Glossary

**Acronyms**

- LCMP: Linearly constrained minimum power
- LCMV: Linearly constrained minimum variance
- MPDR: Minimum power distortionless response
- MVDR: Minimum variance distortionless response
- QCQP: Quadratically-constrained quadratic program
- SNR: Signal-to-noise ratio
- SOCP: Second-order cone programming
- WNG: White noise gain

**Mathematical operators**

- $\| \cdot \|$: 2-norm
- $(\cdot)^*$: Complex conjugate
- $(\cdot)^T$: Transpose
- $(\cdot)^H$: Hermitian or complex transpose
- $(\cdot)^\dagger$: Pseudo matrix inverse
- $(\cdot)!$: Factorial
- $\nabla$: Gradient
- $\nabla^2$: Laplacian in Cartesian coordinates
- $\nabla^2_r$: Laplacian in spherical coordinates
- $E[\cdot]$: Expectation
- $Im\{\cdot\}$: Imaginary part
- $\kappa(\cdot)$: Condition number of a matrix
- $Re\{\cdot\}$: Real part
- $\Lambda(\cdot)$: Rotation operator

**Greek symbols**

- $\alpha_q$, $\alpha_q^{nm}$: Sampling weights
- $\alpha$: Vector of sampling weights
δ_{nm}, δ_{n}  
Kronecker delta function

δ(·)  
Dirac delta function

θ  
Elevation angle

φ  
Azimuth angle

Ω  
Solid angle

Symbols

\(a(·)\)  
Plane-wave decomposition in the space domain

\(a_{nm}\)  
Plane-wave decomposition in the spherical-harmonics domain

\(b_{n}(·)\)  
Function relating pressure to plane-wave decomposition

\(DF\)  
Directivity factor

\(DI\)  
Directivity index

\(d_{n}\)  
Axis-symmetric beamforming weighting function

\(d_{nm}^{n}(·)\)  
Wigner-d function

\(D_{nm}^{n}(·)\)  
Wigner-D function

\(d_{n}\)  
Axis-symmetric beamforming weighting vector

\(F\)  
Front-back ratio

\(h_{n}(·)\)  
Spherical Hankel function of the first kind

\(h_{n}^{(2)}(·)\)  
Spherical Hankel function of the second kind

\(I\)  
Unit matrix

\(j_{n}(·)\)  
Spherical Bessel function of the first kind

\(k\)  
Wave number

\(\tilde{k}\)  
Wave vector denoting arrival direction

\(k\)  
Wave vector denoting propagation direction

\(L_{2}(·)\)  
Space of square-integrable functions

\(N\)  
Order of spherical harmonics

\(\mathbb{N}\)  
Set of all natural numbers

\(n\)  
Noise vector in the space domain

\(n_{nm}\)  
Noise vector in the spherical harmonics domain

\(P_{n}(·)\)  
Legendre polynomial

\(P_{n}^{m}(·)\)  
Associated Legendre function

\(p\)  
Sound pressure in the space domain

\(p_{nm}\)  
Sound pressure in the spherical-harmonics domain

\(p\)  
Sound pressure vector

\(p_{nm}\)  
Sound pressure vector in the spherical-harmonics domain

\(Q\)  
Number of samples or microphones

\(\mathbb{R}\)  
One-dimensional space of real numbers

\(\mathbb{R}^{3}\)  
Three-dimensional space of real numbers

\(r\)  
Vector of spherical coordinates

\(R_{y}\)  
Euler rotation matrix for rotations about the y axis

\(R_{z}\)  
Euler rotation matrix for rotations about the z axis

\(S^{2}\)  
Unit sphere

\(S\)  
Spherical Fourier transform matrix

\(S_{xx}\)  
Cross-spectrum matrix in the space domain
<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$S_{xnm}x_{nm}$</td>
<td>Cross-spectrum matrix in the spherical-harmonics domain</td>
</tr>
<tr>
<td>$S_{nn}$</td>
<td>Noise cross-spectrum matrix in the space domain</td>
</tr>
<tr>
<td>$S_{nmmnm}$</td>
<td>Noise cross-spectrum matrix in the spherical-harmonics domain</td>
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<tr>
<td>$T_M(\cdot)$</td>
<td>Chebyshev polynomial</td>
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<td>$v$</td>
<td>Steering vector in the space domain</td>
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<tr>
<td>$v_{nm}$</td>
<td>Steering vector in the spherical-harmonics domain</td>
</tr>
<tr>
<td>$WNG$</td>
<td>White noise gain</td>
</tr>
<tr>
<td>$w(\cdot)$</td>
<td>Beamforming weighting function in the space domain</td>
</tr>
<tr>
<td>$w_{nm}$</td>
<td>Beamforming weighting function in the spherical-harmonics domain</td>
</tr>
<tr>
<td>$w$</td>
<td>Beamforming weighting vector in the space domain</td>
</tr>
<tr>
<td>$w_{nm}$</td>
<td>Beamforming weighting vector in the spherical-harmonics domain</td>
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<tr>
<td>$y_n(\cdot)$</td>
<td>Spherical Bessel function of the second kind</td>
</tr>
<tr>
<td>$Y_m(\cdot)$</td>
<td>Spherical harmonics</td>
</tr>
<tr>
<td>$Y$</td>
<td>Matrix of spherical harmonics</td>
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<tr>
<td>$\mathbb{Z}$</td>
<td>Set of all integers</td>
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References

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