Appendix
Network Connectivity and Fault-Tolerance Measures in Three-Dimensional Deployment Fields

This appendix computes the connectivity and fault-tolerance of three-dimensional homogeneous and heterogeneous $k$-covered wireless sensor networks. The proposed measures are based on the Reuleaux tetrahedron model, which is used to characterize $k$-coverage of a three-dimensional field. This choice is to make this computation problem more tractable. Moreover, based on the concepts of conditional connectivity and forbidden faulty sensor set, this appendix proposes conditional network connectivity and fault-tolerance measures for the above networks.

1 Introduction

Data accuracy depends on the size of the connected component that contains the sink. It reaches the highest value when the sink belongs to the largest connected component of the network. Thus, high-quality coverage requires all source sensors be connected to the sink. That is why we focus on the sink to compute the connectivity of three-dimensional $k$-covered wireless sensor networks [14]. In other words, connectivity of wireless sensor networks should be defined so as to take into consideration the inherent structure of this type of network. Indeed, sensors have neither the same role nor the same impact on the network performance. Thus, measuring the connectivity of wireless sensor networks should account for their specific morphology, where the sink is the most critical node in the network. Hence, we compute the connectivity of three-dimensional $k$-covered wireless sensor networks based on the size of the connected component that includes the sink.

The remainder of this appendix is organized as follows. Section 2 revisits $k$-coverage in three-dimensional deployment fields. Section 3 computes unconditional connectivity for three-dimensional homogeneous and heterogeneous $k$-covered wireless sensor networks while Section 4 computes their conditional counterparts. Section 5 discusses the results obtained in the previous sections. Section 6 shows how to relax the unit sphere model and account for a convex model for sensing. Section 7 discusses the results for underwater sensor networks while Sect. 8 concludes the appendix.

2 $k$-Coverage Characterization

In this section, we show how to guarantee $k$-coverage of a three-dimensional field and derive the corresponding sensor spatial density. Also, we provide some simulation results showing a perfect match with their theoretical counterparts.
A Sensor Density for \( k \)-Coverage

Lemma A.1 characterizes the breadth of a three-dimensional \( k \)-covered convex region.

**Lemma A.1:** A three-dimensional convex region \( C \) is guaranteed to be \( k \)-covered with exactly \( k \) sensors if its breadth is less than or equal to \( r \), the radius of the sensing spheres of sensors.

**Proof:** (By contradiction) Assume that the breadth of a three-dimensional convex region \( C \) does not exceed \( r \) and \( C \) is not \( k \)-covered when exactly \( k \) sensors are deployed in it. Notice that each of these \( k \) sensors is located on the boundary or inside of \( C \). Thus, there must be at least one location \( \xi \in C \) that is not \( k \)-covered. In other words, there is at least one sensor \( s_i \) located at \( \xi \) such that \(|\xi - \xi'| > r\), which contradicts our hypothesis that the breadth of \( C \) does not exceed \( r \). ■

It is true that the deployment of \( k \) sensors in a 3D convex region, say \( C \), whose breadth is larger than \( r \) cannot guarantee its \( k \)-coverage, where \( r \) is the radius of the sensing range of the sensors. Let \( p_i \) and \( p_j \) be two points in \( C \) such that one sensor \( s_i \) is located at \( p_i \) and \( \delta(p_i,p_j) = b > r \), where \( b \) is the breadth of \( C \) and \( \delta(p_i,p_j) \) is the Euclidean distance between \( p_i \) and \( p_j \). Given that \( b > r \), it is impossible for \( s_i \) to sense any event that occurs at \( p_j \). Thus, there is at least one sensor (i.e., \( s_i \)) amongst those \( k \) sensors, which cannot cover \( p_j \), and hence \( C \) cannot be \( k \)-covered.

Next, we compute the sensor spatial density required for guaranteeing \( k \)-coverage of a three-dimensional field. To this end, we compute the volume of a three-dimensional convex region \( C \) that is guaranteed to be \( k \)-covered when exactly \( k \) sensors are deployed in it. From Helly's Theorem [44], we infer that given \( k \geq 4 \) sensors, a three-dimensional convex region \( C \) is \( k \)-covered by those \( k \) sensors if and only if \( C \) is 4-covered by any four of those \( k \) sensors. Given that the breadth of \( C \) is \( r \), the network induced by sensors located in \( C \) is guaranteed to be connected if \( R \geq r \), where \( r \) and \( R \) are, respectively, the radii of the sensing and communication spheres of sensors. Now, let us address the first question: What is the sensor spatial density necessary to guarantee full \( k \)-coverage of a three-dimensional field? Theorem A.1 computes this sensor density.

**Theorem A.1:** Let \( r \) be the radius of the sensing spheres of sensors and \( k \geq 4 \). The sensor spatial density required to \( k \)-cover a three-dimensional field is computed as

\[
\lambda(r,k) = \frac{k}{0.422 r_0^3}
\]

where \( r_0 = r/1.066 \).

**Proof:** Let \( C_k \) be the intersection of \( k \) sensing spheres and assume that their centres do not coincide in a three-dimensional field. From Lemma A.1, it follows that \( C_k \) is guaranteed to be \( k \)-covered by exactly \( k \) sensors if its breadth does not exceed the radius \( r \) of the sensing spheres of sensors. Thus, the maximum volume of \( C_k \) is obtained when its breadth is equal to \( r \). From Helly’s Theorem [44], it follows that the intersection of \( k \) sensing spheres is not empty if the intersection
of any four of these $k$ spheres is not empty. On the other hand, the intersection set operator requires that the maximum intersection volume of these $k$ sensing spheres be equal to that of four spheres provided that the maximum distance between any pair of these $k$ sensors does not exceed $r$. Let us focus on the analysis of four sensing spheres. The maximum overlap volume of four sensing spheres such that every point in this overlap volume is 4-covered, corresponds to the configuration where the centre of each sensing sphere is at distance $r$ from the centres of all other three sensing spheres. Precisely, the sensing sphere of each of the four sensors passes through the centres of the other three sensing spheres as shown in Fig. A.1. The edges between the centres of these four spheres form a regular tetrahedron and the shape of the intersection volume of these four spheres is known as the Reuleaux tetrahedron [234] and denoted by $RT(r)$. Unfortunately, it was proved that the Reuleaux tetrahedron does not have a constant breadth [234]. Indeed, while the distance between some pairs of points on the boundary of the Reuleaux tetrahedron $RT(r)$ is equal to $r$, the maximum distance between other pairs of points on the boundary of $RT(r)$ is equal to 1.066 $r$ [234], i.e., slightly larger than $r$. This implies that the Reuleaux tetrahedron $RT(r)$ cannot be $k$-covered with exactly $k$ sensors given that the distance between some pairs of points on the boundary of $RT(r)$ is larger than $r$, where $r$ is the radius of the sensing spheres of the sensors. Therefore, the Reuleaux tetrahedron that is guaranteed to be $k$-covered with exactly $k$ sensors should have a side length equal to $r_0 = r/1.066$. The volume of the Reuleaux tetrahedron $RT(r_0)$ is given by [234]

$$vol(r_0) = (3\sqrt{2} - 49 \pi + 162 \tan^{-1}(\sqrt{2}) r_0^3) / 12 = 0.422 r_0^3$$

Thus, $RT(r_0)$ is the maximum volume that can be $k$-covered by exactly $k$ sensors, where $k \geq 4$. We conclude that the maximum volume of $C_k$, denoted by $vol_{max}(C_k)$, is equal to $vol_{max}(C_k) = 0.422 r_0^3$. Given that $vol_{max}(C_k)$ has to contain $k$ sensors to $k$-cover $C_k$, we conclude that the sensor spatial density per unit volume required for full $k$-coverage of a three-dimensional field is computed as

$$\lambda(r,k) = k / vol_{max}(C_k) = k / 0.422 r_0^3$$

Related to our work is the novel result discussed in [234], which proved that the breadth of the Reuleaux tetrahedron is not constant. This shows that the properties of 2D space cannot be directly extended to 3D space. Indeed, the Reuleaux triangle [233] (counterpart of Reuleaux tetrahedron in 2D space) has a constant width. Note that the Reuleaux tetrahedron is the symmetric intersection of four congruent spheres such that each sphere passes through the centres of the other three spheres. However, the Reuleaux triangle [233] corresponds to the symmetric intersection of three congruent disks such that each disk passes through the centres of the other two disks. Thus, its constant width is equal to the radius of these disks. Our work can be viewed as an extension of [8] by considering $k$-coverage in 3D WSNs. Moreover, existing works on coverage and connectivity in WSNs assumed the notion of traditional connectivity, whereas our work considers a more realistic measure, namely conditional connectivity [101], which is based on the concept of forbidden faulty set [81]. Also, our work exploits the result given
in [234] with regard to the breadth of the Reuleaux tetrahedron discussed earlier. This helps us provide correct measures of connectivity and fault-tolerance of 3D WSNs based on an accurate characterization of k-coverage of 3D fields.

We should mention that the notion of the arc length discussed in [234] corresponds to the (maximum) breadth of the Reuleaux tetrahedron. Indeed, it is possible to find two parallel plans that bound the Reuleaux tetrahedron such that the maximum distance between these two plans is equal to \( r \) only.

It is worth emphasizing that the value of \( \lambda(r,k) \) is tight in the sense that it is minimum given that \( k \) sensors should be located within \( \text{vol}_{\text{max}}(C_k) \), such that \( C_k \) is guaranteed to be k-covered with exactly these \( k \) sensors. Also, notice that \( \lambda(r,k) \) depends only on the coverage degree \( k \) dictated by a sensing application and the radius \( r \) of the sensing range of sensors. Moreover, \( \lambda(r,k) \) increases with \( k \) and decreases as \( r \) increases, thus reflecting the expected behaviour.

Lemma A.2 uses Theorem A.1 and states a sufficient condition for k-coverage of a three-dimensional field.

**Lemma A.2:** A three-dimensional field is guaranteed to be k-covered if any Reuleaux tetrahedron region of side \( r_0 \) in the field contains at least \( k \) sensors, where \( r_0 = r/1.066 \) and \( k \geq 4 \).

\[ \lambda(r,k) \]

\[ 10^{-4} \]

\[ r = 30 \]

**Fig. A.1** Sensor spatial density \( \lambda(r,k) \) vs. \( k \)
### B Simulation Results

In this section, we present the simulation results using a high-level simulator written in the C programming language. We consider a cubic field of side length 1000m, where all sensors are randomly and uniformly deployed. Moreover, we randomly decompose the cubic field into overlapping Reuleaux tetrahedra as follows. Two adjacent Reuleaux tetrahedra, say $RT_1$ and $RT_2$, overlap in a way such that one of the faces of the regular tetrahedron corresponding to $RT_1$ entirely coincides with one of the faces of the regular tetrahedron corresponding to $RT_2$. The volume of the intersection of two adjacent Reuleaux tetrahedra forms a three-dimensional lens. Each Reuleaux tetrahedron is adjacent to at most four other Reuleaux tetrahedra. In the first experiment, we fix the radius $r$ of the sensing range to 30m and vary $k$. In the second experiment, we fix $k$ to 4 and vary $r$. All simulations are repeated 100 times the results are averaged.

![Fig. A.2 Sensor spatial density $\lambda(r,k)$ vs. $r$](image)

The plot in Fig. A.1 shows the analytical result of Theorem A.1 and simulations results of the first experiment. This figure demonstrates is a close match between the analytical and simulation results. Note that $\lambda(r,k)$ increases with the coverage degree $k$. Indeed, high coverage degree $k$ requires more sensors to be deployed, and hence a denser wireless sensor network is necessary. The plot in Fig. A.2 shows the analytical result of Theorem A.1 and simulations results of
the second experiment with a close match. Notice that $\lambda(r,k)$ decreases when $r$ increases. When the sensing range gets larger, a fewer number of sensors is needed to fully $k$-cover a three-dimensional field. It is worth noting that the difference between analytical and simulation results is due to the boundary effects. It is impossible to decompose a three-dimensional field into complete Reuleaux tetrahedra such that the Reuleaux tetrahedra close to the border of a three-dimensional field lie entirely inside it. Hence, more than the required number of sensors is used to $k$-cover the border of the three-dimensional field.

Next, based on the minimum sensor spatial density (result of Theorem A.1) and another criterion to be specified in the following section, we compute the network connectivity of homogeneous and heterogeneous three-dimensional $k$-covered wireless sensor networks.

### 3 Unconditional Connectivity

In this section, we compute measures of unconditional (or traditional) connectivity for homogeneous and heterogeneous three-dimensional $k$-covered wireless sensor networks, where any subset of sensors can fail.

#### A Homogeneous Sensors

Theorem A.2 deals with homogeneous wireless sensor networks. 

**Theorem A.2:** Let $G$ be a communication graph of a homogeneous three-dimensional $k$-covered wireless sensor network deployed in a cubic field of volume $V$. The connectivity of $G$ is given by

$$\kappa_1(G) \leq \kappa(G) \leq \kappa_3(G)$$

where

$$\kappa_1(G) = 12.024 \alpha^3 k$$

$$\kappa_3(G) = \frac{RV^{2/3} k}{0.422 n_0^3}$$

$r_0 = r/1.066$, $\alpha = R/r$, and $k \geq 4$.

**Proof:** The optimum position of the sink in terms of energy efficient data gathering is the centre of the cubic field [148]. Let $s_0$ be the location of the sink $s_0$. We consider the following three cases depending on the size of the connected component that includes the sink. Also, given that sensor failure is due to low battery power, we assume that the sink has infinite source of energy, thus excluding the possibility of a faulty sink.

**Case 1:** Single-node connected component. In this case, there are at least two components, one of them being the single-node component containing the sink. Finding the minimum number of nodes to disconnect the network requires that the disconnected network have only two components. To isolate the sink, all its
neighbours must fail. Hence, at least the communication sphere of the sink should contain no sensor but the sink.

Let $N$ be a random variable that counts the number of sensor failures to isolate the sink $s_0$. Given that sensors are randomly and uniformly deployed in a volume $V$ with density $\lambda(r,k)$ per unit volume, where $R \ll V^{1/3}$, the expected number of neighbours of the sink is given by

$$E[N] = \lambda(r,k) \| B(\xi_0, R) \|$$

(2a)

where $\|B(\xi_0, R)\| = 4\pi R^3/3$ is the measure of the volume of the communication sphere $B(\xi_0, R)$ of the sink $s_0$ located at $\xi_0$. Thus, the expected minimum number of sensor failures to isolate $s_0$ is equal to $E[N]$. Substituting Eq. 1 in Eq. 2a, we find that the network connectivity is given by

$$\kappa_1(G) = E[N] = 12.024 \alpha^3 k$$

(2b)

where $\alpha = R/r$. Figures A.3 and A.4 plot the function $\kappa_1(G)$ in Eq. 2b. Clearly, $\kappa_1(G)$ increases with the ratio $\alpha$ and the degree of coverage $k$. More importantly, $\kappa_1(G)$ is much higher than $k$.

Case 2: Non-trivial connected components. As in the case of two-dimensional deployment (Chap. 8), two configurations of the disconnected network are of particular interest where the two connected components of the disconnected network

![Graph](image_url)

**Fig. A.3** Plot of the function $\kappa_1(G)$ (fix $k$ and vary $\alpha = R/r$)
are separated by a vacant region (or gap). Furthermore, any pair of sensors, one from each component, are separated by a distance at least equal to $R$ to prohibit any communication between the two components. In the first configuration, the component including the sink is reduced to its communication sphere. Thus, the volume of the vacant region, denoted by $\text{gap}(\xi_0,R)$, and surrounding the component of the sink, is given by

$$
\| \text{gap}(\xi_0,R) \| = 4\pi (2R)^3/3 - 4\pi R^3/3
$$

Thus, the expected minimum number of sensor failures to isolate the component of the sink is given by

$$
E[N] = \lambda(r,k) \| \text{gap}(\xi_0,R) \|
$$

Substituting Eq. 1 in Eq. 2d, we find that the network connectivity is equal to

$$
\kappa_2(G) = E[N] = 84.164 \alpha^3 k
$$

where $\alpha = R/r$.

In the second configuration, the original network is split into two components such that the vacant region forms a cuboid, denoted by $\text{cub}(R)$, and whose sides...
are $R$, $V^{1/3}$, and $V^{1/3}$. Now, this configuration corresponds to the smallest connected component containing the sink if the field has to be divided into two regions such that none of them surrounds the other. Thus, the expected minimum number of sensor failures to isolate the connected component containing the sink is given by

$$E[N] = \lambda \|\text{cub}(R)\|$$

(2f)

where

$$\|\text{cub}(R)\| = RV^{2/3}$$

(2g)

Substituting Eqs. 1 and 2g in Eq. 2f, it follows that network connectivity is equal to

$$\kappa_3(G) = E[N] = \frac{RV^{2/3}k}{0.422 r_0^3}$$

(2h)

It is easy to check that $\kappa_3(G) > \kappa_2(G)$ since $R \ll V^{1/3}$.

**Case 3: Largest connected component.** This configuration is totally opposite to the one given in Case 1 and has only one isolated sensor. Since we are interested in $k$-coverage of the entire field, such a network is considered as disconnected. The network connectivity is the same as in Case 1. Thus,

$$\kappa_1(G) \leq \kappa(G) \leq \kappa_3(G)$$

It is easy to check that $\kappa(G) > k$.

All these bounds and those to be derived in next sections are based on our fundamental result stated in Theorem A.1 related to the minimum sensor spatial density for full $k$-coverage of a three-dimensional field. Thus, these are lower bounds, and hence tight.

### B Heterogeneous Sensors

Achieving $k$-coverage of a three-dimensional field by heterogeneous sensors would depend on the least powerful ones in terms of their sensing capabilities. Lemmas A.3 and A.4 correspond to Lemma A.1 and Theorem A.1, respectively.

**Lemma A.3:** If the breadth of a three-dimensional convex region $C$ is at most equal to the minimum radius $r_{\text{min}}$ of the sensing spheres of sensors, then $C$ is guaranteed to be $k$-covered if $k \geq 4$ sensors are deployed in it, where $r_{\text{min}} = \min\{r_j/1.066 : s_j \in S\}$.

From Lemma A.4, it follows that connectivity between sensors located in the Reuleaux tetrahedron of side $r_{\text{min}}$ requires that $R_{\text{min}} \geq r_{\text{min}}$. 
Lemma A.4: Let $r_{\min}$ be the minimum radius of the sensing spheres of sensors and $k \geq 4$. The minimum sensor spatial density needed for $k$-coverage of a three-dimensional field by heterogeneous wireless sensor networks is given by

$$\lambda(r_{\min}, k) = \frac{k}{0.422 r_{\min}^3}$$

where $r_{\min} = \min\{r_j / 1.066 : s_j \in S\}$.

Lemma A.5 computes the connectivity measures for heterogeneous three-dimensional $k$-covered wireless sensor networks.

Lemma A.5: Let $G$ be a communication graph of a heterogeneous three-dimensional $k$-covered wireless sensor network with $R_{\min} \geq r_{\min}$ and $k \geq 4$. The connectivity of the graph $G$ is given by

$$\kappa_4(G) \leq \kappa(G) \leq \kappa_3(G)$$

where

$$\kappa_3(G) = \frac{R_{\max} V^{2/3}}{0.422 r_{\min}^3}$$

$$\kappa_4(G) = 12.024 \alpha_2^3 k$$

$$\alpha_2 = R_{\min} / r_{\min}, \quad k \geq 4, \quad r_{\min} = \min\{r_j / 1.066 : s_j \in S\}, \quad R_{\min} = \min\{R_j : s_j \in S\}, \quad \text{and} \quad R_{\max} = \max\{R_j : s_j \in S\}.$$
performance of the network. By Lemma A.2, a three-dimensional field is guaranteed to be fully $k$-covered if each Reuleaux tetrahedron region in the field contains at least $k$ sensors. However, it is impossible to randomly decompose a three-dimensional field into an integer number of Reuleaux tetrahedron regions because of the boundary of the field. Indeed, most of the Reuleaux tetrahedron regions close to the border of a three-dimensional field do not entirely lay inside the deployment area. Therefore, more than necessary sensors would be needed to achieve $k$-coverage of these Reuleaux tetrahedron regions on the border of the field. Simulation results reported in Sect. 2 show that the sensor spatial density necessary to fully $k$-cover a cubic field is a bit higher than the bound given in Theorem A.1, mainly due to the boundary effects.

Next, we introduce new measures of connectivity of three-dimensional $k$-covered wireless sensor networks by placing a specific constraint on a subset of sensors that would fail.

4 Conditional Connectivity

In this section, we compute measures of conditional connectivity for homogeneous and heterogeneous three-dimensional $k$-covered wireless sensor networks based on the concepts of conditional connectivity [101] and forbidden faulty set [81].

As we will see, our results prove that three-dimensional $k$-covered wireless sensor networks can sustain a larger number of sensor failures under the restriction imposed on the faulty sensor set. Similarly to our discussion in Chap. 8, it is easy to show that the traditional connectivity, which does not impose any restriction on the faulty sensor set, is not a useful metric for three-dimensional $k$-covered wireless sensor networks, which are highly dense networks and denser than their two-dimensional counterparts.

Fig. A.5 Two nested concentric Reuleaux tetrahedra
A Homogeneous Sensors

Theorem A.3 computes the conditional connectivity of homogeneous three-dimensional $k$-covered wireless sensor networks.

Theorem A.3: The conditional connectivity of a homogeneous three-dimensional $k$-covered wireless sensor network ($k \geq 4$) is given by

$$\kappa(G : P) = \frac{((n_0 + 2 R)^3 - r_0^3) k}{r_0^3}$$

where $r_0 = r/1.066$.

Proof: We consider the following two cases based on the type of connected component that contains the sink.

Case 1: Smallest connected component. According to our conditional connectivity model, no sensor can be isolated and hence no trivial component can be part of the disconnected network. Under the assumption of forbidden faulty sensor set, the smallest connected component that is disconnected from the rest of the network and contains the sink can be determined as follows. To achieve $k$-coverage of the cubic field, every location must be $k$-covered, including the location $\xi_0$ of the sink. Otherwise, the $k$-coverage property will not be satisfied. Therefore, the smallest connected component that includes the sink consists of $k$ sensors deployed in the Reuleaux tetrahedron of side $r_0$ and centered at $\xi_0$. To disconnect the sink under the forbidden faulty sensor set constraint, the Reuleaux tetrahedron should be surrounded by an empty annulus of width equal to $R$, i.e., sensors located in the annulus have failed (Fig. A.5). The Reuleaux tetrahedron together with this annulus forms a larger Reuleaux tetrahedron of side $r_0 + 2R$. The volume of the annulus, denoted by $A(\xi_0, R)$, is equal to

$$\| A(\xi_0, R) \| = 0.422 (r_0 + 2 R)^3 - 0.422 r_0^3$$

Therefore, the expected conditional minimum number of sensor failures to disconnect the smallest component including the sink can be computed as

$$E[N : P] = \lambda(r, k) \| A(\xi_0, R) \|$$

Substituting Eq. 1 in 3a, we find that the conditional connectivity is given by

$$\kappa(G : P) = E[N : P] = \frac{((n_0 + 2 R)^3 - r_0^3) k}{r_0^3}$$

It is easy to check that the forbidden faulty set constraint is satisfied for both the faulty sensors (located inside the annulus and which have failed) and non-faulty sensors (located outside the annulus). Indeed, any sensor in the inner Reuleaux tetrahedron still has non-faulty neighbours in the inner Reuleaux tetrahedron. Besides, any sensor outside the outer Reuleaux tetrahedron still has
non-faulty neighbours outside the outer Reuleaux tetrahedron. Also, any faulty sensor within the annulus $A(R)$ has non-faulty neighbours located in the inner Reuleaux tetrahedron and outside the outer Reuleaux tetrahedron.

**Case 2: Largest connected component.** This case is similar to the previous one. However, the sink belongs to the largest connected component. Hence, the disconnected network consists of two components: the one including the sink and a second component associated with sensors located in a Reuleaux tetrahedron of side $r$. Using the same analysis as in Case 1, we obtain the same conditional network connectivity:

$$\kappa_2(G : P) = \kappa_1(G : P)$$

From both cases 1 and 2, the conditional connectivity of homogeneous three-dimensional $k$-covered wireless sensor networks is computed as

$$\kappa(G : P) = \kappa_1(G : P)$$

## B Heterogeneous Sensors

We observe that computing the conditional connectivity of heterogeneous three-dimensional $k$-covered wireless sensor networks is not a straightforward generalization of the approach used previously for homogeneous three-dimensional $k$-covered wireless sensor networks. If we use the same process as previously, we either violate the forbidden faulty sensor set constraint or maintain network connectivity. Precisely, if the width of the annulus containing the faulty sensors is $R_{\text{max}}$, then sensors whose communication radii are less than or equal to $R_{\text{max}} / 2$ may be located in the annulus. Thus, the entire neighbour set of this type of sensors would fail at the same time and hence the property $P$ would be violated. Now, if the width of the annulus containing the faulty sensors is less than $R_{\text{max}}$, then the non-faulty sensors of one connected component will be able to communicate with the non-faulty sensors of the other connected component of the disconnected network. Hence, the network is still connected. In this case, it is impossible to find an exact value of conditional connectivity for heterogeneous three-dimensional $k$-covered wireless sensor networks. In the following, we compute their lower and upper bounds based on the types of sensors in and around the annulus.

**Lemma A.6:** The conditional connectivity of the heterogeneous three-dimensional $k$-covered wireless sensor networks is given by

$$\kappa_1(G : F_p) \leq \kappa(G : P) \leq \kappa_2(G : F_p)$$

where

$$\kappa_1(G : P) = \frac{((r_{\text{min}}^0 + 2 R_{\text{min}})^3 - r_{\text{min}}^0 3)}{r_{\text{min}}^0 3} k$$
\[ \kappa_2(G : P) = \frac{((r_{\max} + 2R_{\max})^3 - r_{\min}^3)k}{r_{\min}^3} \]

\[ k \geq 4, \quad r_{\min} = \min\{r_j / 1.066 : s_j \in S\}, \quad r_{\max} = \max\{r_j / 1.066 : s_j \in S\}, \]

\[ R_{\min} = \min\{R_j : s_j \in S\}, \text{ and } R_{\max} = \max\{R_j : s_j \in S\}. \]

Proof: As above, we consider the following two cases depending on the size of the connected component that includes the sink.

Case 1: Smallest connected component. To compute a lower bound on the conditional connectivity of heterogeneous three-dimensional \( k \)-covered wireless sensor networks, we consider the Reuleau tetrahedron centered at location \( \xi_0 \) of the sink \( s_0 \), which will be disconnected from the network. First, we assume that the annulus containing the faulty sensors as well as the volume surrounding it contains only least powerful sensors, and hence the width of this annulus is equal to \( r_{\min} \).

Also, to guarantee that the sink will not be isolated, which would violate the forbidden faulty sensor set constraint, the Reuleaux tetrahedron centred at \( \xi_0 \) should have a side equal to \( r_{\min} \). These two conditions help disconnect the network while satisfying the forbidden faulty sensor set constraint. The volume of the annulus \( A(\xi_0, R_{\min}) \) is given by

\[ \| A(\xi_0, R_{\min}) \| = 0.422 (r_{\min} + 2R_{\min})^3 - 0.422 r_{\min}^3 \]  

(6a)

Hence, the expected conditional minimum number of sensor failures to disconnect the connected component including the sink (or the inner Reuleaux tetrahedron) from the rest of the network is computed as

\[ E[N : P] = \hat{\lambda}(r_{\min}, k) \| A(\xi_0, R_{\min}) \| \]  

(6b)

where \( \hat{\lambda}(r_{\min}, k) = \frac{k}{0.422 r_{\min}^3} \) and \( r_{\min} = \min\{r_j / 1.066 : s_j \in S\} \). Thus, the conditional network connectivity is given by

\[ \kappa_1(G : P) = E[N : P] = \frac{((r_{\min} + 2R_{\min})^3 - r_{\min}^3)k}{r_{\min}^3} \]  

(6c)

where \( k \geq 4 \) and \( R_{\min} = \min\{R_j : s_j \in S\} \).

To compute an upper bound on the conditional connectivity of heterogeneous three-dimensional \( k \)-covered wireless sensor networks, we assume that sensors inside the annulus are the most powerful ones. Thus, the side of the inner Reuleaux tetrahedron should be equal to \( r_{\max} \), while the width of the annulus surrounding it should be equal to \( R_{\max} \). It is easy to check that this set-up will disconnect the network while satisfying the forbidden faulty set constraint. The upper bound on the conditional connectivity is given by
5 Discussion

\[ \kappa_2(G : P) = \mathbb{E}[N : P] = \lambda(r_{\min}, k) \| A(\xi_0, R_{\max}) \| = \frac{(r_{\max} + 2 R_{\max}^3 - r_{\min}^3) k}{r_{\min}^3} \]  

(6d)

where \( \lambda(r_{\min}, k) = \frac{k}{0.422 r_{\min}^3}, \quad k \geq 4, \quad r_{\min} = \min\{r_j / 1.066 : s_j \in S\}, \)

\( r_{\max} = \max\{r_j / 1.066 : s_j \in S\}, \) and \( R_{\max} = \max\{R_j : s_j \in S\}. \)

**Case 2: Largest connected component.** In this case, the sink belongs to the largest connected component of the disconnected network. Hence, the previous analysis applies to any sensor in the network instead of the sink.

Thus, the conditional connectivity of heterogeneous three-dimensional \( k \)-covered wireless sensor networks satisfies

\[ \kappa_1(G, P) \leq \kappa(G, P) \leq \kappa_2(G, P) \]

Next, we relax the assumptions used in our previous analysis to enhance the practicality of our results.

5 Discussion

The analysis of the minimum sensor spatial density necessary for \( k \)-coverage of a three-dimensional field and network connectivity of three-dimensional \( k \)-covered wireless sensor networks is based on the unit sphere model, where the sensing and communication ranges of sensors are modelled by spheres. In other words, sensors are supposed to be typically isotropic. Although this assumption is the basis for most of the protocols for coverage and connectivity in wireless sensor networks, it may not hold universally and thus may not be valid in practice. In Appendix B, we show how to relax this assumption to promote the applicability of our results to real-world three-dimensional wireless sensor network scenarios, and summarize our results for the convex model, where the sensing and communications ranges of sensors are convex but not necessarily spherical. Moreover, we assumed that our results for network connectivity hold for a degree of sensing coverage \( k \), where \( k \geq 4 \).

In this section, we show how to relax the latter assumption. Then, we discuss a sensor placement strategy for full \( k \)-coverage of a three-dimensional field.

A Relaxing the Assumption of \( k \geq 4 \)

The analysis of \( k \)-coverage and connectivity for three-dimensional \( k \)-covered wireless sensor networks are valid for all \( k \geq 4 \). Since the breadth of the Reuleaux tetrahedron is equal to \( r \) (or \( r_{\min} \)), our results can also be used for \( k \leq 3 \). In other words, \( k \)-coverage of the cube can be met by deploying \( k \) sensors in the Reuleaux tetrahedron, where \( k \leq 3 \). However, the network would be denser than necessary
(especially for $k = 1$) and the coverage degree would be higher than that dictated by the application.

**B  Sensor Placement Strategy**

Notice that under the assumption of spherical model, it is impossible to achieve a degree of coverage exactly equal to $k$ in all the locations of the cube. Therefore, a sensor placement strategy to achieve $k$-coverage should be devised in such a way that every location in the cube is $k'$-covered, where $k'$ is very close to $k$. This placement strategy should benefit from the geometry of the Reuleaux tetrahedron. The sensor placement problem can be transformed into a problem of covering a cube with overlapping sets of congruent Reuleaux tetrahedra. An optimal covering consists to use a minimum number of Reuleaux tetrahedra by minimizing the overlap volume between them. More precisely, two adjacent Reuleaux tetrahedra overlap such that the faces of their corresponding regular tetrahedra are entirely coinciding with each other. Thus, the curved edges of the Reuleaux tetrahedra should overlap so the same subset of sensors deployed on these curved edges could participate in covering the space associated with both Reuleaux tetrahedra at the same time, thus minimizing the total number of sensors required to $k$-cover the cube. As in the case of two-dimensional deployment, the sensors located in a three-dimensional lens, which corresponds to the overlap volume of two adjacent Reuleaux tetrahedra of side length $r_0$, are at distance $r_0$ from any point in their volumes, and hence participate in $k$-covering both tetrahedra. The design of duty-cycling protocols to $k$-cover a three-dimensional field with a minimum number of sensors should select sensors based on this observation.

**C  Sink-Independent Connectivity Measures**

Although in centralized algorithms the concept of sink is well defined, it is likely that distributed algorithms, such as the consensus based algorithm, will be implemented for WSNs. In this case, the concept of (fixed) sink would lose value. It would be interesting to revise the definition of connectivity to take this concept into account. We suggest that all the nodes be considered as peer-to-peer. Thus, we define connectivity with respect to all sensors in the network. Given the geometry of the deployment field that we consider (cube), the boundary sensors, i.e., sensors located at the boundary of the cube, have small neighbour sets. In particular, the sensors located at the eight corners of the cube $s_{b_1}, \ldots, s_{b_8}$ as shown in Fig. A.6 are highly likely to have the smallest neighbour set. Our measures of connectivity will be based on one of these boundary sensors to find the minimum number of sensors of its neighbour set that should fail in order to disconnect the network. It is easy to check that compared to the sink, the size of the neighbour set of a boundary sensor is equal to a quarter of that of the neighbour set of the sink. Thus, the connectivity measures computed in the previous sections with respect to the sink remain the same for a boundary sensor but are weighted by a coefficient equal to $\frac{1}{4}$. For more details, the interested reader is referred to Sect. 7.
D Stochastic Sensing and Communication Models

In this section, we exploit the results of Sect. 2 to characterize probabilistic $k$-coverage in 3D WSNs based on our stochastic sensing model. Theorem A.4 computes the minimum $k$-coverage probability $p_{k,min}$ such that every point in a field is probabilistically $k$-covered.

**Theorem A.4**: Let $r$ be the radius of the nominal sensing range of the sensors, $r_0 = r/1.066$, and $k \geq 4$. The minimum $k$-coverage probability so that each point in a 3D field is probabilistically $k$-covered by at least $k$ sensors under our stochastic sensing model defined in (5) is computed as

$$p_{k,\text{min}} = 1 - \left(1 - e^{-\beta r_0^a}\right)^k$$

**Proof**: First, we identify the least $k$-covered point in a 3D field so we can compute $p_{k,\text{min}}$. By Lemma A.2, $k$ sensors should be deployed in a Reuleaux tetrahedron region of side $r_0 = r/1.066$ in the field to achieve $k$-coverage of a 3D field with a minimum number of sensors. It is easy to check that the least $k$-covered point $\xi_{lc}$ is the one that corresponds to the configuration where all deployed $k$ sensors are located at a distance $r_0 = r/1.066$ from $\xi_{lc}$. In other words, $\xi_{lc}$ is the farthest point from all those $k$ sensors. Hence, the distance between $\xi_{lc}$ and each of these $k$ sensors is equal to $r_0 = r/1.066$. Thus, the minimum $k$-coverage probability for the least $k$-covered point $\xi_{lc}$ by $k$ sensors under the stochastic sensing model introduced in Chap. 2 is given by

$$p_{k,\text{min}} = 1 - \prod_{i=1}^{k} (1 - p(\xi, s_i)) = 1 - \left(1 - e^{-\beta r_0^a}\right)^k$$

The stochastic $k$-coverage problem is to select a minimum subset $S_{min} \subseteq S$ of sensors such that each point in a 3D field is $k$-covered by at least $k$ sensors and that the minimum $k$-coverage probability of each point is at least equal to some given threshold probability $p_{th}$, where $0 < p_{th} < 1$. This helps us compute the stochastic sensing range $r_s$, which provides probabilistic $k$-coverage of a field with a probability no less than $p_{th}$. Lemma A.7 computes the value of $r_s$. 

---

**Fig. A.6** Eight boundary sensors located on the corners of a cube
Lemma A.7: Let $k \geq 3$ and $2 \leq \alpha \leq 4$. The stochastic sensing range $r_s$ of the sensors that is necessary to probabilistically $k$-cover a 3D field with a minimum number of sensors and with a probability no lower than $0 < p_{th} < 1$ is given by

$$r_s = \left(-\frac{1}{\beta} \log (1 - (1 - p_{th})^{1/k})\right)^{1/\alpha} \quad (8)$$

where $\beta$ represents the physical characteristic of the sensors’ sensing units.

Proof: From Eq. 7, we deduce that $p_{k_{\min}} \geq p_{th} \Rightarrow r_s \leq \left(-\frac{1}{\beta} \log (1 - (1 - p_{th})^{1/k})\right)^{1/\alpha}$.

Since we are interested in the minimum number of sensors to probabilistically $k$-cover a 3D field, we should consider the maximum value of $r_s$, i.e., the maximum stochastic sensing range of the sensors. This will allow the sensors to probabilistically $k$-cover as much space of the 3D deployment field as possible. Thus,

$$r_s = \left(-\frac{1}{\beta} \log (1 - (1 - p_{th})^{1/k})\right)^{1/\alpha}.$$

The upper bound on the stochastic sensing range $r_s$ of the sensors computed in Eq. 8 will be used to compute our measures of connectivity and fault-tolerance of 3D $k$-covered WSNs under the assumption of more realistic stochastic sensing and communication models. Figures A.7 and A.8 show $r_s$ for different values of $p_{th}$ and $k$ while considering the free-space model ($\alpha=2$) (Fig. A.7) and

![Fig. A.7 Upper bound of $r_s$ vs. $k$ for $\alpha = 2$](image-url)
the multi-path model ($\alpha=4$) (Fig. A.8). Note that $r_s$ decreases as a function of $p_{th}$, $k$, and $\alpha$. This is due to the fact that the minimum probability $p_{k,min}$ of $k$-coverage of the same location by multiple sensors decreases as $p_{th}$, $k$, and $\alpha$ increase.

Lemma A.8 states a sufficient condition for probabilistic $k$-coverage of a 3D field based on our stochastic sensing model given in Chap. 2, the threshold probability $p_{th}$, and the degree $k$ of coverage.

Lemma A.8: Let $k \geq 4$. A 3D field is probabilistically $k$-covered with a probability no lower than $0 < p_{th} < 1$ if any Reuleaux tetrahedron of maximum breadth $r_s/1.066$ in the field contains at least $k$ sensors.

Lemma A.9 states a sufficient condition for connectivity between sensors under our stochastic sensing model.

Lemma A.9: Let $k \geq 4$. The sensors that are selected to $k$-cover a 3D field with a probability no less than $0 \leq p_{th} \leq 1$ under the stochastic sensing model defined in Eq. 5 are connected if the radius of their stochastic communication range $R_s$ is at least equal to their stochastic sensing range $r_s$, $R_s \geq r_s$.

Given that network connectivity is defined as the minimum number of sensors of a neighbour set which need to fail to disconnect the network, we should consider the minimum stochastic communication range of the sensors, i.e., $R_s = r_s$, to minimize the size of the neighbour set of each sensor. Therefore, to find our
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![Graph A.9](image)

**Fig. A.9** Sensor spatial density vs. $k$

![Graph A.10](image)

**Fig. A.10** $k$ vs. number of deployed sensors
unconditional and conditional measures of network connectivity and fault-tolerance for 3D \( k \)-covered WSNs using more realistic scenarios, including stochastic models for sensing and communications, we need to replace \( r \) by \( r_s \) and \( R \) by \( r_s \) in Sects. 2, 3, 4, 5, and 7.

Now, we present the simulation results using a high-level simulator written in the C language. We consider a cubic field of side length 1000 m. All simulations are repeated 200 times and the results are averaged.

Figure A.9 plots the sensor spatial density as a function of the degree of coverage \( k \) for different values of the threshold probability \( p_{th} \) and for a path-loss exponent \( \alpha = 2 \). As expected, the density increases with \( p_{th} \). Indeed, as we increase \( p_{th} \), more sensors would be needed to achieve the same degree of coverage \( k \). Recall that the breadth of the Reuleaux tetrahedron that is guaranteed to be covered with exactly \( k \) sensors decreases as \( p_{th} \) and \( \alpha \) increase. Precisely, this breadth is equal to \( r_s / 1.066 \). However, for \( p_{th} = 0.8 \), the sensor density tends to decrease when \( k \) goes from 4 to 5 and increases afterwards. This behaviour is clearly noticeable for \( p_{th} = 0.9 \). This is mainly due to the stochastic nature of the sensing range of the sensors, which depends on the logarithm of \( p_{th} \), the threshold probability, \( k \), and \( \alpha \).

Figure A.10 plots the achieved degree of coverage \( k \) vs. the total number of deployed sensors. Moreover, we vary both \( p_{th} \) and we fix \( \alpha = 2 \). Definitely, higher number of deployed sensors would yield higher coverage degree. Here also, any increase in \( p_{th} \) would require a larger number of deployed sensors to provide the same degree of coverage. Notice that the same observation holds for \( p_{th} = 0.8 \) and \( p_{th} = 0.8 \) as in the previous experiment.

### E Three-Dimensional Sensing Applications

In the case of WSNs deployed on the trees of different heights in a forest, the sensors could be seen almost everywhere in the space. Pompili et al. [169] proposed different deployment strategies for 2D and 3D communication architectures for underwater acoustic sensor networks, where the sensors are anchored at the bottom of the ocean for the 2D design and float at different depths of the ocean to cover the entire 3D region. Indeed, oceanographic data collection, pollution monitoring, offshore exploration, disaster prevention, and assisted navigation are all typical applications of underwater sensor networks [4, 5]. For WSNs deployed in buildings with multiple floors, sensors are placed on the ground and/or the wall, but the networks seldom contain sensors floating in the middle of the room. The first examples show that our proposed 3D model is valid and can be applied to choose the sensor density in practical problems. The last example, however, shows the limited validity of our model due to the restriction imposed on the placement of sensors inside buildings or rooms.

### 6 Relaxing the Unit Sphere Model: Convex Model

The assumption of spherical sensing and communication ranges of the sensors may not hold in real-world wireless sensor network platforms. It has been observed in [209] that the communication range of MICA motes [230] is asymmetric
and depends on the environments. It is also found in [224] that the communication range of radios is highly probabilistic and irregular.

In this appendix, for problem tractability, we consider a convex model, where the sensing and communication ranges of sensors are convex but not necessarily spherical.

First, we define the notion of largest enclosed sphere of a three-dimensional convex region $C$ as a sphere that lies entirely inside $C$ and whose diameter is equal to the minimum distance between any pair of points on the boundary of the region $C$.

### A Homogeneous Sensors

We consider homogeneous sensors that have the same convex sensing ranges and communication ranges. Lemmas A.10 and A.11 correspond to Lemma A.1 and Theorem A.1, respectively. Their proof is similar to that in Sect. 2 by using the largest enclosed sphere instead of the sensing sphere.

**Lemma A.10:** If $4 \geq k$ homogeneous sensors are deployed in a three-dimensional convex region $C$, then the region $C$ is $k$-covered if its breadth does not exceed $r_{\text{led}}$, the radius of the largest enclosed sphere of the sensing range.

**Lemma A.11:** The minimum sensor spatial density required to guarantee $k$-coverage of a three-dimensional field is given by

$$\lambda(r_{\text{led}}, k) = \frac{k}{0.422r_{\text{led}}^3}$$

where $r_{\text{led}}$ stands for the radius of the largest enclosed sphere of the sensing range, $r_{\text{led}}^0 = r_{\text{led}}/1.066$, and $k \geq 4$.

Now, we discuss how those results can be derived using a convex model, where the sensing and communication ranges of the sensors may not necessarily be spherical.

### B Heterogeneous Sensors

Lemmas A.12 and A.13 correspond to Lemma A.1 and Theorem A.1, respectively. Their proof is also the same as that in Sect. 2 by using the notion of largest enclosed sphere instead of sensing sphere.

**Lemma A.12:** A three-dimensional convex region $C$ is guaranteed to be $k$-covered when exactly $k$ heterogeneous sensors are deployed in it, if the breadth of $C$ does not exceed $r_{\text{led}}^{\text{min}}$, where $r_{\text{led}}^{\text{min}} = \min\{r_{\text{led}}/1.066 : s_j \in S\}$ and $k \geq 4$.

**Lemma A.13:** The minimum sensor spatial density required to $k$-cover a three-dimensional field is given by
The measures of network connectivity can be derived using the same approach as in the previous sections. Thus, the assumption of unit sphere model for sensing and communication ranges of sensors can be relaxed with the aid of largest enclosed sphere of their sensing range.

7 Underwater Sensor Networks

The results in the previous sections are only applicable to the connectivity of sink node. Although the connectivity of sink is critical, in some scenarios, such as underwater wireless sensor networks [4, 5], any sensor may be critical due to the high cost, for instance.

In the following, we extend our network connectivity measures for three-dimensional \( k \)-covered wireless sensor networks to the case where any sensor in the network is critical. Specifically, we consider a boundary sensor, i.e., a sensor located at one corner of the cubic field. Such a boundary sensor has the minimum number of communication neighbours given that all sensors are located within the deployment region—the cube, and hence the actual communication range of a boundary sensor is only a quarter of its communication sphere. In [205], a boundary sensor is considered to compute the connectivity of 2D \( k \)-covered wireless sensor networks.

Theorem A.5 summarizes the connectivity measures with respect to a boundary sensor for homogeneous three-dimensional \( k \)-covered wireless sensor networks. The case of heterogeneous wireless sensor networks and the case of sensors with convex sensing and communication ranges can be treated similar to the previous sections. Thus, we omit the proof of Theorem A.5.

**Theorem A.5:** Let \( G \) be a communication graph of a homogeneous three-dimensional \( k \)-covered wireless sensor network deployed in a cubic field, where the radii of the sensing and communication spheres of sensors are \( r \) and \( R \), respectively. The connectivity of \( G \) is computed as

\[
\kappa(G) = 3.02 \alpha^3 k
\]  

whereas the conditional connectivity of \( G \) is given by

\[
\kappa(G : P) = \frac{3.02 \left( (r_0 + R)^3 - r_0^3 \right) k}{r_0^3}
\]  

where \( r_0 = r/1.066 \), \( \alpha = R/r \), and \( k \geq 4 \).
8 Summary

In this appendix, we investigated coverage and connectivity in three-dimensional 
\( k \)-covered wireless sensor networks with emerging applications, such as underwa-
ter acoustic sensor networks that require three-dimensional design. We proposed 
the Reuleaux tetrahedron model to guarantee \( k \)-coverage of a three-dimensional 
field. Based on the geometric properties of Reuleaux tetrahedron, we derived 
the sensor spatial density for guaranteeing \( k \)-coverage of a three-dimensional 
space. We also computed the connectivity of homogeneous and heterogeneous 
three-dimensional \( k \)-covered wireless sensor networks. Our results on connectivity 
take into consideration an inherent characteristic of wireless sensor networks in 
that the sink has a critical role in terms of data processing and decision making, 
compared to the rest of the network. Therefore, we computed the connectivity of 
three-dimensional \( k \)-covered wireless sensor networks based on the size of the 
connected component that includes the sink. We conclude that the connectivity of 
three-dimensional \( k \)-covered wireless sensor networks is much higher than the de-
gree of sensing coverage \( k \) provided by the network. The traditional connectivity 
metric, however, is defined in an abstract way and does not consider the inherent 
properties of wireless sensor networks because it assumes that any subset of 
nodes can fail at the same time. This assumption is not valid for heterogeneous 
three-dimensional \( k \)-covered wireless sensor networks. To compensate for these 
shortcomings, we used proposed more realistic measures of connectivity based on 
the concept of forbidden faulty set. We found that three-dimensional \( k \)-covered 
wireless sensor networks can sustain a large number of sensor failures provided 
that the neighbour set of a sensor cannot fail at the same time.

We believe that our results have practical significance for sensor network de-
signers to develop three-dimensional applications with prescribed degrees of cov-
erage and connectivity. These connectivity measures can be exploited in the de-
sign of fault-tolerant topology control protocols for three-dimensional \( k \)-covered 
wireless sensor networks. Our future work will focus on the design of efficient 
sensor deployment strategies for three-dimensional \( k \)-covered wireless sensor networks. We are also interested in the design of data routing protocols on 
duty-cycled three-dimensional \( k \)-covered wireless sensor networks, which pose 
major challenges due to the time-varying connectivity of the network as sensors are 
turned on or off to save energy and extend the network lifetime.
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