## Appendix A

### Covariance Formulae

\[
\begin{align*}
\frac{n}{S_n} \rightarrow G(s) \rightarrow \frac{x}{\sigma_x^2}
\end{align*}
\]

\(n\) : white noise \quad \(x\) : colored noise

Spectral density \(S_n[(\text{dim } n)^2/\text{Hz}]\) \quad Variance \(\sigma_x^2[(\text{dim } x)^2]\)

<table>
<thead>
<tr>
<th>System</th>
<th>Formula for (G(s))</th>
<th>Formula for (\sigma_x^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-T(_1)</td>
<td>(G(s) = \frac{K}{1 + T_1 s})</td>
<td>(\sigma_x^2 = \frac{K^2}{2 T_1} S_n)</td>
</tr>
<tr>
<td>PD-T(_2)</td>
<td>(G(s) = K \frac{1 + T_D s}{(1 + T_1 s)(1 + T_2 s)})</td>
<td>(\sigma_x^2 = \frac{K^2}{2} \frac{1 + \frac{T_D^2}{T_1 T_2}}{T_1 + T_2} S_n)</td>
</tr>
<tr>
<td>P-T(_2)</td>
<td>(G(s) = \frac{K}{(1 + T_1 s)(1 + T_2 s)})</td>
<td>(\sigma_x^2 = \frac{K^2}{2(T_1 + T_2)} S_n)</td>
</tr>
<tr>
<td>D-T(_2)</td>
<td>(G(s) = \frac{K_D s}{(1 + T_1 s)(1 + T_2 s)})</td>
<td>(\sigma_x^2 = \frac{K_D^2}{2} \frac{1}{T_1 T_2 (T_1 + T_2)} S_n)</td>
</tr>
</tbody>
</table>

\[
G(s) = \frac{K \left(1 + T_{D_1}s \right) \left(1 + T_{D_2}s \right)}{(1 + T_1s)(1 + T_2s)(1 + T_3s)}
\]

\[
\sigma_x^2 = \frac{K^2}{2} \cdot \frac{T_1T_2 + T_1T_3 + T_2T_3 + \left( T_{D_1}^2 + T_{D_2}^2 \right) + T_{D_1}T_{D_2} \frac{T_1 + T_2 + T_3}{TT_2TT_3}}{S_n}
\]

\[
G(s) = \frac{K \left(1 + T_{D_1}s \right) \left(1 + T_{D_2}s \right)}{\left(1 + \frac{2D}{\omega_0} s + \frac{1}{\omega_0^2} s^2 \right)(1 + T_1s)}
\]

\[
\sigma_x^2 = \left( \frac{K}{2} \right)^2 \left[ \frac{(1 + 2D\omega_0 T_1 + \omega_0^2 \left( T_{D_1}^2 + T_{D_2}^2 \right) + \left( T_{D_1}T_{D_2}\omega_0^4 \right)^2 \left(1 + \frac{2D}{\omega_0 T_1} \right)}{1 + 2D\omega_0 T_1 + \left( \omega_0 T_1 \right)^2} \right] \frac{T_1T_2 + T_1T_3 + T_2T_3 + \left(2D_z^2 - 1\right)}{TT_2TT_3 \omega_0^4} \frac{T_1 + T_2 + T_3}{S_n}
\]

\[
G(s) = \frac{K \left(1 + \frac{2D}{\omega_0 z} s + \frac{1}{\omega_0^2 z} s^2 \right)}{\left(1 + \frac{2D}{\omega_0} s + \frac{1}{\omega_0^2} s^2 \right)(1 + T_1s)}
\]

\[
\sigma_x^2 = \left( \frac{K}{2} \right)^2 \left[ \frac{(1 + 2D\omega_0 T_1 + 2 \left( \frac{\omega_0}{\omega_0 z} \right)^2 \left(2D_z^2 - 1\right) + \left( \frac{\omega_0}{\omega_0 z} \right)^4 \left(1 + \frac{2D}{\omega_0 T_1} \right)}{1 + 2D\omega_0 T_1 + \left( \omega_0 T_1 \right)^2} \right] \frac{T_1T_2 + T_1T_3 + T_2T_3 + \left(2D_z^2 - 1\right)}{TT_2TT_3 \omega_0^4} \frac{T_1 + T_2 + T_3}{S_n}
\]
\[ G(s) = \frac{K}{(1 + T_1 s)(1 + T_2 s)(1 + T_3 s)} \]

\[ \sigma_x^2 = \frac{K^2}{2} \cdot \frac{T_1 T_2 + T_1 T_3 + T_2 T_3}{(T_1 + T_2)(T_1 + T_3)(T_2 + T_3)} S_n \]

\[ G(s) = \frac{K}{\left( 1 + \frac{2D}{\omega_0} s + \frac{1}{\omega_0^2} s^2 \right) \left( 1 + T_1 s \right)} \]

\[ \sigma_x^2 = \left( \frac{K}{2} \right)^2 \frac{\omega_0}{D} \cdot \frac{1 + 2D\omega_0 T_1}{1 + 2D\omega_0 T_1 + \left( \omega_0 T_1 \right)^2} S_n \]
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