References


[59] Hecht, F., Borouchaki, H.: Mesh Adaption by Metric Control for Multi-scale Phenomena and Turbulence, AIAA-97-0859-
[81] Novozhilov, V.V.: The theory of thin shells. Walters Noordhoff Publ., Groningen (1959)
Characteristics of the Membrane System

In order to prove that the characteristic equation of the membrane system (1.68) expressed in terms of the displacements is equivalent to equation (1.71), we shall proceed as follows. First, let us replace system (1.68) by another equivalent one, by considering as unknowns \(u_1, u_2, u_3\), and the supplementary unknowns \((T_{11}, T_{22}, T_{12})\). Inverting the matrix of stiffness \(A^{\alpha\beta\lambda\mu}\) involved in (1.69), system (1.68) is clearly equivalent to:

\[
\begin{align*}
-D_1 T_{11} + b_{11} u_3 - B_{11\alpha\beta} T^{\alpha\beta} &= f_1 \\
-D_1 T_{12} + b_{12} u_3 - B_{12\alpha\beta} T^{\alpha\beta} &= f_2 \\
-b_{11} T_{11} - b_{22} T_{22} - 2b_{12} T_{12} &= f_3 \\
\end{align*}
\]

(A.1)

with

\[
\begin{align*}
D_1 u_1 - b_{11} u_3 - B_{11\alpha\beta} T^{\alpha\beta} &= 0 \\
D_2 u_2 - b_{22} u_3 - B_{22\alpha\beta} T^{\alpha\beta} &= 0 \\
\frac{1}{2} (D_1 u_2 + D_2 u_1) - b_{12} u_3 - B_{12\alpha\beta} T^{\alpha\beta} &= 0 \\
\end{align*}
\]

(A.2)

where \(B_{\alpha\beta\lambda\mu}\) are the coefficients of the compliance matrix (inverse of the stiffness matrix of \(A^{\alpha\beta\lambda\mu}\), see (1.57). With that order of the unknowns and equations, we obtain a system of 6 equations with the 6 unknowns \((u_1, u_2, u_3, T_{11}, T_{22}, T_{12})\). We recognize (see (1.65) and (1.44) for comparison) the membrane tension system in (A.1) and the rigidity system in (A.2), unless concerning the membrane tensions \(T^{\alpha\beta}\). By analogy with the results of sections 1.5.2 and 1.7.2, we should define the indices \((1, 1, 0, 0, 0, 0)\) both for unknowns and equations. Then replacing again the derivatives \(\partial_\alpha\) with \(dz_\alpha\), and taking the determinant of the system obtained, we have a determinant of order 6 with the structure:

\[
\begin{vmatrix}
0 & C_{12} \\
C_{21} & C_{22}
\end{vmatrix} = 0 
\]

(A.3)
where the $C_{\alpha\beta}$ are 3x3 matrices, and where 0 denotes the zero 3x3 matrix. Moreover, $C_{12}$ and $C_{21}$ are precisely those of the membrane tension (1.65) and of the rigidity system (1.44), respectively. Obviously, $C_{22}$ comes from the terms in $T^{\alpha\beta}$ of (A.2). But it follows immediately from the definition and elementary properties of determinants that the determinant of (A.3) is given by the product of the determinants of $C_{11}$ and $C_{22}$. The conclusion follows immediately.
This appendix contains the detailed calculations which lead to the reduced formulation of the problem \[1.64\] comprising three PDEs respectively for \(u_1\), \(u_2\) and \(u_3\). A similar development is then carried out for the full Koiter problem (section \[B.2\]) but only for \(u_3\).

B.1 Membrane Problem

First, we start from membrane system (obtained after integration by parts of the variational formulation of the membrane problem):

\[
\begin{align*}
-D_\alpha T^{\alpha\beta} &= f^\beta \\
-b_\alpha^\beta T^{\alpha\beta} &= f^3
\end{align*}
\]  

(B.1)

Using the constitutive law, we get:

\[
\begin{align*}
-D_\alpha \left( A^{\alpha\beta\lambda\mu} \gamma_{\lambda\mu} \right) &= f^\beta \\
-b_\alpha^\beta A^{\alpha\beta\lambda\mu} \gamma_{\lambda\mu} &= f^3
\end{align*}
\]  

(B.2)

As our aim is to study the singularities and their propagations, according to the microlocal analysis \[43\], it is sufficient to keep only the higher order terms for the displacements \(u_1\), \(u_2\) and \(u_3\) and to consider the geometrical coefficients \(a_{\alpha\beta}\) and \(b_{\alpha\beta}\) as constants (at least locally at the point considered). Thus, we obtain the following system which only involves the displacements \(u_1\), \(u_2\) and \(u_3\):

\[
\begin{align*}
-A^{1\beta\gamma_1} \partial_\beta \partial_\gamma u_1 - A^{1\beta\gamma_2} \partial_\beta \partial_\gamma u_2 + A^{1\beta\gamma_3} b_{\gamma\delta} \partial_\beta u_3 + \cdots &= f^1 \\
-A^{2\beta\gamma_1} \partial_\beta \partial_\gamma u_1 - A^{2\beta\gamma_2} \partial_\beta \partial_\gamma u_2 + A^{2\beta\gamma_3} b_{\gamma\delta} \partial_\beta u_3 + \cdots &= f^2 \\
-A^{3\beta\gamma_1} b_{\gamma\delta} \partial_\beta u_1 - A^{2\beta\gamma_2} b_{\gamma\delta} \partial_\beta u_2 + A^{3\beta\gamma_3} b_{\alpha\beta} b_{\gamma\delta} u_3 + \cdots &= f^3
\end{align*}
\]  

(B.3)
where \( + \ldots \) denotes lower orders terms of the form \( \Gamma^\xi_{\mu\delta} \partial_\gamma u_\beta \) and \( \Gamma^\eta_{\mu\delta} \Gamma^\xi_{\beta\eta} u_\beta \) or \( \Gamma^\xi_{\mu\delta} b_{\eta\mu} u_3 \) in the first two lines of the system, and of the form \( \Gamma^\xi_{\mu\delta} b_{\eta\mu} u_\beta \) in the third one. In the sequel, we only keep the highest order derivatives.

Now, let us write the simplified system \((B.3)\) as follows:

\[
Au = f
\]

with

\[
A = \begin{pmatrix}
-A^1_{1\gamma 1} \partial_\beta \partial_\gamma & -A^1_{1\gamma 2} \partial_\beta \partial_\gamma & A^1_{1\gamma 3} b_{\gamma\delta} \partial_\beta \\
-A^2_{1\gamma 1} \partial_\beta \partial_\gamma & -A^2_{1\gamma 2} \partial_\beta \partial_\gamma & A^2_{1\gamma 3} b_{\gamma\delta} \partial_\beta \\
-A^1_{2\gamma 3} b_{\gamma\delta} \partial_\beta & -2A^2_{2\gamma 3} b_{\gamma\delta} \partial_\beta & A^0_{3\beta\gamma} b_{\alpha\beta} b_{\gamma\delta}
\end{pmatrix}
\]

(B.5)

Developing the terms of the matrix \(A\), we get:

\[
A = \begin{pmatrix}
-A^{1111} \partial_1^4 - A^{1212} \partial_1^2 \partial_2^2 & -A^{1122} \partial_1^2 - A^{1222} \partial_2^2 & B \partial_1 \\
-A^{1112} \partial_1^2 \partial_1 \partial_2 & -2A^{1112} \partial_1 \partial_2 - (A^{1122} + A^{1212}) \partial_1 \partial_2 & +C \partial_2 \\
-A^{1122} \partial_1^2 \partial_1 \partial_2 & -A^{1222} \partial_2^2 - A^{1212} \partial_1^2 & C \partial_1 \\
-(A^{1122} + A^{1212}) \partial_1 \partial_2 & -2A^{1222} \partial_1 \partial_2 & +D \partial_2 \\
-B \partial_1 & -C \partial_1 & B b_{11} + D b_{22} \\
-C \partial_2 & -D \partial_2 & +2C b_{12}
\end{pmatrix}
\]

(B.6)

with \( B = A^{11\alpha\beta} b_{\alpha\beta} \), \( C = A^{12\alpha\beta} b_{\alpha\beta} \) and \( D = A^{22\alpha\beta} b_{\alpha\beta} \), where the Einstein summation convention for the indexes \( \alpha \) and \( \beta \) is used.

**B.1.1 Case of the Normal Displacement \( u_3 \)**

In order to obtain a reduced equation for \( u_3 \), we need to compute the cofactor \( A_{33}^C \) (see section 2.5 of chapter 2). By definition, \( A_{33}^C = A_{11} A_{22} - A_{12} A_{21} \), so that:

\[
A_{33}^C = \left[ A^{1111} A^{1212} - (A^{1112})^2 \right] \partial_1^4 + \left[ A^{2222} A^{1212} - (A^{1222})^2 \right] \partial_2^4
\]

\[
+ \left[ A^{1111} A^{2222} + (A^{1212})^2 + 4A^{1112} A^{1222} - 2A^{1112} A^{1222} - (A^{1112} + A^{1212})^2 \right] \partial_1^2 \partial_2^2
\]

\[
+ \left[ 2A^{1111} A^{1222} + 2A^{1112} A^{1212} - 2A^{1112} (A^{1112} + A^{1212}) \right] \partial_1 \partial_2^3
\]

\[
+ \left[ 2A^{2222} A^{1112} + 2A^{2222} A^{1212} - 2A^{1222} (A^{1112} + A^{1212}) \right] \partial_1^3 \partial_2
\]
It then follows that:

\[ A_{33}^C = [A^{1111}A^{1212} - (A^{1112})^2] \partial_1^4 + [A^{2222}A^{1212} - (A^{1222})^2] \partial_2^4 \]

\[ + [A^{1111}A^{2222} + 2A^{1112}A^{1222} - (A^{1122})^2 - 2A^{1222}A^{1212}] \partial_1^2 \partial_2^2 \]

\[ [2A^{1111}A^{1222} - 2A^{1112}A^{1122}] \partial_1^3 \partial_2 + [2A^{2222}A^{1112} - 2A^{1222}A^{1122}] \partial_1 \partial_2^3 \]

Let us now recall the expression of the coefficients of the linear elastic isotropic constitutive law:

\[ A^{\alpha\beta\lambda\delta} = \frac{E}{2(1+\nu)} \left[ a^{\alpha\lambda}a^{\beta\delta} + a^{\alpha\delta}a^{\beta\lambda} + Ja^{\alpha\beta}a^{\lambda\delta} \right] \quad \text{with} \quad J = \frac{2\nu}{1-\nu} \quad (B.7) \]

Taking the symmetries into account, we have:

\[ A^{1111} = \frac{E}{2(1+\nu)}(2 + J)(a^{11})^2 \quad (B.8) \]

\[ A^{1112} = \frac{E}{2(1+\nu)}(2 + J)a^{11}a^{12} \quad (B.9) \]

\[ A^{2222} = \frac{E}{2(1+\nu)}(2 + J)(a^{22})^2 \quad (B.10) \]

\[ A^{1222} = \frac{E}{2(1+\nu)}(2 + J)a^{22}a^{12} \quad (B.11) \]

\[ A^{1212} = \frac{E}{2(1+\nu)}((1 + J)(a^{12})^2 + a^{11}a^{22}) \quad (B.12) \]

\[ A^{1122} = \frac{E}{2(1+\nu)}((2)(a^{12})^2 + Ja^{11}a^{22}) \quad (B.13) \]

In the expression of \( A_{33}^C \), we shall compute separately the different terms:

- Terms in \( \partial_1^4 \):

\[ \frac{E^2}{4(1+\nu)^2} \left[ (2 + J)(a^{11})^2 (a^{11}a^{22} + (1 + J)a^{12}a^{12}) - (2 + J)^2 (a^{11}a^{12})^2 \right] \]

\[ = \frac{E^2}{4(1+\nu)^2} (2 + J)(a^{11})^2 \left( a^{11}a^{22} - (a^{12})^2 \right) \quad (B.14) \]

- Terms in \( \partial_2^4 \) (obtained symmetrically):

\[ \frac{E^2}{4(1+\nu)^2} (2 + J)(a^{22})^2 \left( a^{11}a^{22} - (a^{12})^2 \right) \quad (B.15) \]
• Terms in $\partial_1^2 \partial_2^2$

$$
\frac{E^2}{4(1+\nu)^2} \left[ (2 + J)^2 (a_1^{11} a_2^{22})^2 + 2(2 + J)^2 (a_1^{12} a_2^{22})^2 - \left( 2a_1^{12} a_2^{12} + Ja_1^{11} a_2^{22} \right)^2 \right]
- 2 \left( 2a_1^{12} a_2^{12} + Ja_1^{11} a_2^{22} \right) (a_1^{11} a_2^{22} + (1 + J)a_1^{12} a_2^{12})
= \frac{E^2}{4(1+\nu)^2} \left[ (2 + J)^2 (a_1^{11} a_2^{22})^2 + 2(a_2^{12})^2 - \left( 4(a_1^{12})^4 + 2Ja_1^{11} a_2^{22} (a_1^{12})^2 \right) \right]
- 2 \left( 2a_1^{12} a_2^{22} (a_1^{12})^2 + 2(1 + J)(a_1^{12})^2 + Ja_1^{11} a_2^{22} (a_1^{12})^2 \right)
= \frac{E^2}{4(1+\nu)^2} \left[ 2(2 + J)(a_1^{11} a_2^{22})^2 - 4(2 + J)(a_1^{12})^2 \right]
+ 2(2 + J)a_1^{11} a_2^{22} (a_1^{12})^2
= \frac{E^2}{4(1+\nu)^2} \left[ 2(2 + J) \left( a_1^{11} a_2^{22} - (a_1^{12})^2 \right) (a_1^{11} a_2^{22} + 2(a_1^{12})^2) \right]
$$
\hfill (B.16)

• Terms in $\partial_1^3 \partial_2$

$$
\frac{E^2}{4(1+\nu)^2} \left[ 2(2 + J)(a_1^{11} a_2^{22})^2 - 2(2 + J)a_1^{11} a_2^{12} (a_2^{12} a_2^{12}) + Ja_1^{11} a_2^{22} \right]
= \frac{E^2}{4(1+\nu)^2} \left[ 4(2 + J)a_1^{11} a_2^{12} \left( a_1^{11} a_2^{22} - (a_1^{12})^2 \right) \right]
$$
\hfill (B.17)

• Terms in $\partial_1 \partial_2^3$ (obtained symmetrically)

$$
\frac{E^2}{4(1+\nu)^2} \left[ 4(2 + J)a_2^{22} a_1^{12} \left( a_1^{11} a_2^{22} - (a_1^{12})^2 \right) \right]
$$
\hfill (B.18)

After simplification, we obtain:

$$
A_{33}^C = \frac{E^2}{4(1+\nu)^2} (2 + J) \left( a_1^{11} a_2^{22} - (a_1^{12})^2 \right) \left[ (a_1^{11})^2 \partial_1^4 + (a_2^{22})^2 \partial_2^4 + 2a_1^{11} a_2^{22} + 2(a_1^{12})^2 \right] \partial_1^2 \partial_2^2
+ 4a_1^{11} a_2^{12} \partial_1^3 \partial_2 + 4a_2^{22} a_1^{12} \partial_1 \partial_2^3
$$
\hfill (B.19)

Replacing $J$ by its expression, we get finally:

$$
A_{33}^C = \frac{E^2}{2(1+\nu)^2 (1-\nu)} \left( a_1^{11} a_2^{22} - (a_1^{12})^2 \right) \left[ a_1^{11} \partial_1^2 + a_2^{22} \partial_2^2 + 2a_1^{12} \partial_1 \partial_2 \right]^{(2)}
$$
\hfill (B.20)
As \( a^{11}a^{22} - (a^{12})^2 = (a_{11}a_{22} - (a_{12})^2)^{-1} = \frac{1}{a} \), expression (B.20) reduces to:

\[
A_{33}^C = \frac{E^2}{2(1 + \nu)(1 - \nu)a} \left[ a^{11}\partial_1^2 + a^{22}\partial_2^2 + 2a^{12}\partial_1\partial_2 \right]^{(2)} \quad (B.21)
\]

On the other hand, the term \( \det(A) \) may be obtained in a similar way:

\[
\det(A) = \frac{E^3}{2(1 + \nu)(1 - \nu)a^3} \left[ b_{22}\partial_1^2 + b_{11}\partial_2^2 - 2b_{12}\partial_1\partial_2 \right]^{(2)} \quad (B.22)
\]

Replacing the expressions (B.21) and (B.22) of \( A_{33}^C \) and \( \det(A) \) in equation (2.57), we obtain the reduced membrane PDE (2.60) accounting for the displacement \( u_3 \):

\[
E \left[ b_{22}\partial_1^2 + b_{11}\partial_2^2 - 2b_{12}\partial_1\partial_2 \right]^{(2)} u_3 = a^2 \left[ a^{11}\partial_1^2 + a^{22}\partial_2^2 + 2a^{12}\partial_1\partial_2 \right]^{(2)} f^3 \quad (B.23)
\]

B.1.2 Reduced Equation for the Tangential Displacements \( u_1 \) and \( u_2 \)

In the same way, we can exhibit two PDEs for the displacements \( u_1 \) and \( u_2 \). We present here the details for the case of \( u_1 \), that of \( u_2 \) being obtained symmetrically.

Let us start from the equation (2.62) characterizing \( u_1 \):

\[
\det(A)u_1 = A_{31}^C f^3 \quad (B.24)
\]

We only have to compute \( A_{31}^C \) whose expression is given by:

\[
A_{31}^C = A_{12}A_{23} - A_{22}A_{13}
= -A^{1112}C\partial_1^3 - A^{1222}C\partial_1\partial_2^2 - (A^{1122} + A^{1212})C\partial_1\partial_2^2 - A^{1112}D\partial_1^2\partial_2 - A^{1222}D\partial_2^3
-(A^{1122} + A^{1212})D\partial_1\partial_2^2 + A^{1212}B\partial_1^3 + A^{2222}C\partial_2^3 + A^{2222}B\partial_1\partial_2^2 + A^{1212}C\partial_1\partial_2^2
+2A^{1222}B\partial_1^2\partial_2 + 2A^{1222}C\partial_1\partial_2^2
= (-A^{1112}C + A^{1212}B)\partial_1^3 + (A^{2222}C - A^{1222}D)\partial_2^3
+(A^{1212}C + 2A^{1222}B - (A^{1212} + A^{1122})C - A^{1112}D)\partial_1^2\partial_2
+(A^{2222}B + 2A^{1222}C - (A^{1212} + A^{1122})D - A^{1222}C)\partial_1\partial_2^2 \quad (B.25)
\]
After some long technical calculations, we obtain:
\[
A_{31}^C = \frac{1}{a} \frac{E^2}{4(1+\nu)^2} \left\{ \left( (2+J)b_{11}(a_{11}^1)^2 + (a_{11}^1 a_{22}^2 - (2+2J)(a_{12}^2)^2 \right) b_{22} \right\} \partial_3^3 + \left\{ (2+J)(a_{22}^2)^2 b_{12} + 2(2+J)a_{22}^2 a_{12} b_{11} \right\} \partial_2^3 + \left\{ 4(2+J)a_{11} a_{12} b_{11} + (4(2+J)(a_{12}^2)^2 - 2a_{11} a_{22}^2) b_{12} - 2(2+J)a_{22} a_{12} b_{22} \right\} \partial_1^3 \partial_2 + \left\{ ((3J+4)a_{11} a_{22}^2 + (6+2J)(a_{12}^2)^2) b_{11} + (2+J)(a_{22}^2)^2 b_{22} + (8+4J)a_{22} a_{12} b_{12} \right\} \partial_1 \partial_2^2 \right] 
\]

B.2 Koiter Problem

In order to obtain a reduced PDE of the Koiter model for \( u_3 \), we use the local formulation (1.63) of the Koiter model:
\[
\begin{align*}
-D_\alpha T^{\alpha \beta} - \varepsilon^2 \left[ b^{\alpha \beta} D_\alpha M^{\alpha \gamma} + D_\gamma (b^{\alpha \gamma} M^{\alpha \beta}) \right] &= f^{\beta} \\
-b_{\alpha \beta} T^{\alpha \beta} + \varepsilon^2 \left[ D_\alpha D_\beta M^{\alpha \beta} - b^{\alpha \gamma} b^{\beta \delta} M^{\alpha \beta} \right] &= f^3
\end{align*}
\]

where \( T^{\alpha \beta} = A^{\alpha \beta \lambda \delta} \gamma_{\lambda \delta} \) and \( M^{\alpha \beta} = \frac{1}{12} A^{\alpha \beta \lambda \delta} \rho_{\lambda \delta} \) denote, respectively, the membrane stresses and the bending moments.

In the equation involving \( u_3 \), we only consider the most important bending terms, i.e. those with the lowest power in \( \varepsilon \), and the highest order derivatives. We can write system (B.26) under the form (B.4) with:
\[
A = \begin{pmatrix}
-A^{1111} \partial_1^2 - A^{1212} \partial_2^2 & -A^{1112} \partial_1^2 - A^{1222} \partial_2^2 & -A^{1122} \partial_1^2 - A^{1212} \partial_2^2 & B \partial_1 \\
-2A^{1112} \partial_1 \partial_2 & -(A^{1122} + A^{1212}) \partial_1 \partial_2 & -2A^{1212} \partial_1 \partial_2 & +C \partial_2 \\
+\varepsilon^2 (\partial^2 + \ldots) & +\varepsilon^2 (\partial^2 + \ldots) & +\varepsilon^2 (\partial^2 + \ldots) & +\varepsilon^2 (\partial^2 + \ldots) \\
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
-B \partial_1 & -C \partial_1 & +B b_{11} + D b_{22} + 2C b_{12} \\
-C \partial_2 & -D \partial_2 & +\varepsilon^2 F \\
+\varepsilon^2 (\partial^2 + \ldots) & +\varepsilon^2 (\partial^2 + \ldots) & +\varepsilon^2 (\partial^2 + \ldots) \\
\end{pmatrix}
\]

with \( F = \left[ A^{1111} \partial_1^4 + A^{2222} \partial_2^4 + (2A^{1122} + 4A^{1212}) \partial_1^2 \partial_2^2 + 4A^{1112} \partial_1^2 \partial_2 + 4A^{1222} \partial_1 \partial_2^2 \right] \) and where \( +\varepsilon^2 (\partial^n + \ldots) \) denote the ignored bending terms, \( n \) being the highest
order of differentiation contained in these terms. Let us quote that $F$ is a fourth-order operator.

Thus, the most important bending term of $det(A)$ comes from:

$$(A_{11}A_{22} - A_{12}A_{12}) \frac{\varepsilon^2}{12} F =$$

$$\frac{\varepsilon^2}{12} A_{33}^C \left[ A^{1111} \partial_1^4 + A^{2222} \partial_2^4 + (2A^{1122} + 4A^{1212}) \partial_1^2 \partial_2^2 + 4A^{1112} \partial_1^3 \partial_2 + 4A^{1222} \partial_1 \partial_2^3 \right]$$

(B.28)

This term is in $\varepsilon^2$ and comprises $8^{th}$ order derivatives. The other terms in $\varepsilon^2$ involve lower order derivatives. The full expression of $Det(A)$ then writes:

$$Det(A) = E^3 \left( \frac{\varepsilon^2}{24} \frac{1}{(1 + \nu)^3 (1 - \nu)^2} a \left[ a^{11} \partial_1^2 + a^{22} \partial_2^2 + 2a^{12} \partial_1 \partial_2 \right]^{(4)} + \frac{1}{2(1 + \nu)^2 (1 - \nu) a^3} \left[ b_{22} \partial_1^2 + b_{11} \partial_2^2 - 2b_{12} \partial_1 \partial_2 \right]^{(2)} + O(\varepsilon^2) \right)$$

(B.29)

where $O(\varepsilon^2)$ denotes terms containing lower order derivatives or being factors of terms in $\varepsilon^n$ with $n > 2$.

On the other hand, we always have:

$$A_{33}^C = E^2 \frac{1}{2(1 + \nu)^2 (1 - \nu) a} \left[ a^{11} \partial_1^2 + a^{22} \partial_2^2 + 2a^{12} \partial_1 \partial_2 \right]^{(2)} + O(\varepsilon^2)$$

(B.30)

so that, when $\varepsilon \searrow 0$, the reduced PDE of the Koiter model involving only $u_3$ writes:

$$E \left[ \frac{\varepsilon^2}{12} a^2 \left[ a^{11} \partial_1^2 + a^{22} \partial_2^2 + 2a^{12} \partial_1 \partial_2 \right]^{(4)} + (1 + \nu) \left[ b_{22} \partial_1^2 + b_{11} \partial_2^2 - 2b_{12} \partial_1 \partial_2 \right]^{(2)} + O(\varepsilon^2) \right] u_3 = \left[ a^2 (1 + \nu) \left[ a^{11} \partial_1^2 + a^{22} \partial_2^2 + 2a^{12} \partial_1 \partial_2 \right]^{(2)} + O(\varepsilon^2) \right] f^3$$

(B.31)
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