \omega^2 \mu \epsilon} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right)$  
(A4.2)

$H_z = \frac{-j\omega \mu}{\gamma^2 + \omega^2 \mu \epsilon} \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)$  
(A4.3)

$E_x = \frac{1}{j\omega \epsilon} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right)$  
(A4.4)

$E_y = \frac{1}{j\omega \epsilon} \left( \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right)$  
(A4.6)

$E_z = \frac{1}{j\omega \epsilon} \left( \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} \right)$  
(A4.7)

On looking above equations, we get that $H_z, E_z$ in 2-D Helmholtz equation
Here, \( k \) is the wave number

\[
\nabla^2 \psi + k^2 \psi = 0
\]

\( \Psi = X(x)Y(y)Z(z) \)

\[
\left( \frac{1}{X} \frac{d^2 X}{dx^2} \right) + \frac{1}{Y} \left( \frac{d^2 Y}{dy^2} \right) + \frac{1}{Z} \left( \frac{d^2 Z}{dz^2} \right) + k^2 = 0
\]

Separating the independent terms, we get

\[
\frac{1}{X} \left( \frac{d^2 X}{dx^2} \right) = -k_x^2
\]

\[
\frac{1}{Y} \left( \frac{d^2 Y}{dy^2} \right) = -k_y^2
\]

\[
\frac{1}{Z} \left( \frac{d^2 Z}{dz^2} \right) = -k_z^2
\]

\[
k^2 = k_x^2 + k_y^2 + k_z^2
\]

\[
\Psi = \{ (A \sin k_x \cdot x + B \cos k_x \cdot x)(C \sin k_y \cdot y + D \cos k_y \cdot y) \} e^{-jk_z z}
\]

Solving above function and keeping propagation in +z-direction only, we get TE mode

\[
H_z = \sum_{mn} \left\{ C_{mn} \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} \right) \right\} e^{-jk_z z}; \quad C_{mn} \text{ Fourier Coefficients } \quad (A4.8)
\]

TM mode

\[
E_z = \sum_{mn} \left\{ D_{mn} \left( \sin \frac{m\pi x}{a} \right) \left( \sin \frac{n\pi y}{b} \right) \right\} e^{-jk_z z}; \quad D_{mn} \text{ Fourier Coefficients } \quad (A4.9)
\]

These Fourier coefficients are resultant of mode amplitude and propagation constant at any instant.

Let \( \gamma = -jk_z \) and \( m, n \) are integers and \( a, b \) are dimensions;

\[
\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 = (k_z)_{mn}; \quad \text{cut off frequency}
\]

\[
k_z^2 = \omega^2 \mu \varepsilon - \left( \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right)
\]
Hence, EM wave will propagate in $z$-direction if:

$$\omega^2 \mu \varepsilon - \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 > 0$$

This gives cutoff frequency as follows:

$$\omega_c = \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2}$$

It means, waveguide will support all waves having $\omega$ greater than $\omega_c$ to propagate.

Now, rewriting $H_z$ and $E_z$

$$H_z = \sum_{mn} \left\{ C_{mn} \left( \cos \frac{m\pi x}{a} \right) \left( \cos \frac{n\pi y}{b} \right) \right\} e^{-jkz}$$

$$E_z = \sum_{mn} \left\{ D_{mn} \left( \sin \frac{m\pi x}{a} \right) \left( \sin \frac{n\pi y}{b} \right) \right\} e^{-jkz}$$

Here $C_{mn}$ and $D_{mn}$ are coefficients of $H_z$ and $E_z$ fields

$E_{ix(x,y)}$ Incident EM wave in $x$-direction;

$E_{iy(x,y)}$ Incident EM wave in $y$-direction;

$$E_{ix(x,y)} = \sum \left[ j\omega \mu D_{mn} \left( \frac{n\pi}{b} \right) + \gamma_{mn} C_{mn} \left( \frac{m\pi}{a} \right) \right] \cos \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \exp(-\gamma_{mn}z);$$

Similarly,

$$E_{iy(x,y)} = \sum \left[ j\omega \mu D_{mn} \left( \frac{m\pi}{a} \right) + \gamma_{mn} C_{mn} \left( \frac{n\pi}{b} \right) \right] \cos \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \exp(-\gamma_{mn}z);$$

On simplification

$$E_{ix(m,n)} = \frac{j\omega \mu D_{mn} \left( \frac{n\pi}{b} \right) + \gamma_{mn} C_{mn} \left( \frac{m\pi}{a} \right)}{h_{m,n}^2}$$
Similarly

\[
E_{iy(m,n)} = \frac{j \omega \mu D_{mn} \left( \frac{m \pi}{a} \right) + \gamma_{mn} C_{mn} \left( \frac{n \pi}{b} \right)}{h_{m,n}^2}
\]

\[
\left[ E_{ix(m,n)} \right] = \left[ \begin{array}{c}
\frac{m \pi \frac{\gamma_{mn}}{a}}{h_{m,n}^2} - \frac{n \pi \frac{\gamma_{mn}}{b}}{h_{m,n}^2} \\
\frac{m \pi \frac{\gamma_{mn}}{a}}{h_{m,n}^2} + \frac{n \pi \frac{\gamma_{mn}}{b}}{h_{m,n}^2}
\end{array} \right] \left[ \begin{array}{c}
C_{mn} \\
D_{mn}
\end{array} \right];
\]

we can now get the value of \( C_{mn}, D_{mn} \) after substitution of \( E_{ix(m,n)}, E_{iy(m,n)} \) values.

Where \( h_{m,n}^2 = \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \) and

\[
\gamma_{mn} = \sqrt{h_{m,n}^2 - \omega^2 \mu e}
\]

Hence, \( C_{mn} \) and \( D_{mn} \) gives us relative amplitudes of \( E_z \) and \( H_z \) fields in TM or TE modes.

Hence, we get solution of possible amplitudes and phase of wave propagating through rectangular waveguide called as modes of propagation.

Half-wave Fourier expansion in waveguide is given as follows:

\[
f_{mn} = \int_0^a \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right) dx = \int_0^b \sin \left( \frac{m \pi y}{b} \right) \sin \left( \frac{n \pi y}{b} \right) dy;
\]

even or odd terms, i.e., \( f(x) = f(-x) \) for even term (all cosine terms) or even modes.

Where \( m, m' \) and \( n, n' \geq 1 \)

\[
E_{ix(m,n)} = \frac{2}{ab} \int_0^a \int_0^b E_{ix(x,y)} \left( \cos \frac{m \pi x}{a} \right) \left( \cos \frac{n \pi y}{b} \right) dxdy
\]

\[
E_{iy(m,n)} = \frac{2}{ab} \int_0^a \int_0^b E_{iy(x,y)} \left( \sin \frac{m \pi x}{a} \right) \left( \sin \frac{n \pi y}{b} \right) dxdy
\]

Half-wave Fourier analysis will have odd or even terms, i.e., sine–sine or cosine–cosine.

If \( f(x) = f(-x) \), even harmonics will take place and only cosine terms will occur, i.e.,

\[
f(x) = \sum_{n=1}^{\infty} C_n \cos \left( \frac{n \pi x}{a} \right)
\]
where

\[ C_n = \frac{2}{a} \int_{0}^{a} f(x) \cos \left( \frac{n\pi x}{a} \right) dx \]

Similarly for odd terms, \( f(x) \neq f(-x) \);

\[ f(x) = \sum_{n=1}^{\infty} B_n \sin \left( \frac{n\pi x}{a} \right) \]

where

\[ B_n = \frac{2}{a} \int_{0}^{a} f(x) \sin \left( \frac{n\pi x}{a} \right) dx \]

Solving wave equation with boundary conditions \( E_{tan} = 0 \), we find \( E \) fields and then \( H \) fields. Now shape and size of resonator is given, wave equation shall give solution of characteristic frequencies \( \omega(mnp) \) called eigenvalues or eigenfrequencies of e-m oscillations of cavity resonator.

Lowest eigenfrequency \( \omega_1 \) is \( \xi_1 \); where \( l \) is the dimension of resonator.

Higher frequency \( (\omega \gg \xi_1) \); then \( \omega \) is \( \frac{\omega_0^2}{2\pi c^2} \).

Hence, it depends on volume and net on shape of resonator.

For resonator:

\[
\sum_{mnr} f_{mnr} \sin \left( \frac{\pi m x}{a} \right) \sin \left( \frac{\pi n y}{b} \right) \sin \left( \frac{\pi r z}{d} \right) = f(x, y, z)
\]

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z) + k^2 \psi(x, y, z) = f(x, y, z);
\]

Helmholtz equation

\[
\psi(x, y, z) = \sum_{mnr} C_{mnr} \sin \left( \frac{\pi m x}{a} \right) \sin \left( \frac{\pi n y}{b} \right) \sin \left( \frac{\pi r z}{d} \right)
\]

\[
\left[ k^2 - \pi^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{r^2}{d^2} \right) \right] C_{mnr} = f_{mnr}
\]

Amplitude coefficient, \( C_{mnr} = \frac{f_{mnr}}{k^2 - \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{r^2}{d^2} \right) / \omega_{mnr}} \)
\[ k = \omega \sqrt{\mu \epsilon} \]

\[ \omega = \omega(mnp) + \delta; \text{ where } \delta \text{ small deviation and } r \text{ is different from } p. \]

Hence,

\[ C_{mnr} = \frac{f_{mnr}}{(\omega(mnp) + \delta)^2 - \omega(mnr)^2} \]

\[ = \frac{f_{mnr}}{\delta(\omega(mnp) - \omega(mnr))} \]

**A4-3 Solution of Single-String Resonator**

\[ x''(t) + \omega_0^2 x(t) = B e^{i\omega t} \]

\[ x(t) = A e^{i\omega t} \]

\[ (\omega_0^2 - \omega^2)A = B \]

Hence,

\[ A = \frac{B}{\omega_0^2 - \omega^2} \]

\[ x(t) = \frac{B e^{i\omega t}}{\omega_0^2 - \omega^2}, \text{ if } \omega_0 = \omega; \text{ then } x(t) \text{ will be } \infty \]

Now, \( \omega = \omega_0 + \delta \) when \( \delta \) is small deviation

\[ = \frac{B e^{i\omega t}}{(\omega_0 + \omega)(\omega_0 - \omega)} \]

Hence, the solution of spring resonator is in one dimension

\[ = \frac{B e^{i\omega t}}{\delta(2\omega_0)} \]
\( \left( \frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) u(x,t) = 0 \); at boundaries

\[ u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0 \]

Taking Fourier transform of above equation

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\omega^2}{c^2} \right) \hat{u}(x, \omega) = 0 \]

Writing above terms in sine and cosine form, we have

\[ A \sin \left( \frac{\omega x}{c} \right) + B \cos \left( \frac{\omega x}{c} \right) = 0 \]

\[ \hat{u}(0, \omega) = 0 \]

\[ \hat{u}(L, \omega) = 0 \]

\( \sin \left( \frac{\omega L}{c} \right) = 0 \); Hence \( kL = n\pi \); sine values to be zero.

\( \omega = kc = \frac{n\pi}{L} \), when \( n = 1, 2, 3 \) where \( k = \omega/c \);

when \( 2L \), it is fundamental frequency \( \omega_1 \)
when \( L \), the frequency is \( 2\omega_1 \)
when \( 2L/3 \), the frequency \( 3\omega_1 \);
which can be generalized as:

\[ \sum_n C(n) \sin \left( \frac{n\pi x}{L} \right) \]

**A-4 Solution of Two-Dimensional Resonator**

General Helmholtz equation is given below (Fig. A4.2):

**Fig. A4.2** Rectangular resonator
\[
\frac{\partial^2 \psi(x, y, t)}{\partial x^2} + \frac{\partial^2 \psi(x, y, t)}{\partial y^2} - \frac{1}{c^2} \frac{\partial^2 \psi(x, y, t)}{\partial t^2} = 0
\]

Applying boundary conditions

\[
\psi(0, y, t) = \psi(a, y, t) = 0
\]

\[
\psi(x, 0, t) = \psi(x, b, t) = 0
\]

Let input excitation be some tension \( T \)

\[
\sigma \mathrm{d}x \mathrm{d}y \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial x} \left( T \frac{\mathrm{d}y}{\mathrm{d}x} \frac{\partial \psi}{\partial x} \right) \mathrm{d}x + \frac{\partial}{\partial y} \left( T \frac{\mathrm{d}x}{\mathrm{d}y} \frac{\partial \psi}{\partial y} \right) \mathrm{d}y
\]

\[
\frac{Y''}{Y} = -k_Y^2; \quad \frac{X''}{X} = -k_X^2;
\]

Now from Helmholtz equation:

Using separation of variables:

\[
\psi(x, y, t) = X(x)Y(y)T(t)
\]

\[
-\omega^2 = \frac{T''(t)}{T(t)} = c^2 \left( \frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} \right)
\]

Let

\[
X(x) = \sin(k_x x)
\]

\[
Y(y) = \sin(k_y y)
\]
\[ k_x^2 + k_y^2 = \frac{\omega^2}{c^2} \]

where \( k_x \) and \( k_y \) can be

\[ k_x = \frac{m\pi}{a}; \quad k_y = \frac{n\pi}{b} \]

Equation (A4.1) can be written as \( \omega(mn) = c\pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \).

From Fourier series analysis

\[
\omega(m, n) = \sum_{mn=1}^{\infty} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left[ C(m, n)e^{j\omega(mn)t} + D(m, n)e^{-j\omega(mn)t} \right]
\]

(A4.12)

At \( t = 0 \):
\[ \psi(x, y, 0) = \psi_0(x, y) \]

On differentiating equation \( \psi_0(x, y) \), we get \( \psi_1(x, y, 0) = \psi_1(x, y) \).

When \( t \neq 0 \);

\[
\psi_0(x, y) = \sum_{mn=1}^{\infty} (C(m, n) + D(m, n)) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)
\]

(A4.13)

\[
\psi_1(x, y) = \sum_{mn=1}^{\infty} j\omega(m, n)(C(m, n) - D(m, n)) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)
\]

(A4.14)

\[
\frac{2}{\sqrt{ab}} \int_{0}^{a} \int_{0}^{b} \psi_0(x, y) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \, dx \, dy = [C(m, n) + D(m, n)]
\]

(A4.15)

Similarly,

\[
\frac{2}{\sqrt{ab}} \int_{0}^{a} \int_{0}^{b} \psi_1(x, y) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \, dx \, dy = [C(m, n) - D(m, n)]
\]

(A4.16)

Hence, obtain the value of \( C(m, n), D(m, n) \) from Eqs. (A4.3) and (A4.4)
\[ [C(m, n), D(m, n)] = \frac{1}{\sqrt{ab}} \left[ \int \int \psi_0(x, y) \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \, dx \, dy \right. \\
+ \left. \frac{1}{j\omega(m, n)} \int \psi_1(x, y) \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \, dx \, dy \right] \quad (A4.17) \]

Hence, from Eq. (A4.17),

\[ \psi_0(x, y) = A \sin \left( \frac{m_0\pi x}{a} \right) \sin \left( \frac{n_0\pi y}{b} \right) \]
\[ \psi_1(x, y) = B \sin \left( \frac{m_0\pi x}{a} \right) \sin \left( \frac{n_0\pi y}{b} \right) \]

due to force, perturbation occurs (Fig. A4.3)

Solving equation (A4.17)

\[ (C(m, n), D(m, n)) = \delta[m - m_0]\delta[n - n_0] \]
\[ = \left( \frac{A}{\sqrt{ab}} \left( \frac{a}{2} \times \frac{b}{2} \right) \pm \left( \frac{1}{\sqrt{ab} j\omega(m_0, n_0)} \left( \frac{a}{2} \times \frac{b}{2} \right) \right) \right) \]
\[ (C(m, n), D(m, n)) = \sqrt{ab} \left( \frac{A}{4} \pm \frac{j \cdot B}{4} \right) \frac{\sqrt{ab}}{4} (A \pm jB) \delta[m - m_0]\delta[n - n_0] \]

\[ \psi(x, y, t) = \frac{\sqrt{ab}}{2} \text{Re}(A - jB) \sin \left( \frac{m_0\pi x}{a} \right) \sin \left( \frac{n_0\pi y}{b} \right) e^{j\omega(m_0, n_0)t} \]

Hence, we complete solution of two-dimensional resonator.

\[ \psi(x, y, t) = \frac{\sqrt{ab}}{2} (A \cos(\omega(m_0, n_0)t) + B \sin(\omega(m_0, n_0)t)) \sin \left( \frac{m_0\pi x}{a} \right) \sin \left( \frac{n_0\pi y}{b} \right) \quad (A4.18) \]

**Fig. A4.3** Deformation due to excitation \( T(x, y) \)
Alternate method

\[ m = 2b \sin \theta; \]
\[ n = 2a \cos \theta; \]

Dividing both sides of above equations by \( 2a \) and \( 2b \) and adding them gives us

\[ \frac{1}{\lambda_2} = \frac{n^2}{4a^2} + \frac{m^2}{4b^2}; \quad \text{where} \quad k^2 = k_x^2 + k_y^2 \]

Thus, resonant frequency of resonator can be determined.

Half-wave Fourier analysis:

\[ f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos \left( \frac{2n\pi}{a} x \right) + b_n \sin \left( \frac{2n\pi}{a} x \right) \right] \]

\[ a_n = \frac{2}{a} \int_{0}^{a} f(x) \cos \left( \frac{2n\pi}{a} x \right) dx \]

\[ b_n = \frac{2}{a} \int_{0}^{a} f(x) \sin \left( \frac{2n\pi}{a} x \right) dx \]

Half-wave Fourier analysis will have odd or even terms, i.e., sine–sine or cosine–cosine.

If \( f(x) = f(-x) \), even harmonics will take place and only cosine terms will occur, i.e.,

\[ f(x) = \sum_{n=1}^{\infty} C_n \cos \left( \frac{n\pi x}{a} \right) \]

where

\[ C_n = \frac{2}{a} \int_{0}^{a} f(x) \cos \left( \frac{n\pi}{a} x \right) dx \]

Similarly for odd terms, \( f(x) \neq f(-x) \);

\[ f(x) = \sum_{n=1}^{\infty} B_n \sin \left( \frac{n\pi x}{a} \right) \]
where

\[ B_n = \frac{2}{a} \int_0^a f(x) \sin \left( \frac{n\pi x}{a} \right) dx \]

**Spectral resolution of EM waves**

Every wave can be subjected to the process of spectral resolution, i.e., can be represented as a superposition of monochromatic waves of various frequencies. The character of this expansion varies according to the character of the time dependence of the fields.

One category consists of those cases, where the expansion contains frequencies forming a discrete sequence of values. The simplest case of this type arises in the resolution of a purely periodic field. This is the usual expansion in Fourier series. It contains the frequencies which are integral multiples of the “fundamental” frequency \( \omega_0 = \frac{2\pi}{T} \), where \( T \) is the period of the field. We therefore write it in the form as follows:

\[ f = \sum_{n=-\infty}^{\infty} f_n e^{i\omega_0 nt} \]

(where \( f \) is any of the quantities describing the field). The quantities \( f_n \) are defined in terms of the function \( f \) by the integrals

\[ f_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)e^{i\omega_0 t} dt. \]

Because \( f(t) \) must be real

\[ f_n = f_n^*, \]

in more complicated cases, the expansion may contain integral multiples of several different incommensurable fundamental frequencies. When the sum is squared and averaged over the time, the product of terms with different frequencies is given zero because they contain oscillating factors.

Only terms of the form \( f_n f_{-n} = |f_n|^2 \) remain. Thus, the average of the square of the field, i.e., the average intensity of the wave, is the sum of the intensities of its monochromatic components. \( \overline{f^2} = \sum_{n=-\infty}^{\infty} |f_n|^2 = 2 \sum_{n=1}^{\infty} |f_n|^2 \); where it is assumed that the average of the function \( f \) over a period is zero. Another category consists of fields which are expandable in a Fourier integral containing a continuous distribution of different frequencies. For this to be possible, the function \( f(t) \) must satisfy certain definite conditions; usually we consider functions which vanish for \( t \to \pm \infty \).
Similarly, $f_{-\omega} = f^*_\omega$. Let us express the total intensity of the wave, i.e., the integrals of $f^2$ over all time, in terms of the intensity of the Fourier components. Now, we have

$$
\int_{-\infty}^{\infty} f^2 \, dt = \int_{-\infty}^{\infty} \left\{ f \int_{-\infty}^{\infty} f_{\omega} e^{-j\omega t} \, d\omega \frac{d\omega}{2\pi} \right\} \, dt = \int_{-\infty}^{\infty} \left\{ f_{\omega} \int_{-\infty}^{\infty} f_{-\omega} e^{j\omega t} \, d\omega \frac{d\omega}{2\pi} \right\} \, dt
$$

$$
= \int_{-\infty}^{\infty} f_{\omega} f_{-\omega} \frac{d\omega}{2\pi},
$$
or

$$
\int_{-\infty}^{\infty} f^2 \, dt = \int_{-\infty}^{\infty} |f_{\omega}|^2 \frac{d\omega}{2\pi} = 2 \int_{0}^{\infty} |f_{\omega}|^2 \frac{d\omega}{2\pi}.
$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_{\omega} e^{-j\omega t} \, d\omega; \text{ where the Fourier components are given in terms of the function } f(t) \text{ by the integrals, } f_{\omega} = \int_{-\infty}^{\infty} f(t) e^{j\omega t} \, dt.$$

**Power and Energy Signals:**

Let $x(t)$ is the input signal, i.e., voltage signal. As per Parseval’s power theorem, energy associated with this signal be

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt; \text{ in time domain}
$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega; \text{ in frequency domain}
$$

The amount of energy radiated by this signal, when applied across Antenna having radiation resistance $R_r$ shall be

$$E = \frac{1}{R_r} \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi R_r} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega
$$

Now if input signal is $x(t)$ having current signal

$$E = R_r \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{R}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega.$$
ESD energy spectral density; energy spread per unit volume across 1 Ω resister

$$\text{ESD} = |X(\omega)|^2$$

Discrete Fourier transform (DFT) in time domain into frequency domain spectral analysis

$$x(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}; \quad k = 0, 1, \ldots, N - 1.$$  

$$X(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{-j2\pi kn/N}; \quad n = 0, 1, 2, \ldots, N - 1.$$ 

$X(n)$ finite sequence.

DFT has finite length $N$, period $N$

$$\psi(\theta, \phi) = k(\vec{r}_n \cdot \vec{r}_0) = (n - 1)kd \sin \theta$$

$$E(\sin \theta) = \sum_{n=1}^{N} e^{i(n-1)(kd \sin \theta)}$$
Annexure-5

Resonant Mode Generation and Control in RDRA

In this annexure, resonant modes TE and TM have been generated inside RDRA whose dimensions are $a$, $b$, and $d$. Two parallel plates are attached along with dielectric slab in between these plates to RDRA. This slab forms non-resonant part and RDRA is main resonant. This is shown in Fig. A5.1a, b. The resonant modes dominant and higher-order modes are being generated by maintaining appropriate aspect ratio of RDRA. Then, the non-resonant slab inductance and capacitance is introduced into main RDRA. This lumped value of inductance and capacitance is seen in the resonant frequency.

(a) The increase in the length of internal strip introduce shift in the higher resonant modes frequency, as they shift toward lower side and vice versa. Hence, resonant frequency is reduced.

(b) On the other side, increase in the length of external strip introduces shift in the lower resonant modes frequency shifts toward higher side and vice versa. Hence, frequency is increased with strip length.

(c) Increase in spacing between parallel plates introduces the combined effect of internal as well as external strip length variation, i.e., higher- and lower-order resonant modes shift toward the centre frequency which can be seen as mode-merging effect.

(d) Finally, the effect of placing a lumped varactor diode between parallel plates is seen. The increase in the capacitance value of lumped varactor diode causes shift in the higher resonant frequency toward lower resonant frequency side.

These results have been investigated using HFSS and they shown with $S_{11}$ results along with each RDRA model. By varying length, “$a$,” width “$b$,” and height “$d$” of RDRA modes are generated. The internal strip, external strip, and dielectric slab and dielectric constant provided several degrees of freedom in the RDRA design. This has extended the control on the amount of coupling, hence resonant frequency. This shall have large impact on resonant modes, compactness of antenna, radiation pattern, and polarization.
**A5.1 Effect of Change of Aspect Ratio (a/b) and (a/d) of RDRA on Resonant Modes**

See Fig. A5.2.

**A5.2 Effect of Strip Length, Separation, $\varepsilon_r$ on the Modes Developed Inside the RDRA**

See Fig. A5.3 and Table A5.1.
**Fig. A5.2**  
(a) Higher-order modes generated in RDRA with square base.  
(b) Higher-order modes generated in the rectangular base RDRA.
A5.2(a) Effect of Internal Strip Length Variation on Resonant Modes Inside RDRA

The effect of the internal strip length is seen on resonance frequency and resonant modes of RDRA.

The reflection coefficient plot can be seen for the possible changes as given in Fig. A5.4a.

The effective electrically length of RDRA is changed by introducing change in length of internal strip as given below.

Changing the effective dimension of the dielectric resonator changes the resonant frequency.

**Table A5.1** Effect of strip length

<table>
<thead>
<tr>
<th>Structure</th>
<th>x (mm)</th>
<th>y (mm)</th>
<th>z (mm)</th>
<th>εr</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRA</td>
<td>4.6</td>
<td>9</td>
<td>10.8</td>
<td>9.8</td>
</tr>
<tr>
<td>substrate</td>
<td>20</td>
<td>30</td>
<td>0.8</td>
<td>2.2</td>
</tr>
<tr>
<td>Micro-strip</td>
<td>2.4 (width)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rect. SLAB</td>
<td>1</td>
<td>9</td>
<td>10.8</td>
<td>1</td>
</tr>
<tr>
<td>External strip</td>
<td>2.4</td>
<td></td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>Internal strip</td>
<td>2.4</td>
<td></td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
Fig. A5.4  

(a) External strip (fixed) = 10.5 mm and variation in internal strip from (2 mm).

(b) External strip (fixed) = 10.5 mm and variation in internal strip from (2.5 mm).

(c) External strip (fixed) = 10.5 mm and variation in internal strip from (3 mm)
A5.2(b) Effect of External Strip Length Variation on Resonant Modes Inside RDRA

The effect of the external strip length on resonance frequency and resonant modes is shown in Fig. A5.5b. Internal strip (fixed) = 3 mm and variation in external strip from 10.5, 7, 0 mm is investigated. Contrary to the previous case, the third resonance stays mainly fixed at the same frequency, while the first and second resonant frequencies are considerably decreased with increasing external strip length.

A5.2(c) Effect of Separation Width Between the Two Parallel Standing Strips and \( \varepsilon_r \)

The effect of the spacing between parallel plates and permittivity of the rectangular slab between parallel plates is seen on resonance frequency and modes (Figs. A5.6 and A5.7).

- The separation width variation ranges as 0.5, 1.5, and 2.5 mm \([\varepsilon_r = 1, \text{external strip} = 10.5 \text{ mm and inner strip} = 3.5 \text{ mm (fixed)}] \).
- we will change the variable separation Width (0.5, 1.5, 2.5) for \( \varepsilon_r = 2 \) keeping external strip = 10.5 mm, inner strip = 3.5 mm constant (Fig. A5.8).
Fig. A5.5  

a  Internal strip (fixed) = 3 mm and variation in external strip from (0 mm).  
b  Internal strip (fixed) = 3 mm and variation in external strip from (7 mm).  
c  Internal strip (fixed) = 3 mm and variation in external strip from (10.5 mm)
Fig. A5.5  (continued)

Fig. A5.6  RDRA with separated plates
Fig. A5.7  
(a) Separation width (0.5 mm) for $\varepsilon_r = 1$, external strip = 10.5 mm, inner strip = 3.5 mm.
(b) Separation width (1.5 mm) for $\varepsilon_r = 1$, external strip = 10.5 mm, inner strip = 3.5 mm.
(c) Separation width (2.5 mm) for $\varepsilon_r = 1$, external strip = 10.5 mm, inner strip = 3.5 mm.
Fig. A5.8  a Separation width (0.5) for \( \varepsilon_r = 2 \), external strip = 10.5 mm, inner strip = 3.5 mm. b Separation width (1.5) for \( \varepsilon_r = 2 \), external strip = 10.5 mm, inner strip = 3.5 mm. c Separation width (2.5) for \( \varepsilon_r = 2 \), external strip = 10.5 mm, inner strip = 3.5 mm
A.5.2(d) Effect of Variable Capacitance (Varactor Diode) in Between the Plate

The effect of the varactor diode capacitance placed in between the parallel standing strips is seen. The resonant modes get shifted lower side (Fig. A5.9).

The separation width = 1.0, $\varepsilon_r = 1$, external strip = 10.6 mm, inner strip = 3.0 mm, varactor diode (variation from 1 to 5 $\mu$F with step of 1 $\mu$F) at position ($z = 2.3$) in vertical direction. The resulting effect is shown in Fig. A5.10.

A5.3 Designing Steps

HFSS steps_Project1

1. Open HFSS.
2. Create file name project1.
3. Define in the Cartesian co-ordinate system origin as ($x = 0$, $y = 0$, $z = 0$).
4. Choose 3-D rectangular box for substrate by defining the desired substrate material and its dimensions such as (RT Duroid and $x = 20$ mm, $y = 30$ mm, $z = 0.8$ mm).
5. Create DRA structure with desired material and dimensions on the substrate top surface (e.g., If substrate dimension from origin was 0.8 mm in z-direction. Then choose DRA #d dimension keeping substrate dimension as reference).
6. Create two parallel strips adjacent to DRA above the substrate surface with rectangular slab in between them keeping substrate dimension as reference.

7. Apply micro-strip feeding to the DRA structure by defining the micro-strip port with appropriate length and width for impedance matching (e.g., wave port) assigning in the desired direction of input excitation.

8. Variation in height of external strip keeping the internal strip height fixed and vice versa.

9. Effect of the permittivity of rectangular slab can be seen by varying the material property and thickness of the slab in between two fixed parallel plates.

10. Placing a lumped capacitor between two parallel standing strips with desired value (e.g., 2 μF) and perform parametric analysis for variable capacitance value of lumped element.

11. Performing the simulation for the steps 8, 9, 10 mentioned above separately and for mode analysis of DRA which give modal frequency response and effect of the variation of radiation parameters associated with DRA and non-resonant slab with parallel standing strip geometry.

12. Analysis of the simulated structure can be performed by taking various response quantities such as S11, radiation pattern, gain, and field distribution.

13. The above mechanism can also be validated in RDRA by VNA with anechoic chamber on prototype model after structure is simulated.

**Fig. A5.10** Variation of resonant frequency with lumped capacitance
Cartesian, Cylindrical, and Spherical Coordinate System

There are three different coordinate systems, i.e., Cartesian, cylindrical, and spherical systems. Cartesian are \((x, y, z)\), cylindrical are \((\rho, \phi, z)\), and spherical are \((r, \theta, \phi)\) representation (Figs. A6.1 and A6.2).

<table>
<thead>
<tr>
<th>(a) Cylindrical to cartesian</th>
<th>(b) Cartesian to cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = \rho \cos \phi)</td>
<td>(\rho = \sqrt{x^2 + y^2})</td>
</tr>
<tr>
<td>(Y = \rho \sin \phi)</td>
<td>(\phi = \tan^{-1} \frac{y}{x})</td>
</tr>
<tr>
<td>(Z = z)</td>
<td>(Z = z)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Spherical to cartesian</th>
<th>(d) Cartesian to cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = r \sin \theta \cos \phi)</td>
<td>(r = \sqrt{x^2 + y^2 + z^2})</td>
</tr>
<tr>
<td>(Y = r \sin \theta \sin \phi)</td>
<td>(\theta = \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right))</td>
</tr>
<tr>
<td>(Z = r \cos \theta)</td>
<td>(\phi = \tan^{-1} \left( \frac{y}{x} \right))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(e) Cylindrical to spherical</th>
<th>(f) Spherical to cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = \sqrt{\rho^2 + z^2})</td>
<td>(\rho = r \sin \theta)</td>
</tr>
<tr>
<td>(\theta = \tan^{-1} \left( \frac{\rho}{z} \right))</td>
<td>(\phi = \phi)</td>
</tr>
<tr>
<td>(\phi = \phi)</td>
<td>(z = r \cos \theta)</td>
</tr>
</tbody>
</table>

1. **DEL** \((\nabla)\) derivation in cylindrical system:

The Cartesian \(\nabla\) (Del) is given as follows:

\[
\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}
\]
Cylindrical $\nabla$ (Del) is given below:

$$\nabla = \hat{\rho} \frac{\partial}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{z} \frac{\partial}{\partial z}$$

Converting differential operators in terms of the cylindrical system by chain rule:

$$\frac{\partial}{\partial x} = \left( \frac{\partial}{\partial \rho} \frac{\partial \rho}{\partial x} \neq \frac{\partial}{\partial \phi} \frac{\partial \phi}{\partial x} \neq \frac{\partial}{\partial z} \frac{\partial z}{\partial x} \right) \frac{\partial}{\partial x}$$

$$\frac{\partial \rho}{\partial x} = \frac{\partial}{\partial x} \left( \sqrt{x^2 + y^2} \right) = \frac{1}{z} \frac{2x}{\sqrt{x^2 + y^2}} = \frac{x}{\sqrt{x^2 + y^2}} = \cos \phi$$

$$\rho = \sqrt{x^2 + y^2}$$

Hence,

$$\cos \phi = \frac{x}{\rho}$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \left[ \tan^{-1} \left( \frac{y}{x} \right) \right] = \frac{1}{1 + \frac{y^2}{x^2}} \left( \frac{x(0) - y1}{x^2} \right) = \frac{x^2}{x^2 + y^2} = \frac{-y}{x^2}$$

$$\frac{\partial \phi}{\partial x} = \frac{-1}{\sqrt{x^2 + y^2}} \frac{y}{\sqrt{x^2 + y^2}} = \frac{-1}{\rho} \sin \phi$$

$$\frac{\partial \phi}{\partial x} = 0 (\because \text{z is the same as in Cartesian system it doesn't depend on x})$$
As per chain rule

Thus, we have

\[
\dot{x} \frac{\partial}{\partial x} = \dot{x} \left( \frac{\partial P}{\partial p} \frac{\partial}{\partial x} + \frac{\partial \phi}{\partial \phi} \frac{\partial}{\partial x} + \frac{\partial z}{\partial \phi} \right) \\
= \dot{x} \left( \frac{\partial}{\partial \phi} \cos \phi + \frac{\partial}{\partial \phi} \left( -\frac{1}{p} \sin \phi \right) + 0 \right) \\
\]

(A6.1)

Using the same technique to convert the differential for \( y \):

\[
\dot{y} \frac{\partial}{\partial y} = \dot{y} \left( \frac{\partial P}{\partial \rho} \frac{\partial}{\partial y} + \frac{\partial \phi}{\partial \phi} \frac{\partial}{\partial y} + \frac{\partial z}{\partial y} \right) \\
\frac{\partial P}{\partial y} = \frac{\partial \phi}{\partial \phi} \sqrt{x^2 + y^2} = \frac{2y}{2\sqrt{x^2 + y^2}} = \frac{y}{\sqrt{x^2 + y^2}} = \sin \phi \\
\frac{\partial \phi}{\partial \phi} \frac{\partial}{\partial y} (\tan^{-1}(y)x) = \frac{1}{1 + \frac{x^2}{y^2}} = \frac{y}{\sqrt{x^2 + y^2}} = \frac{1}{\rho} \cos \phi \\
\frac{\partial z}{\partial y} = 0
\]

Thus,

\[
\dot{y} \frac{\partial}{\partial y} = \dot{y} \left( \frac{\partial \rho}{\partial \rho} \frac{\partial}{\partial y} + \frac{\partial \phi}{\partial \phi} \frac{\partial}{\partial y} + \frac{\partial z}{\partial y} \right) \\
\dot{y} \left( \sin \phi \frac{\partial}{\partial \rho} + \frac{1}{\rho} \cos \phi \frac{\partial}{\partial \phi} + 0 \right) \\
\]

(A6.2)

Finally, since \( z \) is not transformed between coordinate systems

\[
\frac{\partial}{\partial z} = \frac{\partial}{\partial z} \\
\nabla = \dot{x} \frac{\partial}{\partial x} + \dot{y} \frac{\partial}{\partial y} + \dot{z} \frac{\partial}{\partial z} \\
\nabla = \dot{x} \left( \cos \phi \frac{\partial}{\partial \rho} - \frac{1}{\rho} \sin \phi \frac{\partial}{\partial \phi} \right) + \dot{y} \left( \sin \phi \frac{\partial}{\partial \rho} + \frac{1}{\rho} \cos \phi \frac{\partial}{\partial \phi} \right) + \dot{z} \frac{\partial}{\partial z}
\]

Cylindrical

\[
\nabla = (\dot{x} \cos \phi + \dot{y} \sin \phi) \frac{\partial}{\partial \rho} + \frac{1}{\rho} (\dot{y} \cos \phi - \dot{x} \sin \phi) \frac{\partial}{\partial \phi} + \dot{z} \frac{\partial}{\partial z} \\
\]

(A6.4)
Hence, definition to cylindrical unit vector is given as follows:

\[
\hat{p} = \hat{x} \cos \phi + \hat{y} \sin \phi = \hat{p} \\
\hat{\phi} = \hat{x} \sin \phi + \hat{y} \cos \phi = \hat{\phi} \\
\hat{z} = \hat{z} = \hat{z}
\]

Thus, Del cylindrical can be written as follows:

\[
\nabla = \hat{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho} \hat{\phi} \frac{\partial}{\partial \phi} + \hat{z} \frac{\partial}{\partial z}
\]

which is the desired solution of \(\nabla\) in cylindrical coordinates.

2. **DEL \((\nabla)\) expression as spherical system** (Figs. A6.3, A6.4 and A6.5):

<table>
<thead>
<tr>
<th>Spherical to Cartesian</th>
<th>Cartesian to Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = r \sin \theta \cos \phi)</td>
<td>(r = \sqrt{x^2 + y^2 + z^2})</td>
</tr>
<tr>
<td>(Y = r \sin \theta \sin \phi)</td>
<td>(\theta = \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right))</td>
</tr>
<tr>
<td>(Z = r \cos \theta)</td>
<td>(\phi = \tan^{-1} \left( \frac{Y}{X} \right))</td>
</tr>
</tbody>
</table>

\[
\nabla = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}
\]

(A6.5)

\[
\hat{x} \frac{\partial}{\partial x} = \hat{x} \left[ \frac{\partial}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial x} + \frac{\partial}{\partial \phi} \frac{\partial \phi}{\partial x} \right]
\]

(A6.6)

---

*Fig. A6.3  Spherical system*
Fig A6.4  Spherical components

\[ y \frac{\partial}{\partial y} = y \left[ \frac{\partial \theta}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial \theta}{\partial \theta} \frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial \phi} \frac{\partial \phi}{\partial y} \right] \]  \hfill (A6.7)

\[ z \frac{\partial}{\partial z} = z \left[ \frac{\partial \theta}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial \theta}{\partial \theta} \frac{\partial \theta}{\partial z} + \frac{\partial \theta}{\partial \phi} \frac{\partial \phi}{\partial z} \right] \]  \hfill (A6.8)

Fig. A6.5  Spherical subcomponents
Now, partially differentiate $r$ with respect to $x$

\[ \frac{\partial r}{\partial x} = \frac{\partial}{\partial x} \sqrt{x^2 + y^2 + z^2} = \frac{2x}{2 \sqrt{x^2 + y^2 + z^2}} = \frac{x}{\sqrt{x^2 + y^2 + z^2}} = \frac{r \sin \theta \cos \phi}{r} = \sin \theta \cos \phi \]  

(A6.9)

Similarly partially differentiate $r$ with respect to $y$

\[ \frac{\partial r}{\partial y} = \frac{\partial}{\partial y} \sqrt{x^2 + y^2 + z^2} = \frac{2y}{2 \sqrt{x^2 + y^2 + z^2}} = \frac{y}{\sqrt{x^2 + y^2 + z^2}} = \frac{r \sin \theta \sin \phi}{r} = \sin \theta \sin \phi \]  

(A6.10)

Partially differentiate $r$ with respect to $z$

\[ \frac{\partial r}{\partial z} = \frac{\partial}{\partial z} \sqrt{x^2 + y^2 + z^2} = \frac{2z}{2 \sqrt{x^2 + y^2 + z^2}} = \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{r \cos \theta}{r} = \cos \theta \]  

(A6.11)
Partially differentiate $\theta$ with respect to $x$

$$\frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x} \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$= \frac{1}{1 + \frac{x^2 + y^2}{z^2}} \frac{2x}{2\sqrt{x^2 + y^2}}$$

$$= \frac{x}{z^2 + x^2 + y^2 \sqrt{x^2 + y^2}}$$

$$= \frac{(\sqrt{x^2+y^2})}{z} (x^2 + y^2 + z^2)$$

$$= \frac{r \sin \theta \cos \phi}{r^2 \tan \theta}$$

$$= \frac{\cos \theta \cos \phi}{r}$$

(A6.12)

Partially differentiate $\theta$ with respect to $y$

$$\frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y} \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$= \frac{1}{1 + \frac{x^2 + y^2}{z^2}} \frac{2y}{2\sqrt{x^2 + y^2}}$$

$$= \frac{y}{z^2 + x^2 + y^2 \sqrt{x^2 + y^2}}$$

$$= \frac{(\sqrt{x^2+y^2})}{z} (x^2 + y^2 + z^2)$$

$$= \frac{r \sin \theta \sin \phi}{r^2 \tan \theta}$$

$$= \frac{\cos \theta \sin \phi}{r}$$

(A6.13)
Partially differentiate $\theta$ with respect to $z$

$$
\frac{\partial \theta}{\partial z} = \frac{\partial}{\partial z} \tan^{-1} \left( \frac{\sqrt{x^2 + y^2}}{z} \right)
$$

$$
= -\frac{1}{1 + \frac{x^2 + y^2}{z^2}} \frac{\sqrt{x^2 + y^2}}{z^2}
$$

$$
= -\frac{z}{(x^2 + y^2 + z^2)} \left( \frac{\sqrt{x^2 + y^2}}{z} \right)
$$

$$
= -\frac{\sin \theta}{r}
$$

(A6.14)

Partially differentiate $\phi$ with respect to $x$

$$
\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \tan^{-1} \left( \frac{y}{x} \right)
$$

$$
= \frac{1}{1 + \frac{y^2}{x^2}} \left[ x(0) - y(1) \right]
$$

$$
= \frac{x^2}{x^2 + y^2} \left[ -\frac{y}{x^2} \right]
$$

$$
= -\frac{y}{x^2 + y^2}
$$

$$
= -\frac{\sin \phi}{r \sin \theta}
$$

(A6.15)

Partially differentiate $\phi$ with respect to $y$

$$
\frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y} \tan^{-1} \left( \frac{y}{x} \right)
$$

$$
= \frac{1}{1 + \frac{y^2}{x^2}} \left[ 1 \right]
$$

$$
= \frac{x}{x^2 + y^2}
$$

$$
= \frac{\cos \phi}{r \sin \theta}
$$

(A6.16)
Partially differentiate $\phi$ with respect to $z$

$$\frac{\partial \phi}{\partial z} = \frac{\partial}{\partial z} \tan^{-1}\left(\frac{y}{x}\right) = 0$$  \hspace{1cm} (A6.17)


$$\hat{x} \frac{\partial}{\partial x} = \hat{x}\left[\frac{\partial}{\partial r} \sin \theta \cos \phi + \frac{\partial}{\partial \theta} \frac{\cos \theta \cos \phi}{r} - \frac{\sin \phi}{r \sin \theta} \frac{\partial}{\partial \phi}\right]$$  \hspace{1cm} (A6.18)

$$\hat{y} \frac{\partial}{\partial y} = \hat{y}\left[\frac{\partial}{\partial r} \sin \theta \sin \phi + \frac{\partial}{\partial \theta} \frac{\cos \theta \sin \phi}{r} + \frac{\cos \phi}{r \sin \theta} \frac{\partial}{\partial \phi}\right]$$  \hspace{1cm} (A6.19)

$$\hat{z} \frac{\partial}{\partial z} = \hat{z}\left[\frac{\partial}{\partial r} \cos \theta - \frac{\partial}{\partial \theta} \frac{\sin \theta}{r}\right]$$  \hspace{1cm} (A6.20)

Put Eqs. (A6.13), (A6.14) and (A6.15) in Eq. (A6.1).

And by using original definition to Spherical unit vector,

$$\hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$$

$$\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$$

$$\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$$

We get

$$\vec{\nabla} = \frac{\hat{r}}{r} \frac{\partial}{\partial r} + \frac{\hat{\theta}}{r \sin \theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{r \sin \theta} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$$

Fig. A6.6  $E$ and $H$ fields pattern in RDRA
Hence \( \vec{V} \) from Cartesian to spherical converted.

3. **E and H fields in RDRA**

Fields converting into TE and TM modes inside rectangular DRA (Fig. A6.6).

4. **Transcendental equation solution using MATLAB programs (simulated rectangular DRA)** (Fig. A6.7; Table A6.1).

### Table A6.1 Transcendental equation solution

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Permittivity</th>
<th>Dimension ((a\text{length}) \times b\text{ (width)} \times d\text{ (depth)}) (mm)</th>
<th>Resonant frequency</th>
<th>Effective width ((b'))</th>
<th>Multiple factor</th>
<th>Percentage change in width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>10.0</td>
<td>(14.3 \times 25.4 \times 26.1)</td>
<td>3.5</td>
<td>34.22</td>
<td>1.3474</td>
<td>34.7381</td>
</tr>
<tr>
<td>2.</td>
<td>10.0</td>
<td>(14 \times 8 \times 8)</td>
<td>5.5</td>
<td>14.13</td>
<td>1.7665</td>
<td>76.6535</td>
</tr>
<tr>
<td>3.</td>
<td>10.0</td>
<td>(15.24 \times 3.1 \times 7.62)</td>
<td>6.21</td>
<td>8.33</td>
<td>2.8872</td>
<td>168.7230</td>
</tr>
<tr>
<td>4.</td>
<td>20.0</td>
<td>(10.2 \times 10.2 \times 7.89)</td>
<td>4.635</td>
<td>15.31</td>
<td>1.5014</td>
<td>50.1419</td>
</tr>
<tr>
<td>5.</td>
<td>20.0</td>
<td>(10.16 \times 10.2 \times 7.11)</td>
<td>4.71</td>
<td>15.15</td>
<td>1.4858</td>
<td>48.5797</td>
</tr>
<tr>
<td>6.</td>
<td>35.0</td>
<td>(18 \times 18 \times 6)</td>
<td>2.532</td>
<td>24.12</td>
<td>1.34</td>
<td>33.9973</td>
</tr>
<tr>
<td>7.</td>
<td>35.0</td>
<td>(18 \times 18 \times 9)</td>
<td>2.45</td>
<td>25.64</td>
<td>1.4244</td>
<td>42.4423</td>
</tr>
<tr>
<td>8.</td>
<td>100.0</td>
<td>(10 \times 10 \times 1)</td>
<td>7.97</td>
<td>11.24</td>
<td>1.1242</td>
<td>12.4237</td>
</tr>
</tbody>
</table>
Program 1

```matlab
%%Dimensions of DRA
%%length
d=[14.3, 14.0, 15.24, 10.2, 10.16, 18, 18, 10];
%%width
w=[25.4, 8, 3.1, 10.2, 10.2, 18, 18, 10];
%%height
h=[26.1, 8, 7.62, 7.89, 7.11, 6, 9, 1];
%%Mode
m=1;
M=1;
p=1;
c=3e8;
cons=[10.0, 10.0, 10.0, 20, 20, 35, 35, 100];
syms y real
for i=drange(1:8)
kx(i)=pi/d(i);
kz(i)=pi/2/h(i);
k0=sqrt((kx(i)^2+y.^2+kz(i)^2)/cons(i));
f=real(y.*tan(y*w(i)/2)-sqrt((cons(i)-1)*k0.^2-y.^2));
ky(i)=fzero(inline(f),[0,(pi/w(i))-0.01]);
%%Resonant frequency
fre(i)=c/2*pi*sqrt((kx(i).^2+ky(i).^2+kz(i).^2)/cons(i))*1e3;
Effwidth(i)=pi/ky(i);
factor(i)=Effwidth(i)./w(i);
perchangwidth(i)=((Effwidth(i)-w(i))/w(i))*100;
end
```

Results:
Program2

\[ m=1; \]
\[ n=1; \]
\[ p=1; \]
\[ E_r=10; \]
\[ a=15.24e-03; \]
\[ b=3.1e-03; \]
\[ d=7.62e-03; \]
\[ c=3e+08; \]
\[ \text{ky}=n*\pi/b; \]
\[ \text{kx}=m*\pi/a; \]
\[ \text{kz}=p*(\pi/d)/2; \]
\[ \text{ko} = \frac{\sqrt{\text{kx}^2+\text{ky}^2+\text{kz}^2}}{\sqrt{E_r}}; \]
\[ \text{fo} = \frac{(c*\text{ko}/\pi)/2}{1e+09}; \]
\[ \text{foghz} = \frac{\text{fo}}{1e+09}; \]

Results:
MATLAB programs taking parameters a, b, d same and comparing frequency using:

Program 1: Characteristic Equation

\[
m = 1 \\
n = 1 \\
p = 1 \\
E_r = 10 \\
a = 14.3 \times 10^{-3} \\
b = 25.4 \times 10^{-3} \\
d = 26.1 \times 10^{-3} \\
c = 3 \times 10^8 \\
k_x = m \pi / a \\
k_y = n \pi / b \\
k_z = p \pi / d / 2 \\
k_o = \sqrt{k_x^2 + k_y^2 + k_z^2} / \sqrt{E_r} \\
f_o = (c \times k_o / \pi) / 2 \\
f_o \text{GHz} = f_o / 10^9
\]

output:

![Output Image]
Program 4
Transcendental Equation for same dimensions:

m=1;
n=1;
p=1;
E_r=10;
a=14.3e-03;
b=25.4e-03;
d=26.1e-03;
c=3e+08;
syms y real
kx=pi/a;
kz=pi/d/2;
k0=sqrt(kx^2+y^2+kz^2)/sqrt(E_r);
f=real(y*tan(y*b/2)-sqrt((E_r-1)*k0^2-y^2));
ky=fzero(inline(f),[0,(pi/b)-0.01]);
f0=c/2/pi*sqrt((kx^2+ky^2+kz^2)/E_r)*1e3;
effwidth=pi/ky;
factor=effwidth/b;
perchangwidth=((effwidth-b)/b)*100;

output:

```
<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>x_2</td>
<td>0.1014</td>
<td>0.1007</td>
<td>0.1017</td>
</tr>
<tr>
<td>x_3</td>
<td>0.2146</td>
<td>0.2144</td>
<td>0.2146</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

ky = 0.1026
factor = 0.1026
perchangwidth = 0.1026
```

Image of program output with values and calculations.
MATLAB programs taking parameters a, b, d same and comparing frequency using:

: Characteristic Equation

Where

\[ a = 17 \text{mm} \]
\[ b = 25 \text{mm} \]
\[ c = 10 \text{mm} \]
\[ m = 1; \]
\[ n = 1; \]
\[ p = 1; \]
\[ E_r = 10; \]
\[ a = 17 \times 10^{-3}; \]
\[ b = 25 \times 10^{-3}; \]
\[ d = 10 \times 10^{-3}; \]
\[ c = 3 \times 10^{8}; \]
\[ k_x = m \times \pi / a; \]
\[ k_y = n \times \pi / b; \]
\[ k_z = p \times (\pi / d) / 2; \]
\[ k_o = \sqrt{k_x^2 + k_y^2 + k_z^2} / \sqrt{E_r}; \]
\[ f_o = (c \times k_o / \pi) / 2; \]
\[ f_o \text{GHz} = f_o / 1 \times 10^9; \]

Output

![MATLAB output](image-url)
Program 6

Transcendental Equation

m=1;
n=1;
p=1;
E_r=10;
a=17e-03;
b=25e-03;
d=10e-03;
c=3e+08;
syms y real
kx=pi/a;
kz=pi/d/2;
ko=sqrt((kx^2+y^2+kz^2)/sqrt(E_r));
f=real(y*tan(y*b/2)-sqrt((E_r-1)*ko^2-y^2));
ky=fzero(inline(f),[0,(pi/b)-0.01]);
fre=c/2/pi*sqrt((kx^2+ky^2+kz^2)/E_r)*1e3;
effwidth=pi/ky;
factor=effwidth/b;
perchangwidth=((effwidth-b)/b)*100;
Program 7

MATLAB programs taking parameters a, b, d same and comparing frequency using: Characteristic Equation

\[ m=1 \]
\[ n=1 \]
\[ p=1 \]
\[ E_r=10 \]
\[ a=14.3\times10^{-3} \]
\[ b=25.4\times10^{-3} \]
\[ d=26.1\times10^{-3} \]
\[ c=3\times10^{8} \]
\[ k_x=m\pi/a \]
\[ k_y=n\pi/b \]
\[ k_z=p\pi/(pi/d)/2 \]
\[ k_o=sqrt(k_x^2+k_y^2+k_z^2)/sqrt(E_r) \]
\[ f_o=(c*k_o/pi)/2 \]
\[ f_oGHz=f_o/1e+09 \]

Output:

![MATLAB Output Image]
Program 8
Tanscandental Equation

```matlab
m=1;
n=1;
p=1;
E_r=10;
a=14.3e-03;
b=25.4e-03;
d=26.1e-03;
c=3e+08;
syms y r e a l
kx=pi/a;
kz=pi/d/2;
ko=sqrt((kx^2+y^2+kz^2)/sqrt(E_r));
f=real(y*tan(y*b/2)-sqrt((E_r-1)*ko^2-y^2));
ky=fzero(inline(f),[0,(pi/b)-0.01]);
fre=c/2/pi*sqrt((kx^2+ky^2+kz^2)/E_r)*1e3;
effwidth=pi/ky;
factor=effwidth/b;
perchangwidth=((effwidth-b)/b)*100;
```

Output:
Program 9
1 - m=3;
2 - n=3;
3 - a=10;
4 - b=5;
5 - x=linspace(-5,5,51);
6 - y=linspace(-2.5,2.5,51);
7 - [xi,yi]=meshgrid(x,y);
8 - Ez=cos(m*pi*xi/a).*cos(n*pi*yi/b);
9 - Ez=Ez.^2;
10 - Ez=sqrt(Ez);
11 - surf(xi,yi,Ez);
12 - view([-45,60])
13 - view([180,0])
14 - drawnow

Workspace

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ez</td>
<td>&lt;51x51 double&gt;</td>
<td>3.37..</td>
<td>1</td>
</tr>
<tr>
<td>a</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>b</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>m</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>n</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>x</td>
<td>&lt;1x51 double&gt;</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>xi</td>
<td>&lt;51x1 double&gt;</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>y</td>
<td>&lt;1x51 double&gt;</td>
<td>-2.50..</td>
<td>250..</td>
</tr>
<tr>
<td>yi</td>
<td>&lt;51x1 double&gt;</td>
<td>-2.50..</td>
<td>250..</td>
</tr>
</tbody>
</table>
Program 10:

```matlab
1 d=[14.3,14.0,15.24,10.2,10.16,18,18,10];
2 w=[25.4,8,3.1,10.2,10.2,18,18,10];
3 h=[26.1,8,7.62,7.89,7.11,6,9,1];
4 m=1;
5 n=1;
6 p=1;
7 c=3e8;
8 cons=[10,0,10,0,10,20,20,35,35,100];
9 syms y real
10 for i=range(1:8)
11 xk(i)=pi/d(i);
12 xz(i)=pi/2/h(i);
13 ko=sqrt((xk(i).^2+y.^2+2+2x(i).^2)/cons(i));
14 f=real (y.*tan(y.*w(i)/2)-sqrt((cons(i)-1)*ko.^2-y.^2));
15 ky(i)=feval (inline(f),1,0,100);[pi/4/y(i));
16 f=real (y.*tan(y.*w(i)/2)-sqrt((cons(i)-1)*ko.^2-y.^2))/cons(i);=1e3;
17 Effwidth(i)=pi/ky(i);
18 factor(i)=Effwidth(i)/w(i);
19 pchwidth(i)=((Effwidth(i)-w(i))/w(i))^100;
20 end
```
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