References

References


Wold H (1938) A study in the analysis of stationary time series (2nd edn. 1954) Almqvist & Wiksells, Uppsala

Notational Conventions, Symbols and Acronyms

The following notational conventions will be used throughout the text:

- Bold lower case letters indicate vectors.
- Bold upper case letters indicate matrices.
- Both notations \([A \ B]\) and \([A, B]\) will be used, depending on convenience, for column-wise partitioned matrices.
- Both notations \(\begin{bmatrix} A & B \\ C & D \end{bmatrix}\) and \(\begin{bmatrix} A \vdash B \\ \cdots \cdots \\ C \vdash D \end{bmatrix}\) will be used, depending on convenience, for block matrices.

Symbols and Acronyms

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<td>(A^\dagger)</td>
<td>generalized inverse of (A)</td>
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<td>(r(A))</td>
<td>rank of (A)</td>
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<td>(A_{p}^\perp)</td>
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<td>(A')</td>
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<td>(det(A))</td>
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<td>(A_{\perp})</td>
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