Appendices

Abstract. For better understandings of readers, several important mathematics and derivation of relations used in the text are given here. They are (A) Dirac delta function and Fourier transform, (B) uniaxial strain and strain components in cubic semiconductors, (C) boson operators, (D) random phase approximation, (E) density matrix, and (F) derivation of spontaneous and stimulated emission rates.

A Delta Function and Fourier Transform

A.1 Dirac Delta Function

The Delta function is very important for understanding semiconductor physics and some important relations will be discussed in this section. The Dirac delta function is defined by

\[ \delta(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} dt. \]  \hspace{1cm} (A.1)

In the integral with respect to \( \omega \) the following relation holds for \( \varepsilon > 0 \)

\[ \lim_{\varepsilon \to +0} \frac{1}{\omega - i\varepsilon} = \mathcal{P} \frac{1}{\omega} + i\pi \delta(\omega), \]  \hspace{1cm} (A.2)

where \( \mathcal{P}[1/\omega] \) is the Cauchy principal value of \( 1/\omega \). This relation is called the Dirac identity.

First, we consider the integral

\[ \int_{0}^{\infty} e^{i\omega t} dt. \]

This integral does not converge when \( \omega \) is real. Then we introduce an infinitesimal positive value \( \varepsilon \), and replace \( \omega \) by \( \omega + i\varepsilon \). The integral may be equivalently written as

1 Strictly speaking, we need to prove the validity of the interchange between the limit (lim) and the integral (\( \int \)).

In a similar fashion, we obtain
\[
\int_{-\infty}^{0} e^{i\omega t} dt = \lim_{\varepsilon \to 0} \left( -i / (\omega - i\varepsilon) \right) = \delta(\omega).
\] (A.4)

These expressions lead to the following relation:
\[
\int_{-\infty}^{\infty} F_L(\omega) d\omega = \int_{-\infty}^{\infty} \frac{\varepsilon/\pi}{\omega^2 + \varepsilon^2} d\omega = \int_{-\infty}^{\infty} \delta(\omega) d\omega = 1.
\] Using this result we find the following expression for the Dirac delta function:
\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} dt = \lim_{\varepsilon \to 0} \left( \frac{\varepsilon/\pi}{\omega^2 + \varepsilon^2} \right) = \delta(\omega).
\] (A.6)

Next, the second relation, the Dirac identity, is understood in the following way. When we rewrite \(1 / (\omega - i\varepsilon)\) as
\[
\frac{\omega}{\omega^2 + \varepsilon^2} + i \frac{\varepsilon}{\omega^2 + \varepsilon^2},
\]
we easily find the following result:
\[
\frac{\varepsilon}{\pi} \int_{-\infty}^{+\infty} \frac{d\omega}{\omega^2 + \varepsilon^2} = \frac{\varepsilon}{\pi} \left[ \int_{-\infty}^{0} \frac{d\omega}{\omega^2 + \varepsilon^2} + \int_{0}^{+\infty} \frac{d\omega}{\omega^2 + \varepsilon^2} \right] = \frac{2\varepsilon}{\pi} \int_{0}^{\infty} \frac{d\omega}{\omega^2 + \varepsilon^2}.
\]
the first term on the right-hand side has the value $1/\omega$ in the limit $\varepsilon \to 0$. The second term may be replaced by a delta function. Therefore, we obtain
\[
\lim_{\varepsilon \to 0} \frac{1}{\omega - i \varepsilon} = P \frac{1}{\omega} + i\pi \delta(\omega),
\]
where $P$ is the Cauchy principal value and is defined by
\[
\int_{-\infty}^{+\infty} f(\omega') P \left[ \frac{1}{\omega - \omega'} \right] d\omega' = \int_{-\infty}^{+\infty} \frac{f(\omega')}{\omega - \omega'} d\omega' = \lim_{\varepsilon \to 0} \left( \int_{-\infty}^{-\varepsilon} \frac{f(\omega')}{\omega - \omega'} d\omega' + \int_{\varepsilon}^{+\infty} \frac{f(\omega')}{\omega - \omega'} d\omega' \right). \tag{A.7}
\]

### A.2 Cyclic Boundary Condition and Delta Function

In Appendix A.1 the Dirac delta function is defined in the region of integration $[-\infty, +\infty]$. In solid state physics, however, the dimension of a crystal of length $L$ is defined by the region $[-L/2, L/2]$ and the cyclic boundary condition is adopted. For example, considering the one-dimensional case and letting the lattice constant be $a$, the wave vectors $q = 2\pi n/L$ ($n = 0, \pm 1, \pm 2, \ldots$) are those in the first Brillouin zone $[-\pi/a, +\pi/a]$, which correspond to the lattice points $N$ ($n = -N/2, \ldots, N/2$). The calculations with respect to the wave vectors, therefore, can be carried out in the first Brillouin zone $[-\pi/a, +\pi/a]$ of the reciprocal lattice vector $2\pi/a$. Here we take account of the wave vector $q$ of the lattice vibrations, but we may draw the same conclusion for the wave vectors of an electron in a crystal.

With this definition we find
\[
\frac{1}{L} \int_{-L/2}^{L/2} e^{i(q-q')x} dx = \delta_{q,q'}, \tag{A.9}
\]
\[
\sum_{q} e^{i(q(x-x'))} = L \delta(x-x'). \tag{A.10}
\]

In the case of a crystal with $d$ dimensions, we obtain
\[
\frac{1}{L^d} \int_{V} e^{i(q-q') \cdot \mathbf{r}} d^d \mathbf{r} = \delta_{q,q'}, \tag{A.11}
\]
\[
\sum_{q} e^{i(q \cdot (\mathbf{r-r'}))} = L^d \delta(\mathbf{r-r'}). \tag{A.12}
\]

First, we will prove the one-dimensional case. The cyclic boundary condition leads to the following relation
\[
e^{iqL} = 1, \quad q = \frac{2\pi n}{L}.\]
For \( q = \neq q' \) we find
\[
\int_{-L/2}^{L/2} e^{i(q-q')x} dx = \left[ \frac{e^{i(q-q')x}}{i(q-q')} \right]_{-L/2}^{L/2} = \frac{2\sin[(q-q')L/2]}{(q-q')}, \tag{A.13}
\]
and putting \( q = 2\pi n/L \) and \( q' = 2\pi m/L \) \((m \neq n)\), the above equation becomes equal to zero. On the other hand, for \( q = q' \) we have
\[
\int_{L/2}^{L/2} e^{i(q-q')x} dx = \int_{-L/2}^{L/2} 1 dx = L,
\]
and thus we obtain the following relation
\[
\frac{1}{L} \int_{-L/2}^{L/2} e^{i(q-q')x} dx = \delta_{q,q'},
\]
and (A.9) is proved.

The relation given by (A.10) is just the inverse Fourier transform of (A.9), which will be understood from the following discussion of the Fourier transform. For simplicity we put \( x' = 0 \) and prove the following relation:
\[
\sum_q e^{iqx} = L\delta(x).
\]
Multiplying both sides of this equation by \((1/L) \exp(-iq'x)\) and integrating over the region \([-L/2, L/2]\) leads to the following result with the help of (A.9):
\[
\text{Left-hand side} = \sum_q \frac{1}{L} \int_{-L/2}^{L/2} e^{i(q-q')x} dx = \sum_q \delta_{q,q'} = 1,
\]
\[
\text{Right-hand side} = \int_{-L/2}^{L/2} e^{-iqx} dx \delta(x) = 1,
\]
and therefore the relation (A.10) is proved. The same result is obtained when the summation is replaced by the integral
\[
\sum_q = \frac{L}{2\pi} \int dq,
\]
which leads to the following relation
\[
\sum_q e^{iq(x-x')} = \frac{L}{2\pi} \int e^{iq(x-x')} dq = L\delta(x-x').
\]
From these results we may understand the relations between the delta function and the cyclic boundary condition.
In the case of the three-dimensional lattice we may deduce the same result. Consider a crystal with \( N \) lattice points and let the position vector be \( \mathbf{R}_j \). Assuming the cyclic boundary condition, we rewrite the integral over the crystal as the sum of the integral over the unit cell \( \Omega \):

\[
I = \frac{1}{L^3} \int_V \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{r}] d^3 r
\]

\[
= \frac{1}{N\Omega} \sum_j^N \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_j] \int_{\Omega} \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{r}] d^3 r
\]

\[
= \delta_{\mathbf{q},\mathbf{q}'} \frac{1}{\Omega} \int_{\Omega} \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{r}] d^3 r
\]

\[
= \begin{cases} 
1 & \text{(for } \mathbf{q} = \mathbf{q}'\text{)} \\
0 & \text{(for } \mathbf{q} \neq \mathbf{q}'\text{)}
\end{cases}, \tag{A.14}
\]

where

\[
\sum_j^N \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_j] = \sum_{j=0}^{N-1} \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_j]
\]

\[
= \frac{1 - \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_N]}{1 - \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_j]}
\]

\[
= 0 \quad \text{(for } \mathbf{q} - \mathbf{q}' \neq 0\text{)}, \tag{A.15}
\]

which is shown in the following. Rewriting \( \mathbf{q} \) and \( \mathbf{R} \) in their vector components such as \( q_x = (2\pi/L)n_x \) (\( n_x = 0, \pm 1, \pm 2, \pm 3, \ldots \)) and \( R_x = am_x \) (\( m_x = 0, 1, 2, \ldots, N - 1 \)), then we find

\[
\frac{2\pi}{L} n_x a N = 2\pi n_x. \tag{A.16}
\]

For \( \mathbf{q} - \mathbf{q}' = 0 \), we obtain

\[
\sum_{j=0}^{N-1} \exp[i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_j] = N. \tag{A.17}
\]

In general, therefore, the following relations hold for the wave vectors of electrons and phonons, \( \mathbf{k} \) and \( \mathbf{q} \):

\[
\sum_j \exp i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{R}_j = N \delta_{k,k'}, \tag{A.18}
\]

\[
\sum_j \exp i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{R}_j = N \delta_{q,q'}. \tag{A.19}
\]
In addition, we have the relation
\[ \frac{1}{\Omega} \int_{\Omega} \exp[i(q' - q) \cdot r] d^3 r = 1 \quad (\text{for } q = q'), \tag{A.20} \]
and therefore we obtain the final result.
\[ \frac{1}{L^d} \int \exp[i(q - q') \cdot r] d^d r = \delta_{q,q'} \tag{A.21} \]
\[ = \begin{cases} 1 & (\text{for } q = q') \\ 0 & (\text{for } q \neq q') \end{cases}. \]

### A.3 Fourier Transform

As is well known in mathematics, the Fourier transform is expressed as
\[ f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) \exp(ikx) dk, \tag{A.22} \]
\[ F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \exp(-ikx) dx, \tag{A.23} \]
where the function \( F(k) \) is called the Fourier transform of the function \( f(x) \) and the function \( f(x) \) is the Fourier transform of the function \( F(k) \).

The Fourier transform given by (A.23) is realized when the function \( f(x) \) satisfies the following condition
\[ \int_{-\infty}^{\infty} |f(x)|^2 dx < \infty. \tag{A.24} \]

The Fourier transform discussed above is shown for the one-dimensional case. It is easy to extend it the three-dimensional case, which is written as
\[ f(x,y,z) = \left( \frac{1}{2\pi} \right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k_x,k_y,k_z) \times \exp[i(k_x x + k_y y + k_z z)] dk_x dk_y dk_z. \tag{A.25} \]
When we introduce the vector notation \((x, y, z) = r, dx dy dz = d^3 r, (k_x, k_y, k_z) = k, \) and \( dk_x dk_y dk_z = d^3 k, \) the Fourier transform may be rewritten as
\[ f(r) = \left( \frac{1}{2\pi} \right)^{3/2} \int_{-\infty}^{\infty} F(k) \exp(i k \cdot r) d^3 k, \tag{A.26} \]
\[ F(k) = \left( \frac{1}{2\pi} \right)^{3/2} \int_{-\infty}^{\infty} f(r) \exp(-i k \cdot r) d^3 r. \tag{A.27} \]

It is very important in solid state physics to express the Fourier transform under the cyclic boundary condition. For one-dimensional space we find the following relations:
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\[ f(x) = \sum_{q} F(q)e^{iqx}, \quad (A.28) \]

\[ F(q) = \frac{1}{L} \int f(x)e^{-iqx}dx. \quad (A.29) \]

Here again (A.28) is called the Fourier transform of the function \( f(x) \), and the coefficient \( F(q) \) is the Fourier coefficient, or (A.29) is called the Fourier transform of \( f(x) \). In general, the Fourier transform in \( d \) dimensional space is given by the following relations:

\[ f(r) = \sum_{q} F(q)e^{iq\cdot r}, \quad (A.30) \]

\[ F(q) = \frac{1}{L^d} \int f(r)e^{-iq\cdot r}d^d r. \quad (A.31) \]

When we replace the summation in \( q \) space by an integral, (A.30) is written as follows.

\[ f(r) = \frac{L^d}{(2\pi)^d} \int F(q)e^{iq\cdot r}d^d r. \quad (A.32) \]

It may be proved as follows that the Fourier coefficient of (A.28) is given by (A.29). Inserting (A.28) into (A.29) and using (A.9), the following result follows:

\[ F(q) = \sum_{q'} \frac{1}{L} \int F(q')e^{i(q'-q)x}dx = \sum_{q'} F(q')\delta_{q',q} = F(q). \]

In contrast, inserting (A.29) into (A.28), we find

\[ f(x) = \sum_{q} \frac{1}{L} \int f(x')e^{iq(x-x')}dx' = \int f(x')\delta(x-x')dx = f(x) \]

and thus (A.10)

\[ \sum_{q} e^{iq(x-x')} = L\delta(x-x') \]

should hold.

B Uniaxial Stress and Strain Components in Cubic Crystals

Denoting the displacement of a crystal by \( u \) the strain tensor is defined by
When we define the force per unit area in the direction along the \( i \) axis in the plane perpendicular to the \( j \) axis by the stress tensor \( T_{ij} \), Hooke’s law is expressed as
\[
T_{ij} = c_{ijkl} e_{kl},
\]
where \( c_{ijkl} \) is called the elastic constant. Let us define the notations
\[
ij : \ xy \ yz \ yx \ zx \ zx \ \alpha : \ 1 \ 2 \ 3 \ 4 \ 5 \ 6.
\]
We may therefore rewrite (B.2) as
\[
T_\alpha = c_{\alpha\beta} e_\beta, \quad \alpha, \beta = 1, 2, 3, 4, 5, 6
\]
where \( s_{\alpha\beta} \) is the elastic compliance constant. When we define
\[
e_{\alpha\beta} = c_{ijkl},
\]
we find for strain tensors the relations
\[
2e_{yz} = 2e_{zy} = e_4, \quad 2e_{zx} = 2e_{xz} = e_5, \quad 2e_{xy} = 2e_{yx} = e_6,
\]
and for the elastic compliance constants the relations
\[
2s_{xyyz} = s_{14}, \quad 2s_{xxyy} = s_{12}, \quad 2s_{zxzz} = s_{13},
\]
Let us calculate the strain components under the application of a uniaxial stress in the (110) plane. We consider coordinates \((x', y', z')\) such that a uniaxial stress \( X \) is applied in the \( z' \) direction, and the directions \( x', y' \) are perpendicular to the stress. Then the stress tensor is written as
\[
||T'|| = \begin{vmatrix}
0 \\
0 \\
X \\
0 \\
0
\end{vmatrix}.
\]
\[ x_i = (a^{-1})_{ij} x'_j, \]  

(B.9)

the transform matrix is given by

\[
||a^{-1}|| = \begin{vmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \theta & -\frac{1}{\sqrt{2}} \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{vmatrix},
\]

(B.10)

where \( \theta \) is the angle between the \( z \) and \( z' \) axes. The transform of the stress is written as

\[ T_{ik} = (a^{-1})_{ij} (a^{-1})_{kl} T'_{jl}, \]

(B.11)

and thus the stress in the coordinates \((x, y, z)\) is given by the following relation:

\[
||T|| = \begin{vmatrix}
T_{xx} & \frac{1}{2} \sin^2 \theta \\
T_{yy} & \frac{1}{2} \sin^2 \theta \\
T_{zz} & \frac{1}{2} \sin^2 \theta \\
T_{yz} & \frac{1}{2} \sin \theta \cos \theta \\
T_{zx} & \frac{1}{2} \sin \theta \cos \theta \\
T_{xy} & \frac{1}{2} \sin \theta \cos \theta
\end{vmatrix},
\]

(B.12)

From these results the strain tensor components are expressed as

\[
||e|| = X \begin{vmatrix}
e_1 & s_{11} \left( \frac{1}{2} \sin^2 \theta + s_{12} \left( \frac{1}{2} \sin^2 \theta + \cos^2 \theta \right) \right) \\
e_2 & s_{11} \left( \frac{1}{2} \sin^2 \theta + s_{12} \left( \frac{1}{2} \sin^2 \theta + \cos^2 \theta \right) \right) \\
e_3 & s_{11} \cos \theta \sin \theta \\
e_4 & \frac{1}{\sqrt{2}} s_{44} \cos \theta \sin \theta \\
e_5 & \frac{1}{\sqrt{2}} s_{44} \cos \theta \sin \theta \\
e_6 & \frac{1}{2} s_{44} \sin^2 \theta
\end{vmatrix}.
\]

(B.13)

Since the relation between the strain tensors \( e_\alpha \) and \( e_{ij} \) is given by (B.6), we obtain the following result:

\[
e_{xx} = e_{yy} = X \left[ \frac{1}{2} s_{11} \sin^2 \theta + s_{12} \left( \frac{1}{2} \sin^2 \theta + \cos^2 \theta \right) \right] \\
e_{zz} = X \left[ s_{11} \cos^2 \theta + s_{12} \sin^2 \theta \right] \\
e_{xy} = \frac{X}{4} s_{44} \sin^2 \theta \\
e_{zx} = e_{yz} = \frac{X}{2 \sqrt{2}} s_{44} \cos \theta \sin \theta.
\]

(B.14)

As stated above, the strain tensor is a second-rank tensor and its six independent components are \( e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{zx} = e_{zy}, e_{xy} = e_{yx} \). The strain tensor is related to the symmetry of the crystal and the analysis of
the Raman scattering tensor and the deformation potentials is classified with the help of the group theory analysis of the strain tensor. In this Appendix we briefly describe the irreducible representation of the strain tensor for a crystal with cubic symmetry, where we use the notation of group theory for the zinc blende-type crystals of the $T_d$ group. In Table B.1 the character table for the $T_d$ group is shown. From Table B.1 a symmetric strain tensor $e_{ij}$ is classified into the irreducible representations of one-dimensional $\Gamma_1$, two-dimensional $\Gamma_3$ and three-dimensional $\Gamma_4$. In other words, we obtain

$$\Gamma_1 : e_{xx} + e_{yy} + e_{zz},$$

$$\Gamma_3 : e_{xx} - e_{yy}, \quad e_{zz} - (e_{xx} + e_{yy})/2,$$

$$\Gamma_4 : e_{xy}, \quad e_{xz}, \quad e_{zx}.$$  \hspace{1cm} (B.15)

It is also possible to express the strain tensor $e_{ij}$ by the following three matrices:

$$[e_{ij}(\Gamma_1)] = \frac{1}{3} \begin{bmatrix} e_{xx} + e_{yy} + e_{zz} & 0 & 0 \\ 0 & e_{xx} + e_{yy} + e_{zz} & 0 \\ 0 & 0 & e_{xx} + e_{yy} + e_{zz} \end{bmatrix}$$

$$[e_{ij}(\Gamma_3)] = \frac{1}{3} \begin{bmatrix} 2e_{xx} - (e_{yy} + e_{zz}) & 0 & 0 \\ 0 & 2e_{yy} - (e_{zz} + e_{xx}) & 0 \\ 0 & 0 & 2e_{zz} - (e_{xx} + e_{yy}) \end{bmatrix}$$

$$[e_{ij}(\Gamma_4)] = \begin{bmatrix} 0 & e_{xy} & e_{xz} \\ e_{xy} & 0 & e_{yz} \\ e_{xz} & e_{yz} & 0 \end{bmatrix}.$$
C Boson Operators

In the main text we have discussed the quantization of the lattice vibrations, where boson operators appear. Here we will describe the boson operators to supplement the treatment. For simplicity we disregard the subscripts. The Hamiltonian for a simple harmonic oscillator is written as

\[ H = \frac{1}{2M} (p^2 + M^2 \omega^2 q^2) , \]  

(C.1)

where the following commutation relation holds:

\[ [q, p] = i\hbar . \]  

(C.2)

Defining new variables by

\[ P = \sqrt{\frac{1}{M}} p , \]  

(C.3)

\[ Q = \sqrt{M} q , \]  

(C.4)

the Hamiltonian is rewritten as

\[ H = \frac{1}{2} (P^2 + \omega^2 Q^2) \]  

(C.5)

and the commutation relation is expressed as

\[ [Q, P] = i\hbar . \]  

(C.6)

As described in Sect. 6.1.2, we introduce new variables defined by

\[ a = \left( \frac{1}{2\hbar\omega} \right)^{1/2} (\omega Q + iP) , \]  

(C.7)

\[ a^\dagger = \left( \frac{1}{2\hbar\omega} \right)^{1/2} (\omega Q - iP) , \]  

(C.8)

where \( a \) and \( a^\dagger \) are Hermite conjugates. Using these operators we may define \( Q \) and \( P \), which are given by

\[ Q = \left( \frac{\hbar}{2\omega} \right)^{1/2} (a + a^\dagger) , \]  

(C.9)

\[ P = -i \left( \frac{\hbar\omega}{2} \right)^{1/2} (a - a^\dagger) . \]  

(C.10)

From these results we easily find the following relations:

\[ a^\dagger a = \frac{1}{\hbar\omega} \left( H - \frac{1}{2} \hbar\omega \right) , \]  

(C.11)

\[ aa^\dagger = \frac{1}{\hbar\omega} \left( H + \frac{1}{2} \hbar\omega \right) . \]  

(C.12)
The commutation relation between $a$ and $a^\dagger$ is written as
\[ [a, a^\dagger] = aa^\dagger - a^\dagger a = 1. \] (C.13)

Finally, the Hamiltonian is rewritten with new operators as
\[ H = \left( a^\dagger a + \frac{1}{2} \right) \hbar \omega. \] (C.14)

We have to note here that the operators $a^\dagger$ and $a$ are not observable, but that when the operators are applied to a state, they change the state. As shown in Sect. 6.1.2, we use the number operator $\hat{n}$ in the following.

Denoting the eigenstate of a simple harmonic oscillator by $|n\rangle$ and its eigenvalue by $\mathcal{E}_n$, we may write the Schrödinger equation as
\[ H|n\rangle = \hbar \omega \left( a^\dagger a + \frac{1}{2} \right) |n\rangle = \mathcal{E}_n |n\rangle. \] (C.15)

This may be rewritten as follows by using the commutation relation given by (C.13):
\[ \hbar \omega \left( aa^\dagger - 1 + \frac{1}{2} \right) |n\rangle = \mathcal{E}_n |n\rangle. \] (C.16)

Multiplication by $a^\dagger$ from the left to both sides of this equation leads to
\[ \hbar \omega \left( a^\dagger aa^\dagger - a^\dagger + \frac{1}{2} a^\dagger \right) |n\rangle = \mathcal{E}_n a^\dagger |n\rangle. \] (C.17)

Transposition of the second term on the left-hand side to the right-hand side results in
\[ \hbar \omega \left( a^\dagger a + \frac{1}{2} \right) a^\dagger |n\rangle = Ha^\dagger |n\rangle = (\mathcal{E}_n + \hbar \omega) a^\dagger |n\rangle. \] (C.18)

This equation is understood to be an eigenequation with eigenstate $a^\dagger |n\rangle$ and eigenvalue $\mathcal{E}_n + \hbar \omega$. In other words, when $a^\dagger$ operates on the eigenstate $|n\rangle$, the eigenvalue is changed from $\mathcal{E}_n$ to $\mathcal{E}_n + \hbar \omega$. Considering this fact we may write new eigenstate and eigenvalue as
\[ a^\dagger |n\rangle = c_n |n+1\rangle \] (C.19)
\[ \mathcal{E}_n + \hbar \omega = \mathcal{E}_{n+1}. \] (C.20)

Here the constant $c_n$ of (C.19) is introduced to normalize the state $|n+1\rangle$ and is determined later. Inserting these results in (C.18), we obtain
\[ H|n+1\rangle = \mathcal{E}_{n+1} |n+1\rangle. \] (C.21)
In a similar fashion, multiplying $a$ from the left to both sides of (C.16) and applying a similar treatment, we obtain

$$Ha|n\rangle = (\mathcal{E}_n - \hbar \omega) a|n\rangle.$$  
(C.22)

Here the above equation means that the eigenvalue for the eigenstate $a|n\rangle$ is given by $\mathcal{E}_n - \hbar \omega$. Therefore, in a similar fashion to the previous treatment we may rewrite as follows:

$$a|n\rangle = c'_n|n-1\rangle,$$  
(C.23)

$$\mathcal{E}_n - \hbar \omega = \mathcal{E}_{n-1},$$  
(C.24)

where $c'_n$ is a constant to normalize the eigenstate $|n-1\rangle$. These relations lead to

$$H|n-1\rangle = \mathcal{E}_{n-1}|n-1\rangle.$$  
(C.25)

It is evident from the process of deriving (C.21) and (C.25) that starting with an eigenstate $|n\rangle$ and an eigenvalue $\mathcal{E}_n$ all other eigenstates and eigenvalues are calculated. In addition, these energy eigenvalues are equally spaced with the interval $\hbar \omega$. If $|n\rangle$ is not the ground state, the eigenstate $a|n\rangle$ exists and its energy eigenvalue lower than $\mathcal{E}_n$ by $\hbar \omega$. If $a|n\rangle$ is not the ground state, there exists the eigenstate $a^2|n\rangle$ and its energy is lower than that of $|n\rangle$ by $2\hbar \omega$. In this way we find the lowest eigenstate $|0\rangle$ and its eigenenergy $\mathcal{E}_0$, where the eigenenergy $\mathcal{E}_0$ should be positive. The lowest state is understood to be the ground state. When $a$ operates on the ground state, we find

$$Ha|0\rangle = (\mathcal{E}_0 - \hbar \omega)a|0\rangle.$$  
(C.26)

Since the eigenstate with eigenenergy lower than the ground state energy is not allowed, we find

$$a|0\rangle = 0.$$  
(C.27)

Taking account of these results, the eigenequation of (C.25) for the ground state $|0\rangle$ is written as

$$H|0\rangle = \frac{1}{2}\hbar \omega|0\rangle = \mathcal{E}_0|0\rangle,$$  
(C.28)

and thus the energy eigenvalue for the ground state is given by

$$\mathcal{E}_0 = \frac{1}{2}\hbar \omega.$$  
(C.29)

From (C.20) or (C.24) the following relation is deduced.

$$\mathcal{E}_n = \left(n + \frac{1}{2}\right) \hbar \omega, \quad n = 0, 1, 2, \ldots.$$  
(C.30)
Using (C.16) and (C.30) we may obtain
\[ H|n\rangle = \hbar \omega \left( \hat{n} + \frac{1}{2} \right) |n\rangle = \hbar \omega \left( n + \frac{1}{2} \right) |n\rangle, \]  
(C.31)
\[ \hat{n}|n\rangle = a\dagger a|n\rangle = n|n\rangle. \]  
(C.32)
The above equation tells us that the eigenvalue for the operator \( \hat{n} = a\dagger a \) is \( n \).

In a similar fashion, using the commutation relation of (C.13) to (C.16), the following relation is derived:
\[ aa\dagger |n\rangle = (n+1)|n\rangle. \]  
(C.33)

Let us determine the normalization constants \( c_n \) and \( c'_n \). The normalization of the eigenstates \( |n\rangle \), \( |n+1\rangle \) and \( |n-1\rangle \) is written as
\[ \langle n|n\rangle = \langle n+1|n+1\rangle = \langle n-1|n-1\rangle. \]  
(C.34)

Multiplying the Hermite conjugate of (C.19) to both sides and using (C.33) and (C.34), we obtain
\[ \langle n+1|c^*_n c_n |n+1\rangle = \langle n|aa\dagger|n\rangle = (n+1)\langle n|n\rangle = n+1, \]  
(C.35)
which leads to
\[ |c_n|^2 = n+1. \]  
(C.36)
Assuming the phase factor of \( c_n \) to be zero, (C.19) can be rewritten as
\[ a\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \]  
(C.37)

In a similar fashion, from (C.24) we obtain
\[ a|n\rangle = \sqrt{n-1}|n-1\rangle. \]  
(C.38)
Since the eigenfunctions are diagonal, i.e. \( \langle n|n'\rangle = \delta_{n,n'} \), the non-zero matrix elements of the operators \( a\dagger \) and \( a \) are as follows:
\[ \langle n+1|a\dagger|n\rangle = \sqrt{n+1}, \]  
(C.39)
\[ \langle n-1|a|n\rangle = \sqrt{n}. \]  
(C.40)
The operator \( a\dagger \) is called the creation operator and the operator \( a \) is called the annihilation operator.

We have mentioned that a known eigenstate will determine all the other eigenstates. An arbitrary eigenstate \( |n\rangle \) is therefore derived from the ground state \( |0\rangle \). Since (C.37) leads to \( (n!)^{1/2}|n\rangle = (a\dagger)^n|0\rangle \), we find the following relation:
\[ |n\rangle = (n!)^{-1/2}(a\dagger)^n|0\rangle. \]  
(C.41)
As stated in Sect. 6.1.2, the lattice vibrations are expressed by a summation over the modes. Here we use the notation $\mu$ for the mode. The eigenstate of the lattice vibrations is given by $|n_1, n_2, \ldots, n_\mu, \ldots\rangle$ and therefore the following relations hold:

$$a_\mu |n_1, n_2, \ldots, n_\mu, \ldots\rangle = \sqrt{n_\mu} |n_1, n_2, \ldots, n_\mu - 1, \ldots\rangle,$$

$$a^\dagger_\mu |n_1, n_2, \ldots, n_\mu, \ldots\rangle = \sqrt{1 + n_\mu} |n_1, n_2, \ldots, n_\mu + 1, \ldots\rangle,$$

$$a_\mu a_\nu - a_\nu a_\mu = a^\dagger_\mu a^\dagger_\nu - a^\dagger_\nu a^\dagger_\mu = 0,$$

or

$$[a_\mu, a_\nu]_\pm = [a^\dagger_\mu, a^\dagger_\nu]_\pm = 0,$$

$$[a_\mu, a^\dagger_\nu]_\pm = \delta_{\mu\nu},$$

$$a_\mu a^\dagger_\mu |n_1, n_2, \ldots, n_\mu, \ldots\rangle = \sqrt{n_\mu + 1} a_\mu |n_1, n_2, \ldots, n_\mu + 1, \ldots\rangle = (n_\mu + 1) |n_1, n_2, \ldots, n_\mu, \ldots\rangle,$$

$$a^\dagger_\mu a_\mu |n_1, n_2, \ldots, n_\mu, \ldots\rangle = \sqrt{n_\mu} a^\dagger_\mu |n_1, n_2, \ldots, n_\mu - 1, \ldots\rangle = (n_\mu) |n_1, n_2, \ldots, n_\mu, \ldots\rangle.$$

### D Random Phase Approximation and Lindhard Dielectric Function

In this section we follow the treatment of Haug and Koch\textsuperscript{3} and discuss the plasma screening effect. The electron density operator $\langle \rho(q) \rangle$ is defined, as shown in Appendix E by

$$\langle \rho(q) \rangle = -\frac{e}{L^3} \sum_k \langle c^\dagger_k e^{-q_k} c_k \rangle.$$

Denoting the Coulomb potential $V(r)$ and the potential induced by electron fluctuations by $V_{\text{ind}}(r)$, the effective potential energy $V_{\text{eff}}(r)$ for an electron is written as

$$V_{\text{eff}}(r) = V(r) + V_{\text{ind}}(r).$$

This effective potential energy should be determined self-consistently. The Fourier transform of the effective potential energy leads to

\[
V_{\text{eff}}(q) = V(q) + V_{\text{ind}}(q). \tag{D.3}
\]

The electron Hamiltonian is then given by
\[
\mathcal{H} = \sum_k \mathcal{E}(k) c_k^\dagger c_k + \sum_{k,q'} V_{\text{eff}}(q') c_{k+q'}^\dagger c_k. \tag{D.4}
\]

The Heisenberg equation for \(c_{k-q}^\dagger c_k\) gives the following relation:
\[
\frac{d}{dt} c_{k-q}^\dagger c_k = \frac{i}{\hbar} \left[ \mathcal{H}, c_{k-q}^\dagger c_k \right] = \frac{i}{\hbar} (\mathcal{E}(k-q) - \mathcal{E}(k)) c_{k-q}^\dagger c_k \\
- \frac{i}{\hbar} \sum_{q'} V_{\text{eff}}(q') \left( c_{k-q}^\dagger c_{k-q'} - c_{k+q'}^\dagger c_k \right). \tag{D.5}
\]

Here we use the random phase approximation to evaluate the above equation. The random phase approximation is based on the following assumption. We assume that the expectation value \(\langle c_k^\dagger c_{k'} \rangle\) is approximated by \(\langle c_k^\dagger c_{k'} \rangle \propto \exp[i(\omega_k - \omega_{k'}) t]\). In the summation \(\sum_{k,k'} \exp[i(\omega_k - \omega_{k'}) t]\), the term \(k \neq k'\) oscillates and the average contribution is assumed to vanish. Therefore, only the term for \(k = k'\) will contribute to the summation. This assumption is called the random phase approximation. Applying the random phase approximation to the last two terms on the right hand side of (D.5) we obtain (see the reference in the footnote of page 539)
\[
\frac{d}{dt} \langle c_{k-q}^\dagger c_k \rangle = \frac{i}{\hbar} (\mathcal{E}(k-q) - \mathcal{E}(k)) \langle c_{k-q}^\dagger c_k \rangle \\
- \frac{i}{\hbar} V_{\text{eff}}(q) (f(k-q) - f(k)), \tag{D.6}
\]

where we use the following relation
\[
f(k) = \langle c_k^\dagger c_k \rangle. \tag{D.7}
\]

When we assume that the electron density fluctuate as \(\langle c_{k-q}^\dagger c_k \rangle \propto \exp[-i(\omega + i\Gamma/\hbar) t]\), the following relation may be obtained from (D.6)
\[
(h\omega + i\Gamma + \mathcal{E}(k-q) - \mathcal{E}(k)) \langle c_{k-q}^\dagger c_k \rangle \\
= V_{\text{eff}}(q) [f(k-q) - f(k)]. \tag{D.8}
\]

Multiplying by \(-e/L^3\) on both sides, summing up with respect to \(k\), and using the relation given by (D.1), we obtain
\[
\langle \rho(q) \rangle = -\frac{e^2}{L^3} V_{\text{eff}}(q) \sum_k \frac{f(k-q) - f(k)}{h\omega + i\Gamma + \mathcal{E}(k-q) - \mathcal{E}(k)}. \tag{D.9}
\]
Since the potential due to the induced charge follows Poisson’s equation, it is given by
\[ \nabla^2 V_{\text{ind}}(r) = \frac{e \rho(r)}{\epsilon_0}. \] (D.10)

The Fourier transform of this equation results in
\[ V_{\text{ind}}(q) = -\frac{e}{\epsilon_0 q^2} \rho(q) = \frac{e^2}{\epsilon_0 q^2 L^3} V_{\text{eff}}(q) \sum_k \frac{f(k-q) - f(k)}{\hbar \omega + i \Gamma + \mathcal{E}(k-q) - \mathcal{E}(k)} \]
\[ = V(q) V_{\text{eff}}(q) \sum_k \frac{f(k-q) - f(k)}{\hbar \omega + i \Gamma + \mathcal{E}(k-q) - \mathcal{E}(k)}. \] (D.11)

Inserting this into (D.3), (6.257) is derived as the following:
\[ \kappa(q,\omega) = 1 - V(q) \sum_k \frac{f(k-q) - f(k)}{\hbar \omega + i \Gamma + \mathcal{E}(k-q) - \mathcal{E}(k)} \]
\[ = 1 - \frac{e^2}{\epsilon_0 q^2 L^3} \sum_k \frac{f(k-q) - f(k)}{\hbar \omega + i \Gamma + \mathcal{E}(k-q) - \mathcal{E}(k)}. \] (D.12)

### E Density Matrix

In this section the density matrix is summarized. A good introduction to the density matrix method is given in the text of Kittel,\(^\text{4}\) which we shall follow here.

First, we assume a complete and orthonormal set of functions \(u_n\). Any function may be expanded by using these functions and therefore an eigenstate for the Hamiltonian \(H\) is expressed as
\[ \psi(x,t) = \sum_n c_n(t) u_n(x), \] (E.1)

where the orthnormality of the functions gives the following relation:
\[ \langle u_n | u_m \rangle = \int u_n^* u_m dx = \delta_{nm}. \] (E.2)

The density matrix is defined by
\[ \rho_{nm} = \overline{c_n^* c_m}. \] (E.3)

We have to note that the order of \(m\) and \(n\) on the two sides of (E.3) is interchangeable. The bar indicates the ensemble average over all the systems in the ensemble. Several important properties of the density matrix are summarized in the following.

1. \[ \sum_n \rho_{nn} = \text{Tr} \{ \rho \} = 1 . \]

This property leads to the following relation:

\[ \langle \psi | \psi \rangle = \sum_n c_n^* c_n = \sum_n \rho_{nn} = \text{Tr} \{ \rho \} = 1 . \]  \hspace{1cm} (E.4)

2. \[ \langle F \rangle = \text{Tr} \{ F \rho \} . \]

Here \( \langle F \rangle \) represents the ensemble average of the expectation value of an observable \( F \). This relation is derived as follows:

\[ \langle F \rangle = \langle \psi | F | \psi \rangle = \sum_{m,n} F_{mn} c_m^* c_n = \sum_{m,n} F_{mn} \rho_{nm} \]  \hspace{1cm} (E.5)

and thus

\[ \langle F \rangle = \sum_m (F \rho)_{mm} = \text{Tr} \{ F \rho \} . \]  \hspace{1cm} (E.6)

It is very important to point out that traces are independent of the representation and thus that the ensemble average \( \langle F \rangle \) is independent of the representation.

3. \[ i \hbar \frac{\partial \rho}{\partial t} = -[\rho, H] = -(\rho H - H \rho) . \]

The above equation gives the time dependence of the density matrix \( \rho \) of the Hamiltonian \( H \). In order to derive the equation, we begin with the wave function (E.1) and insert it into the Schrödinger equation

\[ i \hbar \frac{\partial \psi}{\partial t} = H \psi . \]  \hspace{1cm} (E.7)

First, we insert (E.1) into (E.7)

\[ i \hbar \frac{\partial \psi}{\partial t} = i \hbar \sum_k \frac{\partial c_k}{\partial t} u_k(x) = H \psi = \sum_k c_k H u_k(x) , \]  \hspace{1cm} (E.8)

and then multiplying by \( u_n(x) \) from the left and integrating over all space we obtain the equation

\[ i \hbar \frac{\partial c_n}{\partial t} = \sum_k H_{nk} c_k , \]  \hspace{1cm} (E.9)

where we have used the orthonormality property (E.2) and

\[ H_{nk} = \langle u_n | H | u_k \rangle = \int u_n^*(x) H u_k(x) dx . \]  \hspace{1cm} (E.10)

In a similar fashion we obtain
\[ -i\hbar \frac{\partial c^*_m}{\partial t} = \sum_k H^*_{mk} c^*_k. \]  
(E.11)

From (E.3) we obtain
\[ i\hbar \frac{\partial \rho_{nm}}{\partial t} = i\hbar \frac{\partial}{\partial t} \left( \frac{\partial c^*_m c_n}{\partial t} + c^*_m \frac{\partial c_n}{\partial t} \right). \]  
(E.12)

Inserting (E.9) and (E.11) into (E.12), we find
\[ i\hbar \frac{\partial \rho_{nm}}{\partial t} = - (\rho H - H \rho)_{nm}. \]  
(E.13)

4. \( Z = \text{Tr} \{ e^{-\beta H} \} \)

Here, \( Z \) is the partition function. For a canonical ensemble (see the reference in the footnote of page 541)
\[ \rho = e^{\beta (F - H)}, \]  
(E.14)

where \( F \) is the Helmholtz free energy and \( H \) is the Hamiltonian. In the quantum mechanical representation the partition function \( Z \) is given by (see the reference of the footnote of page 541)
\[ Z = \sum_i e^{-\beta E_i}, \]  
(E.15)

where \( \beta = k_B T \). Using the relation between the Helmholtz free energy and the partition function \( \log Z = -\beta F \), we find
\[ Z = e^{-\beta F} = \sum e^{-\beta E_n} = \text{Tr} \{ e^{-\beta H} \}. \]  
(E.16)

Since the trace is invariant under unitary transformations, the partition function may be calculated by taking the trace of \( e^{-\beta H} \) in any representation. Using these results one may find
\[ \rho = \frac{e^{-\beta H}}{\text{Tr} \{ e^{-\beta H} \}}, \]  
(E.17)

which is a very important relation and is used to evaluate the ensemble average of an observable quantity.

**F Spontaneous and Stimulated Emission Rates**

Here we will derive the relations between absorption, spontaneous emission, and stimulated emission.
Let us define a vector potential by $A$ and put $B = \text{rot} \ A = \nabla \times A$, then we obtain

$$\nabla \cdot B = \nabla \cdot \nabla \times A = 0,$$

(F.1)

and thus the vector potential satisfies the relation for the magnetic flux $B$ of (9.90d), $(\nabla \cdot B = 0)$. We insert the relation $B = \text{rot} \ A = \nabla \times A$ into Maxwell’s equation (9.90b), we find

$$\nabla \times E = -\frac{\partial}{\partial t} \nabla \times A.$$

(F.2)

Therefore we obtain the following relation

$$E = i \omega A.$$

(F.3)

Since we deal with the squared values of the vector potential and electric field, we put

$$E = \omega A.$$

(F.4)

The electromagnetic field interact with an electron of charge $-e$, and the interaction is given by the Hamiltonian

$$H = \frac{1}{2m} (p + eA)^2 + V(r),$$

(F.5)

where $V(r)$ is the periodic potential of a crystal. This equation is rewritten as

$$H = \frac{p^2}{2m} + V(r) + \frac{e}{m} (A \cdot p) + \frac{1}{2m} (eA)^2$$

(F.6)

and neglecting the last term because of its small contribution, we may rewrite

$$H = H_0 + H_1,$$

(F.7)

$$H_0 = \frac{p^2}{2m} + V(r),$$

(F.8)

$$H_1 = \frac{e}{m} (A \cdot p).$$

(F.9)

Treating $H_1$ as a perturbation term, the transition probability between the initial state $|i\rangle$ and the final state $|f\rangle$ is given by

\[\begin{align*}
\text{(In (F.5), we operate $A \cdot p + p \cdot A$ to a scalar function $f$ and taking account of the vector potential of the electromagnetic field $A = A_0 \exp(ik \cdot r)$ and of the momentum operator $p = -i\hbar \nabla$,)}

(A \cdot p + p \cdot A)f = A \cdot (p + p + \hbar k) f = 2A \cdot pf

\end{align*}\]

is obtained. The last relation was deduced from the fact that the electromagnetic field is transverse wave, resulting in $A \cdot k_p = 0$.\]
\[ w_{if} = \frac{2\pi}{\hbar} |\langle f | H_1 | i \rangle|^2 \delta [\mathcal{E}_f - \mathcal{E}_i - \hbar \omega]. \]  

(F.10)

The transition rate between the valence band state \(|c k\rangle\) and the conduction band state \(|v k'\rangle\) is given by (4.31) in (4.2)

\[ w_{cv} = \frac{2\pi}{\hbar} |\langle c k' | e \cdot p | v k \rangle|^2 \delta [\mathcal{E}_c(k') - \mathcal{E}_v(k) - \hbar \omega]. \]

Writing the vector potential \(A\) using the unit vector of the vector potential \(e\) as \(A = e \cdot A\), the matrix element is given by

\[ |M| = |\langle c k' | \exp(i k_p \cdot r) e \cdot p | v k \rangle|, \]  

(F.11)

and we have

\[ |M|^2 = \frac{1}{3} \left( |M_x|^2 + |M_y|^2 + |M_z|^2 \right), \]  

(F.12a)

\[ M_x = -i\hbar \langle c k' | \exp(i k_p) \frac{\partial}{\partial x} | v k \rangle. \]  

(F.12b)

Since \(k_p\) is very small, we put \(\delta(k' - k - k_p) \equiv \delta(k' - k) = 0\) and thus the summation of the allowed \(k'\) and \(k\) give rise to Kronecker delta \(\delta_{k',k}\).

Using (4.21) and (4.38) or (4.39), the absorption coefficient is given by

\[ \alpha = \frac{\omega \kappa_2}{n_t c} = \frac{2\hbar \omega}{n_t c \epsilon_0 \omega^2 A_0^2} w_{cv} \]

\[ = \frac{\pi \epsilon^2}{n_t c \epsilon_0 m^2 \omega} \sum_{k,k'} |M|^2 \delta [\mathcal{E}_c(k') - \mathcal{E}_v(k) - \hbar \omega] \delta_{k,k'}. \]  

(F.13)

In the case of semiconductor lasers, high densities of electrons and holes are injected into the active region and occupy the conduction band and valence bands. Therefore the absorption coefficient depends on the occupation factors of the electron and holes. Let the occupation factor of the electrons in the upper and lower states as \(f(\mathcal{E}_2)\) and \(f(\mathcal{E}_1)\), respectively, and the net rate of the photon absorption and emission is proportional to \(f(\mathcal{E}_1)[1 - f(\mathcal{E}_2)] - f(\mathcal{E}_2)[1 - f(\mathcal{E}_1)] = f(\mathcal{E}_1) - f(\mathcal{E}_2)\). Then the absorption coefficient is given by

\[ \alpha = \frac{\pi \epsilon^2}{n_t c \epsilon_0 m^2 \omega} \times \sum_k |M|^2 \delta [\mathcal{E}_2(k) - \mathcal{E}_1(k) - \hbar \omega] \delta_{k,k'} [f(\mathcal{E}_1) - f(\mathcal{E}_2)]. \]  

(F.14)

In the above equation \(\sum\) is carried out over the pair states of the valence band \(|v k\rangle\) and the conduction band \(|c k\rangle\), and thus we obtain
\[
\sum_k \delta[\mathcal{E}_{cv}(k) - \hbar \omega] = \frac{1}{(2\pi)^3} \int d^3k \cdot \delta[\mathcal{E}_{cv}(k) - \hbar \omega] \\
\equiv \int \rho_{\text{red}}(\hbar \omega) \cdot d(\hbar \omega), \tag{F.15}
\]

\[
\mathcal{E}_{cv} = \mathcal{E}_c(k) - \mathcal{E}_v(k), \tag{F.16}
\]

where only one direction of spin orientation is considered. The last relation of (F.15) is obtained by putting the photon energy as \(\hbar \omega = \mathcal{E}\) and defining the density of states between the energies \(\hbar \omega = \mathcal{E}\) and \(\hbar \omega + d(\hbar \omega) = \mathcal{E} + d\mathcal{E}\) as \(\rho_{\text{red}}(\mathcal{E})d\mathcal{E}\). When the effective masses of the conduction and valence bands are isotropic, putting the electron effective mass as \(m_c\) and the hole effective mass as \(m_h\), we may write

\[
\mathcal{E}_{cv} = \frac{\hbar^2 k^2}{2m_c} + \frac{\hbar^2 k^2}{2m_h} + \mathcal{E}_G = \frac{\hbar^2 k^2}{2\mu} + \mathcal{E}_G, \tag{F.17}
\]

where \(1/\mu = 1/m_c + 1/m_h\) and \(\mu\) is called reduced mass. Then the density of states is given by

\[
\rho_{\text{red}} \cdot d(\hbar \omega) = \frac{1}{(2\pi)^3} 4\pi k^2 \cdot dk, \\
= \frac{2\pi}{(2\pi)^3} \left( \frac{2\mu}{\hbar^2} \right)^{3/2} \sqrt{\hbar \omega - \mathcal{E}_G} d\hbar \omega. \tag{F.18}
\]

Writing the photon energy as \(\hbar \omega = \mathcal{E}\), we find

\[
\rho_{\text{red}} d(\mathcal{E}) = \frac{2\pi}{(2\pi)^3} \left( \frac{2\mu}{\hbar^2} \right)^{3/2} \sqrt{\mathcal{E} - \mathcal{E}_G} d\mathcal{E}. \tag{F.19}
\]

Next we discuss quantum theory of spontaneous and stimulated emissions[9.6]. First, we deal with the radiation of the electromagnetic waves based on the classical theory. Using Poynting vector of the electromagnetic waves and vector potential \(\mathbf{A}\) given by \(\nabla \times \mathbf{A} = \mathbf{B}\), and (9.90b), the flux of the waves is given by the following relation

\[
\langle \mathbf{S} \rangle = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}) = \frac{1}{2} \text{Re} [(-i\omega \mathbf{A}) \times (i\mathbf{k}_p \times \mathbf{A})] / \mu_0 \\
= \frac{\omega}{2\mu_0} \text{Re} ([\mathbf{A} \cdot \mathbf{k}_p] \mathbf{k}_p - [\mathbf{A} \cdot \mathbf{A}] \mathbf{k}_p) = \frac{n_r \omega^2}{2c\mu_0} |\mathbf{A}|^2 \frac{\mathbf{k}_p}{|\mathbf{k}_p|}, \tag{F.20}
\]

where \(\mu_0\) is the magnetic permeability in free space, \(1/(\epsilon_0\mu_0) = c^2\) (c: the light speed in free space) and \(\mathbf{k}_p/|\mathbf{k}_p|\) is a unit vector in the propagation direction of the vector potential \(\mathbf{A}\). The last relation is derived for the transverse waves in uniform medium, and putting \(\mathbf{k}_p \cdot \mathbf{A} = 0\).

Next, we discuss Planck’s radiation theory. Since the number of plane waves modes in a volume \(V\) in an element \(dk_x dk_y dk_z\) of the \(\mathbf{k}_p\) space is \(V(2\pi)^{-3}dk_x dk_y dk_z\), and is independent of the shape of the sample or of the
boundary conditions provided the dimensions are large compared to the wave-
length. Putting $k_p = |k_p| = n_r \omega / c$, and taking account of two independent
polarization directions for each wave vector of the electromagnetic waves, the
density of modes per unit volume between $\omega$ and $\omega + d\omega$ is given by $G(\omega)d\omega$
with
$$ G(\omega) = \frac{2}{(2\pi)^3} 4\pi k_p^2 \frac{dk_p}{d\omega} = \frac{k_p^2}{\pi^2 v_g} = \frac{n_r^2 \omega^2}{\pi^2 c^2 v_g}. $$

(F.21)

Here the group velocity $v_g = \frac{d\omega}{dk_p}$ is used and assumed to be constant
except strong absorption region with the anomalous dispersion.

The average energy of a mode with angular frequency $\omega$ at temperature $T$
under the thermal equilibrium is given by
$$ \langle \mathcal{E}(\omega) \rangle = \frac{\hbar \omega}{\exp(\hbar \omega/k_B T) - 1}. $$

(F.22)

Thus the energy density of blackbody radiation $u(\omega)d\omega = \langle \mathcal{E}(\omega) \rangle G(\omega)d\omega$ in
the range between $\omega$ and $\omega + d\omega$ is obtained as
$$ u(\omega) = \frac{n_r^2 \hbar \omega^3}{\pi^2 c^2 v_g} \frac{1}{\exp(k_B T) - 1}. $$

(F.23)

Since the velocity of the energy flow of the electromagnetic waves in a
dielectric is the group velocity, the time average of the radiation with wave
vector $k_p$, lying an element of solid angle $d\Omega$, with polarization vector $e$
lying in an angular interval $d\theta$ in a plane perpendicular to $k_p$, and with an angular
frequency in a range $d\omega$, is
$$ |\langle S \rangle| = u(\omega) v_g \frac{d\Omega}{4\pi} \frac{d\theta}{2\pi} d\omega $$
$$ = \frac{n_r^2 \hbar \omega^3}{\pi^2 c^2} \frac{1}{\exp(k_B T) - 1} \frac{d\Omega}{4\pi} \frac{d\theta}{2\pi} d\omega. $$

(F.24)

Averaging (F.24) in the solid angle $\Omega$ and polarization direction, we find
$$ \int \frac{d\omega}{4\pi} \int \frac{d\theta}{2\pi} = 1, $$
and thus we obtain,
$$ |\langle S \rangle| = \frac{n_r^2 \hbar \omega^3}{\pi^2 c^2} \frac{1}{\exp(k_B T) - 1} d\omega. $$

(F.25)

Using (F.11), the spontaneous emission rate from the upper state $\mathcal{E}_2$ to the
lower state $\mathcal{E}_1$ is given by
$$ r_{\text{spon}}(\mathcal{E}) = \frac{\pi e^2 |A|^2}{2m^2 \hbar} |M|^2 \rho_{\text{red}}(\mathcal{E}) [f(\mathcal{E}_2)(1 - f(\mathcal{E}_1))]. $$

(F.26)
Equating (F.20) and (F.25), and eliminating $|A|$, we find

$$r_{\text{spon}}(\mathcal{E}) = \frac{n_e e^2 \mu_0 \omega}{\pi m^2 c} |M|^2 \rho_{\text{red}}(\mathcal{E}) f(\mathcal{E}_2) [1 - f(\mathcal{E}_1)].$$  \hspace{2cm} (F.27)

Using the relation $\epsilon_0 \mu_0 = 1/c^2$, and putting $\hbar \omega = \mathcal{E}$, the spontaneous emission rate between the energy separation $\mathcal{E}$ and $\mathcal{E} + d\mathcal{E}$, is then given by

$$r_{\text{spon}}(\mathcal{E}) = \frac{n_e e^2 \mathcal{E}}{\pi \epsilon_0 m^2 \hbar^2 c^3} |M|^2 \rho_{\text{red}}(\mathcal{E}) f(\mathcal{E}_2) [1 - f(\mathcal{E}_1)],$$  \hspace{2cm} (F.28)

and thus we obtain the spontaneous emission rate given by (9.14). In a similar fashion, the stimulated emission rate (9.15) is given by

$$r_{\text{stim}}(\mathcal{E}) = \frac{n_e e^2 \mathcal{E}}{\pi \epsilon_0 m^2 \hbar^2 c^3} |M|^2 \rho_{\text{red}}(\mathcal{E}) (f_2 - f_1).$$  \hspace{2cm} (F.29)
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