Exercises

Exercise 1 (Sec. 2.5): Make-or-Buy Decisions

A company sells a product that can either be produced by the company itself in causing a variable production cost of 2 $ per unit, or it can be bought from a supplier at a purchasing price of 3 $ per unit. Only part of the demand can be produced inhouse while the remaining amount is purchased. The selling price is 5 $. Manufacturing the product requires two consecutive operations: The first needs 2 and the second 3 periods per unit. The capacity of these two production levels is 4 and 12 periods, respectively. Up to 4 units of the product can be sold while the supplier can deliver a maximum of 3 units.

a) Formulate the pertaining hierarchical model, identify $C^T$, $C^{TT}$, $C^{TB}$, $a^T$, $A^T$, $C^B$, $a^B$, $A^B$, $\hat{a}^B$, $\hat{A}^B$, $IN$, $AF(IN)$, $IN^*$, $a^{B^*}$, and derive a solution.

b) Assume that the purchasing price $q$ is not known in advance and that at the point in time when the top-decision is to be made only stochastic information is available: $P(q = 3) = 0,5$ and $P(q = 1) = 0,5$. However, after the top-decision has been made, price $q$ is revealed. Formulate and solve the problem in fully exploiting this stochastic information.

c) Assume that demand $d$ is not known at the time when the top-decision is to be made. Only stochastic information is available: $P(d = 4) = 0,75$ and $P(d = 1) = 0,25$. However, after the top-decision has been made, demand $d$ is revealed, and subsequently the base-decision is made trying to satisfy demand $d$ completely. The following time line describes the situation:
Let us further assume that production cannot be stored in a warehouse, i.e., if more units are manufactured than being asked for, surplus production is to be disposed causing costs of 3 $ per unit. On the other hand, if one cannot fulfill demand, shortage cost amounts to 4 $ per unit. Again we assume that the purchase price \( q \) is 3 $ per unit at the time when the top-decision is made.

Formulate and solve the problem in using a stochastic dynamic program or a decision tree.

**Exercise 2 (Sec. 4.2.1): Dynamic Lot Sizing**

Consider a 5-period one-product inventory problem characterized by the following general order policy: A lot sizing decision can only be made at the beginning of periods 1 and 3. At period 1 demand of periods 1 and 2 is known deterministically whereas for periods 3, 4, and 5 demand is still stochastic. However, for a possible lot sizing decision in period 3 (for the remainder of the horizon), stochastics is realized.

Every order causes setup cost \( K \) [EUR per period] and an inventory holding cost of \( c^L \) [EUR per period] (related to the amount of stock at the end of a period). Initial inventory is \( x_0^L = x_0^L' \), lead times need not be considered.

a) Illustrate the planning situation with a time line.

b) Identify top-model and base-model verbally. Which class of DDM systems does this problem belong to?

c) Formulate the problem in employing the DDM notation for a perfect anticipation. Identify \( C^T, C^{TT}, C^{TB}, a^T, A^T, IN, \hat{C}^B, \hat{a}^B, \hat{A}^B \), and the anticipation function \( AF(IN) \).
Exercise 3 (Sec. 4.2.1): Location and Production Planning

A company is planning to open several manufacturing plants \((s = 1, \ldots, s)\) in Eastern Europe, at which a centrally manufactured standard product is further refined. The company is planning to supply a larger number of customers \((k = 1, \ldots, k)\) through these plants. For the next 5 years, one intends to determine on the basis of demand forecasts, which plants should be established and which customers should be supplied by particular plants such that total cost be minimized. As a specificity, production cost is of importance while transport costs between the central production location and the plants can be ignored.

The demand of a customer can be satisfied by one or by several plants and, of course, the overall demand of all customers has to be fulfilled. The quantity \(x_{sk}\) describes the entire amount a plant \(s\) delivers during the 5 years to customer \(k\). The adjoint transport costs are denoted by \(c_{sk} [\text{\$ per unit}]\). It is assumed that enough transport capacity is available. In case a new plant is established, an investment expenditure of \(c_s^{\text{invest}}\) is incurred.

Production planning takes place on a quarterly basis with \(d_{kt}^c\) denoting estimated demand of customer \(k\) in quarter \(t\). The production of one metric ton of the product requires \(a_t\) tons of raw material for each quarter \(t\) \((t = 1, \ldots, 20)\). Note that productivity varies within a year, which leads to time-dependent production costs \(c_{st}^p\). An inventory up to a maximum of \(y_s^{L\max}\) units is attainable in plant \(s\). Inventory holding cost amounts to \(h_s [\text{\$ per unit and quarter}]\). No initial inventory is available at the plants. Furthermore, at the plants, \(K_{st}\) tons per quarter of raw material are available, which can be extended at short notice incurring costs of \(k_s [\text{\$ per ton}]\). It is assumed that the capacity of all plants is sufficient to satisfy entire demand.

a) Illustrate the physical location and production conditions with a sketch and describe the decision problem of the company as a DDM problem.

d) Specify an approximate anticipation in dropping different parts of the available information. Give an example.
b) Assuming a perfect anticipation, formulate the models of the top-level and the anticipated base-level. In particular, identify $C^T, C^{TT}, C^{TB}, a^T, A^T, IN, \hat{C}^B, \hat{a}^B, \hat{A}^B$, and $AF$ (Remark: In the base-model, quarterly demand $b_{st}$ at a plant $s$ has to be considered, while in the top-model the five year demand of customer $k$: $\sum_{t=1}^{20} d_{kt}^c$ is of importance. The demand at the base-level $b_{st}$ is given by the top-level's order quantity and is supposed to be determined by

$$b_{st} = \sum_{k=1}^{\bar{k}} \frac{x_{sk}}{20} \sum_{t=1}^{d_{kt}^c} \forall t, s.$$ 

(What is the meaning of this recalculation?).

c) In view of the top-criterion, which class of DDM systems does this problem belong to?

**Exercise 4 (Sec. 4.2.1): Determination of Transfer Prices**

A company, consisting of several subsidiaries $v$ ($v = 1, \ldots, V$), employs a decentralized planning approach for its products $i$ ($i = 1, \ldots, n_v$). The subsidiaries use for their various products a common resource of capacity $M$. For the utilisation, a transfer price $\lambda$ must be paid. This price is determined by the central DMU such that an overall criterion is optimized.

The manufacturing of product $i$ in subsidiary $v$ causes variable unit production cost $k_{vi}$ which is known to the central DMU and the subsidiaries. The manufacturing of one unit of product $i$ in subsidiary $v$ requires $a_{vi}$ units of the common resource. The inverse demand functions are linear and given by $p_{vi} = p_{vi}(x_{vi})$.

For the manufacturing of the products, the subsidiaries have local resources with a capacity of $m_v$. Accordingly, the manufacturing of product $i$ in subsidiary $v$ requires $b_{vi}$ units of the local resource. Demand of product $i$ in subsidiary $v$ is $d_{vi}$. All quantities are common knowledge.
Exercise 5 (Sec. 4.4.1): Capacity Adaptation Problem

In July, a company is planning its required manpower capacity for the next year. The manufacturing process of its two products \((i = 1, 2)\) consists of two operations. These can be performed by employees having two levels \(f\) of experience, incurring production cost of \(c_f\) [EUR per hour] \((f = 1, 2)\). Personnel having abilities of level \(f\) need \(a_{if}\) hours for manufacturing one unit of product \(i\). The products' price is \(p_i\) [EUR per unit] \((i = 1, 2)\), and variable production costs are \(k_i\) [EUR per unit] \((i = 1, 2)\). The production decision maximizes the contribution margin. At the time when the decision on the manpower capacity is to be made, for demand only stochastic information is available. There are four scenarios \(S_r, r = 1, ..., 4: S_r = (d_{1r}, d_{2r})\) having probability \(w_r(r = 1, ..., 4)\). When the production decision is made, one scenario is realized. The allocated manpower capacity should at least be sufficient to satisfy 80% of demand.

a) Formulate a prophetic solution for the two-stage stochastic problem, i.e., describe the situation of one scenario being known in advance.

b) Formulate the problem as a two-stage stochastic program in using the notation \(C^T, C^{TT}, C^{TB}, a^T, A^T, IN, C^B, a^B, A^B, \hat{A}^B\). (Hint: Determine the maximum necessary manpower capacity in order to limit the decision field.)

Why is the formulation of an anticipation function not necessary?
c) Which changes as to the model described under b) have to be made if demand is not stochastic but given by its mean value $\bar{d}_t$?

d) Solve the problem employing two-stage stochastic programming. Use the following data:

- scenarios: $S_1 = (4, 4); S_2 = (6, 4); S_3 = (4, 8); S_4 = (6, 8);$
- $w_r = 0, 25; r = 1, ..., 4,$
- production cost: $c_1 = 2, c_2 = 1,$
- prices: $p_1 = 20; p_2 = 15,$
- variable production cost: $k_1 = 5; k_2 = 3,$
- consumption rates: $a_{11} = 3; a_{21} = 2; a_{12} = 2; a_{22} = 5.$

Assume that at the last step of the negotiation-oriented algorithm only the following three manpower capacities $(X_1 = 30; X_2 = 50), (X_1 = 34; X_2 = 45)$ and $(X_1 = 25; X_2 = 40)$ are still to be considered. (Hint: Solve the linear programs in using an appropriate solver.)

e) Solve the two-stage linear problem for the deterministic case $\bar{d}_1 = 5$ and $\bar{d}_2 = 6.$

Exercise 6 (Sec. 5.3.2): Quality Management

A department of a company manufactures a component of a consumer good. The company gives a warranty for the product, i.e., it carries the cost of repair or complete replacement. Due to the warranty the profit of the company is reduced.

In order to increase profit, management aims to improve the reliability of the component. This depends on the attention the responsible quality inspector is paying in checking the components. Management cannot directly observe his efforts. One is therefore motivating the inspector in sharing with him the company’s profit.

It is assumed that management has full knowledge of the utility function, the decision field, and the aspiration level (participation constraint) of the quality inspector. Furthermore, you can assume that management and quality inspector are risk-neutral.

a) Describe the situation as a principle agent problem. Specify (verbally) its most important constituents: task being delegated, level of activity, disutility of the agent, and information situation.
b) Management offers the quality inspector a contract consisting of a fixum $F$ and a proportional profit share $f$.

How can one determine the optimal contract (i.e., fixum $F$ and share $f$ ($f \in [0, 1]$)) between management and inspector? Formulate the necessary equations and define all used quantities. In particular, specify the meaning of $I^T$ and $I^B$.

c) The gross profit of the company does not exclusively depend on the attention of the quality inspector, but also on external disturbances. It is assumed that three scenarios $S_1$, $S_2$, and $S_3$ occur with equal probability, being revealed after the inspector decides upon his level of activity. He shows low ($a^B = 1$) or high attention ($a^B = 2$). Gross profits (in EUR) depending on the level of activity of the inspector and on scenarios $S_i$ ($i=1,2,3$) may be found in the following table:

<table>
<thead>
<tr>
<th>Profit</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^B = 1$</td>
<td>15000</td>
<td>20000</td>
<td>25000</td>
</tr>
<tr>
<td>$a^B = 2$</td>
<td>25000</td>
<td>31000</td>
<td>34000</td>
</tr>
</tbody>
</table>

The quality inspector estimates his disutility as being 500 EUR for $a^B = 1$ and 3000 EUR for $a^B = 2$. Additionally, he has an aspiration level of 7000 EUR. The fixed amount of his wages (fixum) is given by $F=5500$ EUR, i.e., only profit share $f$ has still to be determined.

c_1) For which values of $f$ is the participation condition fulfilled?

c_2) Determine the anticipation function for a perfect anticipation, proceed as follows:

First assume that the quality inspector chooses the low level of activity ($a^B = 1$). Which values of $f$ are fulfilling this condition? Determine, accordingly, the scope of $f$ for which the quality inspector shows a high attention ($a^B = 2$).

c_3) Using the anticipation function determined in c_2) you can now calculate the optimal proportional profit share $f^*$. Additionally, answer the following questions: Which profit share $f_{1}^*$ does management offer to motivate the inspector to choose
his low level of activity and to maximize the net profit of the company. Derive the value $f^*_2$ accordingly, motivating the quality inspector to choose his high level activity. Which of the two values ($f^*_1$ or $f^*_2$) is (ultimately) maximizing the net profit of the company?

**Exercise 7 (Sec. 6.2): Hierarchical Production Planning**

A company produces three types of tires (winter, summer, and all-year tires). The production process consists of two stages: First a rubber mixture is produced, and subsequently this mixture is cast into individual tire formats. Thus the type of a tire defines a product group and the tire format represents a single product. This is formally expressed by index sets indicating which tire format $j \in N_k$ belongs to product group $k (k = 1, 2, 3)$.

In July the company is setting up the production plan for the coming year. It consists of a medium-term planning procedure which determines production quantities, inventories and additional capacities, and of a short-term detailed production plan determining the output of individual tires.

Planning the medium-term level is based on a time-grid of months ($t = 1, \ldots, 12$) and relies on forecasts of group demand $\hat{D}_{kt}$. On the other hand, detailed weekly planning has a horizon of one month with $\tau = 1, \ldots, 4$ weeks. In planning, one has to take into account the two-stage character of the production process: The production of tire-type $k$ requires energy of $v^T_k$ [kWh per unit]. Altogether there is an amount of energy of $K^T$ [kWh per month] available which cannot be increased. On the medium-term planning level the manpower capacity consumption, which is expressed by the consumption rate $\bar{\omega}^T_k$, is to be anticipated in a non-reactive way. The inventory holding cost of the tire formats belonging to a tire type can be aggregated to average inventory holding cost $\bar{h}^T_k$ [$\$ per period and unit]. The initial inventory is required to be equal to the inventory at the end of the planning horizon and is to be optimized.
The short-term level determines the production plan for the first month (macro period) using detailed deterministically known demand $d^B_{j_T}$ (e.g., for the first 4 weeks of January). For the short-term level, the production of a tire of format $j$ requires $a^B_j$ [periods per unit] and inventory holding cost is given by $h^B_j$ [$ per period and unit]. Moreover, setup time is $s^B_j$ [periods per setup], and setup cost is $c^B_j$ [$ per setup]. The available manpower capacity of the second stage amounts to $K$ [hours per month]. It is possible to allocate additional manpower capacity $\Delta K_r$ [hours per week] at a cost of $c$ [$ per hour]. Shortages are not allowed on the medium-term and on the short-term level. Initial inventory is $x^T_{j_0}$ [units].

a) Describe the problem of the company as a hierarchical production planning problem and identify (in general terms) top-level and base-level. What can be said about the information situation? Is this problem a particular case of a constructional or an organizational DDM system?

b) Formulate the top-level and the base-level using (for the top-level) a non-reactive anticipation. Identify, in particular, $C^T$, $C^{TT}$, $a^T$, $A^T$, $I^T_{t_0}$, $IN$, $C^B$, $a^B$, $A^B$, and $I^B$.

Remarks: (1) Note that $\bar a^T_k$ and $\bar h^T_k$ are determined by the anticipation, while $v^T_k$ exclusively belongs to the upper level. (2) Short-term weekly planning uniformly divides the planned, monthly manpower capacities into weekly manpower capacities of the first macro period. Furthermore, it is assumed that the final inventory of tires cannot be larger than the final aggregated inventory at the end of the short term planning horizon.

c) How can one determine the aggregate capacity consumption rate and inventory holding cost in case of a non-reactive anticipation?

Exercise 8 (Sec. 6.2): Integrative Hierarchical Production Planning

A company is planning its production for the next year. This comprises a medium-term plan as for production capacities and a short-term disaggregated production plan. Medium-term planning is accomplished in July of the previous year (i.e., at $t_0$) using forecasts of group demand
on a quarterly basis. Short-term production planning is executed at the beginning of the year (i.e., at \( t_1 \)) using improved forecasts of item demand on a monthly basis.

For medium-term planning, the \( J \) products are pooled into \( K \) product groups. This is formally expressed by index sets \( N_k(k = 1, \ldots , K) \) indicating which product \( j \) is belonging to group \( k \), \( j \in N_k \). At the beginning of each quarter \( t = 1, \ldots , 4 \), the available production capacity can be extended or reduced by exactly \( P \) capacity units. The initial production capacity amounts to \( P_0' \). Capacity unit costs are \( c^K \) [EUR per period], and a capacity extension incurs costs of \( c^+ \) and a reduction of \( c^- \) [EUR per period].

The production of a unit of product group \( k \) requires on average \( A_k \) periods. If (quarterly) production exceeds demand, the surplus can be stored, with (average) inventory holding cost of \( H_k \) [EUR per unit and period].

Medium-term planning minimizes capacity adaptation cost and aggregated inventory holding cost. At any time stockouts are not allowed.

Short-term planning uniformly divides the quarterly planned capacities into monthly capacities. The allocation of capacity corresponds to the usage of the provided capacity. The criterion of short-term planning is given by inventory holding (\( h_j \)) and setup costs (\( c^s_j \)). One can allocate additional capacity at a cost of \( C^z \) [EUR per period]. Furthermore, shortages are not allowed on the short-term level.

Generally, the disaggregated values \( h_j \) (inventory holding cost of product \( j \)), \( a_j \) (consumption rate of product \( j \)), \( s_j \), and \( c^s_j \) (sequence-independent setup time and cost of product \( j \), respectively) are known.

a) Formulate the top-level and anticipated base-model. Identify, in particular, \( C^T \), \( C^{TT} \), \( C^{TB} \), \( a^T \), \( A^T \), \( IN \), \( \hat{C}^B \), \( \hat{a}^B \), and \( \hat{A}^B \). 

b) How are the aggregate consumption rates \( A_k \) and aggregate inventory holding cost \( H_k \) determined in case of a non-reactive anticipation (\( b_1 \)) and a perfect anticipation (\( b_2 \)) in using a linear aggregation rule?

c) Solve the above problem in applying a perfect anticipation and in using the following data and structure: Three products (\( P_1 \), \( P_2 \) and \( P_3 \)) are manufactured with \( P_1 \) belonging to product group \( G_1 \) and \( P_2 \) and \( P_3 \) belonging to product group \( G_2 \).
Forecast at time $t_0$ for months 1, \ldots, 12:

<table>
<thead>
<tr>
<th>Demand</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P1$</td>
<td>10</td>
<td>30</td>
<td>140</td>
<td>150</td>
<td>200</td>
<td>200</td>
<td>220</td>
<td>200</td>
<td>150</td>
<td>100</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>$P2$</td>
<td>10</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>140</td>
<td>160</td>
<td>180</td>
<td>150</td>
<td>150</td>
<td>90</td>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td>$P3$</td>
<td>15</td>
<td>40</td>
<td>150</td>
<td>170</td>
<td>160</td>
<td>170</td>
<td>160</td>
<td>100</td>
<td>80</td>
<td>60</td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>

Data of the aggregate model:

$P_0 = 800$ [hours per quarter],

$P = 400$ [hours],

$c^+ = 10000$ [EUR], $c^- = 11000$ [EUR]

$c^K = 500$ [EUR per hour].

Data of the detailed model

<table>
<thead>
<tr>
<th>Product</th>
<th>$h_j$ [EUR/unit]</th>
<th>$s_j$ [hours/setup]</th>
<th>$c_j$ [EUR/setup]</th>
<th>$a_j$ [hours/unit]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P1$</td>
<td>10</td>
<td>5</td>
<td>2000</td>
<td>0.5</td>
</tr>
<tr>
<td>$P2$</td>
<td>20</td>
<td>10</td>
<td>4000</td>
<td>4</td>
</tr>
<tr>
<td>$P3$</td>
<td>5</td>
<td>1</td>
<td>500</td>
<td>0.5</td>
</tr>
</tbody>
</table>

$c^z = 1200$ [EUR per hour]

Remarks:

- Use the values of the non-reactive anticipation for the initial values of the aggregated consumption rates $A_k$ and aggregated inventory holding costs $H_k$.
- Aggregate demand in adding units.
- Consider intermediary results up to 5 decimal places
- Compute a non-reactive anticipation and solve the top-level as well as the base-level. Subsequently determine better aggregated consumption rates and aggregated inventory holding cost and solve the top-level once again.
A dealer sells a product which he buys exclusively from a particular manufacturer. One of his tasks is to determine the weekly orders for the next 52 weeks to be placed with the manufacturer.

Selling the product, the dealer obtains a price of $p \ [\$ \text{ per unit}]. The product can be unrestrictedly stored at the dealer with holding cost $h \ [\$ \text{ per unit and week}]. At the beginning of the year there is an initial inventory of $x_0^L$. If the dealer cannot completely fulfill demand within one week, (non-fulfilled) demand is lost incurring additional shortage cost of $f \ [\$ \text{ per unit}].

The dealer anticipates the decision of the manufacturer in considering the model of the manufacturer which has the following structure: The production capacity of the manufacturer allows the production of a maximum of $z_t^{\text{max}}$ units in week $t \ (t = 1, \ldots, 52)$. There are no inventory restrictions, holding cost amounts to $k \ [\$ \text{ per unit and week}], and initial inventory is zero. Selling the product, the manufacturer can realize a price of $q \ [\$ \text{ per unit}]. Variable production cost is $c \ [\$ \text{ per unit}].

The manufacturer can deviate from the weekly order quantities in both directions. However, the deviations cause penalty cost of $K \ [\$ \text{ per unit}]. Hence, penalty cost in week $t$ is $K$. The manufacturer supplies at most the entire ordered quantity during the complete planning horizon of 52 weeks. The manufacturer decides on his output $z_t$ and the quantity $y_t$ to be delivered depending on the quantities $b_t$ ordered by the dealer. Manufacturer and dealer maximize their profit.

It is assumed that at the point in time when the dealer plans his orders weekly demand $d_t$ is known to him.

a) Draw a sketch of the relation between dealer and manufacturer and indicate the most important quantities.

b) Formulate top-level and base-level considering a perfect anticipation. Especially identify $CT, CTT, CTB, aT, AT$, and $IN(aT)$, and formulate the anticipated base-level, i.e., identify $\hat{C}B, \hat{a}B, \hat{A}B$, and $AF(IN)$.

c) How does the anticipated base-level influence the top-level?
d) How does the instruction of the top-level influence the decision of the base-level?

e) Assume the dealer does not know the base-model exactly: Which non-reactive anticipation could be used in that case?

f) How does the relation between penalty cost $K$ and inventory holding cost $h$ influence the decision of the manufacturer?
Solutions to the Exercises

Exercise 1: Make-or-Buy Decisions

a) The top-level decides on the number of units to be manufactured by the company itself, and the base-level decides on the number of units to be bought from an external supplier, i.e., the base-model describes the purchasing department.

Top-model:

Decision variables:
- \( a^T = x \): amount produced (decision variable of the top-level)
- \( a^B = y \): amount purchased (decision variable of the base-level)

Parameters:
- \( p = 5 \) : selling price
- \( q = 3 \) : purchasing price
- \( k = 2 \) : variable unit production cost
- \( a_1 = 2, a_2 = 3 \) : capacity consumption rates
- \( T_1 = 4, T_2 = 12 \) : capacities of the two production stages
- \( B = 3 \) : maximum amount to be purchased
- \( A = 4 \) : maximum demand

Top-criterion:

\[
C^T(a^T) = C^{TT}(a^T) + C^{TB}(AF(IN))
\]
\[
C^{TT}(a^T) = C^{TT}(x) = (p - k)x = (5 - 2)x = 3x
\]
\[
C^{TB} = C_B = (p - q)\hat{y}^* = 2\hat{y}^*
\]

Top-decision field:

\[
A^T := \{a^T : (I), (II), (III)\}
\]
\[(I) \quad 2x \leq 4\]  \[\implies x \leq 2\]

\[(II) \quad 3x \leq 12\] \[\implies x \geq 0\]

**Instruction:** \(IN = a^T = x\)

**Anticipated base-level:**

\[C^{TB} = \hat{C}^B(\hat{a}^B) = \hat{C}^B(\hat{y}) = (p - q)\hat{y} = (5 - 3)\hat{y} = 2\hat{y} \rightarrow \max\]

\[\hat{A}^B_{IN} := \{\hat{a}^B : (i), (ii)\}\]

\[(i) \quad 0 \leq \hat{y} \leq 3\] \[(ii) \quad \hat{y} \leq 4 - x \text{ (coupling condition)}\]

**Anticipation function:**

\[AF(a^T) = AF(x) = \hat{a}^B^* = \hat{y}^* = \min\{3; 4 - x\} = \begin{cases} 
\hat{y}^* = 3 & \text{if } x < 2 \\
\hat{y}^* = 2 & \text{if } x = 2 
\end{cases}\]

**Optimization of top-criterion:**

\[C^{T^*} = \max_{x \in A^T} \{3x + 2\hat{y}^*\} = \begin{cases} 
6 & \text{if } x = 0 \\
9 & \text{if } x = 1 \\
10 & \text{if } x = 2 
\end{cases}\]

Solution to the top-equation: \(IN^* = x^* = 2, \ \hat{y}^* = 2, \ C^{T^*}(x^*) = 10 \ \[$$]

**Base-model:**

\[C^B(a^B) = C^B(y) = (p - q)y = (5 - 3)y = 2y \rightarrow \max\]

\[A^B_{IN^*} := \{a^B : (i_a), (ii_a)\}\]

\[(i_a) \quad 0 \leq y \leq 3\] \[(ii_a) \quad y \leq 4 - x^* \text{ (coupling equation)}\]

\[a^B^* = y^*(x^*) = \min\{3; 4 - x^*\} = \min\{3; 4 - 2\} = 2\]
b) Stochastic purchasing price \( q \)

**Top-criterion:**

\[
E\{C^{T*}\} = \max_{x \in \{0,1,2\}} E\{3x + (5 - q) \min(3; 4 - x)\} \bigg| T_t^T
\]

Note that since the contribution margins according to both prices are positive, the company purchases up to the highest possible amount.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( q = 3 )</th>
<th>( q = 1 )</th>
<th>( E{C^T(x)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3 \cdot 0 + 2 \cdot 3 = 6</td>
<td>3 \cdot 0 + 4 \cdot 3 = 12</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>3 \cdot 1 + 2 \cdot 3 = 9</td>
<td>3 \cdot 1 + 4 \cdot 3 = 15</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>3 \cdot 2 + 2 \cdot 2 = 10</td>
<td>3 \cdot 2 + 4 \cdot 2 = 14</td>
<td>12</td>
</tr>
</tbody>
</table>

Optimal solution:

\[
E\{C^{T*}(x^*)\} = 12 \, \text{[$\text{\$}]} \quad (x^* = 2, \hat{y}^* = 2) \quad \text{or} \quad (x^* = 1, \hat{y}^* = 3)
\]

**Base-decision:**
e.g., \( x^* = 2 \):

\[
C^B(y) = (p - q')y = (5 - q')y \rightarrow \max
\]

s.t.

\[
0 \leq y \leq 3
\]

\[
y \leq 4 - x^*
\]

\[
y^*(x^*) = \min\{3; 4 - x^*\} = \min\{3; 4 - 2\} = 2
\]

(\( q' \) being a realization of \( q \)).

c) Stochastic demand

**Additional parameters:**

\( c^{sh} = 4 \): shortage cost per unit
\( c^d = 3 \): disposal cost per unit
\( d \): demand
Temporal structure of the decisions (time line):

\[
\begin{array}{c|c|c}
1 & 2 & 3 \\
(0,0,0) & (x-d,x,d) & (x-d+y,x,d) \\
\end{array}
\]

Formulation of the model:

States:

\[
\begin{align*}
z_1 &= (0,0,0) \\
z_2 &= (x-d,x,d) \\
z_3 &= (x-d+y,x,d)
\end{align*}
\]

Decisions:

\[
\begin{align*}
x &\in A^T = \{x : x = 0,1,2\} \\
y &\in A^B = \{y : y = 0,1,2,3\}
\end{align*}
\]

Transformation of states:

\[
\begin{align*}
z_1 &= (0,0,0) \\
z_2 &= z_1 + (x-d,x,d) \\
z_3 &= z_2 + (y,0,0) = (x-d+y,x,d)
\end{align*}
\]

Criteria:

\[
\begin{align*}
\bar{C}^T &= -kx + E\{C^B|d\} \\
C^B &= \text{SALES} (x,y,d) - q \cdot y - SC(x,y,d) - DC(x,y,d)
\end{align*}
\]

\[
\begin{align*}
\text{SALES}(x,y,d) &:= \begin{cases} 
p \cdot (x+y) & \text{if } d > x+y \\
p \cdot d & \text{if } d \leq x+y 
\end{cases} \\
SC(x,y,d) &:= \begin{cases} 
c^{sh} \cdot (d-x-y) & \text{if } d > x+y \\
0 & \text{if } d \leq x+y 
\end{cases} \\
DC(x,y,d) &:= \begin{cases} 
c^d \cdot (x+y-d) & \text{if } d < x+y \\
0 & \text{if } d \geq x+y 
\end{cases}
\end{align*}
\]
Graphical structure of the decision problem:

![Graphical representation of the decision problem](image)

_Solution to the functional equations:_

Step 1: (period 2: purchase decision)

\[
f_2(z_2) = \max_{\hat{y} \in \hat{A}^B} \{ \text{SALES}(x, \hat{y}, d) - q \cdot \hat{y} - \text{SC}(x, \hat{y}, d) - \text{DC}(x, \hat{y}, d) \}
\]
Numerical calculations:

<table>
<thead>
<tr>
<th>$z_2$ State</th>
<th>$\hat{y}$ Decision</th>
<th>$C^B$ Criterion</th>
<th>$f_2(z_2)$</th>
<th>$\hat{y}^*$ Opt. Dec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4,0,4)</td>
<td>3</td>
<td>$5 \cdot 3 - 3 \cdot 3 - 4 \cdot 2 - 3 \cdot 0 = 2$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$5 \cdot 2 - 3 \cdot 2 - 4 \cdot 2 - 3 \cdot 0 = -4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$5 \cdot 1 - 3 \cdot 1 - 4 \cdot 3 - 3 \cdot 0 = -10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$5 \cdot 0 - 3 \cdot 0 - 4 \cdot 4 - 3 \cdot 0 = -16$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-3,1,4)</td>
<td>3</td>
<td>$5 \cdot 4 - 3 \cdot 3 - 4 \cdot 0 - 3 \cdot 0 = 11$</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$5 \cdot 3 - 3 \cdot 2 - 4 \cdot 1 - 3 \cdot 0 = 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$5 \cdot 2 - 3 \cdot 1 - 4 \cdot 2 - 3 \cdot 0 = -1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$5 \cdot 1 - 3 \cdot 0 - 4 \cdot 4 - 3 \cdot 0 = -7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-2,2,4)</td>
<td>3</td>
<td>$5 \cdot 4 - 3 \cdot 3 - 4 \cdot 0 - 3 \cdot 1 = 8$</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$5 \cdot 4 - 3 \cdot 2 - 4 \cdot 4 - 3 \cdot 0 = 14$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$5 \cdot 3 - 3 \cdot 1 - 4 \cdot 1 - 3 \cdot 0 = 8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$5 \cdot 2 - 3 \cdot 0 - 4 \cdot 2 - 3 \cdot 0 = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-1,0,1)</td>
<td>3</td>
<td>$5 \cdot 1 - 3 \cdot 3 - 4 \cdot 0 - 3 \cdot 2 = -10$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$5 \cdot 1 - 3 \cdot 2 - 4 \cdot 0 - 3 \cdot 1 = -4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$5 \cdot 1 - 3 \cdot 1 - 4 \cdot 1 - 3 \cdot 0 = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$5 \cdot 0 - 3 \cdot 0 - 4 \cdot 1 - 3 \cdot 0 = -4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,1,1)</td>
<td>3</td>
<td>$5 \cdot 1 - 3 \cdot 3 - 4 \cdot 0 - 3 \cdot 3 = -13$</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$5 \cdot 1 - 3 \cdot 2 - 4 \cdot 0 - 3 \cdot 2 = -7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$5 \cdot 1 - 3 \cdot 1 - 4 \cdot 0 - 3 \cdot 1 = -1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$5 \cdot 1 - 3 \cdot 0 - 4 \cdot 0 - 3 \cdot 0 = 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,2,1)</td>
<td>3</td>
<td>$5 \cdot 1 - 3 \cdot 3 - 4 \cdot 0 - 3 \cdot 4 = -16$</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$5 \cdot 1 - 3 \cdot 2 - 4 \cdot 0 - 3 \cdot 3 = -10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$5 \cdot 1 - 3 \cdot 1 - 4 \cdot 0 - 3 \cdot 2 = -4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$5 \cdot 1 - 3 \cdot 0 - 4 \cdot 0 - 3 \cdot 1 = -2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 2: (period 1: production decision)

$$f_1(z_1) = \max_{x \in A^T} E \{ -x \cdot k + f_2(z_2) | d \}$$
Numerical calculations:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\bar{C}^T$</th>
<th>$f_1(z_1)$</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-2 \cdot 0 + (0.75 \cdot 2 + 0.25 \cdot 2) = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$-2 \cdot 1 + (0.75 \cdot 11 + 0.25 \cdot 5) = 11.5$</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>$-2 \cdot 2 + (0.75 \cdot 14 + 0.25 \cdot 2) = 15$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Optimal solution:

$x^* = 2$

$\hat{y}^*(d = 1) = 0$

$\hat{y}^*(d = 4) = 2$.

Optimal profit:

$f_1(z_1) = 15 [\$]$.

**Exercise 2: Dynamic Lot Sizing**

a) Time line

```
Known Demand    Stochastic Demand
```

```
1  2  3  4  5  6
```

$t_0$  $t_1$

```
Known Demand
```

```
Time Period
```
b) Characterization of the DDM model

The top-level decides in period 1 upon the order quantities in periods 1 and 2 while the base-level decides in period 3 upon the order quantities in periods 3 to 5.

Moreover, this problem belongs to the class of tactical-operational DDM systems, especially to decision time hierarchies with weak information asymmetry.

c) Formulation of the DDM model

Indices:
\( \tau: \) period (month) \( \tau = 1, \ldots, 5 \)

Decision variables:
\( x_\tau: \) order quantity in month \( \tau \) [units]
\( x_\tau^L: \) inventory at the end of month \( \tau \) [units]
\( \delta(x_\tau): \) ordering indicator for month \( \tau \)

Data and parameters:
\( c^L: \) inventory holding cost [EUR per unit and month]
\( K: \) order setup cost [EUR per order]
\( d_\tau: \) demand in month \( \tau \) [units per month]
\( x_0^L: \) initial inventory [units]

Top-model:

Action:
\[
a^T = (x_\tau, x_\tau^L)_{\tau=1,2}
\]

Top-criterion:
\[
\bar{C}^T = \{C^{TT}(a^T) + E(C^{TB}(AF(IN)))\} \longrightarrow \min_{a^T \in A^T}
\]
\[
C^{TT} = \sum_{\tau=1}^{2} (c^L x_\tau^L + K \delta(x_\tau))
\]
\[
C^{TB} = \hat{C}^B(\hat{a}^B^*) = \sum_{\tau=3}^{5} (c^L \hat{x}_\tau^L + K \delta(\hat{x}_\tau^*))
\]
\[
\delta(\hat{x}_\tau) = \begin{cases} 
1 & \text{if } \hat{x}_\tau > 0 \\
0 & \text{if } \hat{x}_\tau = 0 
\end{cases}
\]
Decision field:

\[ A^T = \{ a^T : (I), (II), (III) \} \]

(I) \[ x^L_\tau = x^L_{\tau-1} + x_\tau - d_\tau \quad \tau = 1, 2 \]

(II) \[ x^L_0 = x^L_0 \]

(III) \[ x_\tau, x^L_\tau \geq 0 \quad \tau = 1, 2 \]

**Instruction:**

\[ IN(a^T) = x^L_2 \]

**Anticipated base-level**

**Action:**

\[ \hat{a}^B = (\hat{x}_\tau, \hat{x}^L_\tau)_{\tau=3, \ldots, 5} \]

**Anticipation function:**

\[ AF(IN) = \hat{a}^{B*} = \arg \min_{\hat{a}^B \in \hat{A}^B_{IN}} \sum_{\tau=3}^{5} (c^L \hat{x}^L_\tau + K\delta(\hat{x}_\tau)) \]

Decision field:

\[ \hat{A}^B_{IN} = \{ \hat{a}^B : (i), (ii), (iii) \} \]

(i) \[ \hat{x}^L_\tau = \hat{x}^L_{\tau-1} + \hat{x}_\tau - d_\tau, \quad \tau = 3, 4, 5 \]

(ii) \[ \hat{x}^L_2 = x^L_2 \]

(iii) \[ \hat{x}_\tau, \hat{x}^L_\tau \geq 0, \quad \tau = 3, 4, 5 \]

Note that \( d_\tau \) and hence \( AF \) are random variables.

**d) Approximate anticipation**

As an example consider stochastic demand being replaced with the following 3 scenarios:

1) \[ \bar{d}_\tau = E(d_\tau), \quad \tau = 3, 4, 5 \]

2) \[ \bar{d}^+ = \bar{d}_\tau + \sqrt{Var(d_\tau)}, \quad \tau = 3, 4, 5 \]

3) \[ \bar{d}^- = \bar{d}_\tau - \sqrt{Var(d_\tau)}, \quad \tau = 3, 4, 5 \]

To obtain an approximate anticipation, calculate \( AF(IN) \) for these scenarios and replace the expectation operator \( E \) in \( \dot{C}^T \) with an unweighted average.
Exercise 3: Location and Production Planning

a) Sketch for $s = 4$ and $k = 3$

The top-level decides on the opening of plants for the refinement of products and on the anticipated assignment of customers to plants. On a quarterly basis, the base-level determines the production and the required additional capacity.

*Information state*

The top-level does not have full information about the demand of the next years. Only forecasts are available. At the point in time when the production decision is made the demand of the first year is known.

b) Formulation of the DDM model

*Indices*

- $s$: plant $s = 1, \ldots, s$
- $k$: customer $k = 1, \ldots, k$
- $t$: time $t = 1, \ldots, 20$ (quarter)

*Decision variables*

- $x_{sk}$: total amount delivered by plant $s$ to customer $k$ over the next five years
- $b_{st}$: demand satisfied by plant $s$ in quarter $t$
- $y_{st}^L$: inventory at plant $s$ in quarter $t$
- $y_{st}$: production quantity at plant $s$ in quarter $t$
\[ \Delta K_{st} : \text{additional raw material at plant } s \text{ in quarter } t \]
\[ \delta_s = \begin{cases} 
1 & \text{if } \sum_{k} x_{sk} > 0 \\
0 & \text{if } \sum_{k} x_{sk} = 0 
\end{cases} \]

Data and parameters

\( c_{s}^{\text{invest}} \): construction cost of plant \( s \)
\( c_{st}^{p} \): production cost at plant \( s \) in quarter \( t \)
\( c_{sk} \): transport cost per item from plant \( s \) to customer \( k \)
\( d_{kt}^{c} \): demand of customer \( k \) in quarter \( t \)
\( h_{s} \): inventory holding cost per quarter in plant \( s \)
\( k_{s} \): cost of additional raw material at plant \( s \)
\( a_{t} \): required raw material to produce 1 ton of the product in quarter \( t \)

\( K_{st} \): raw material available at plant \( s \) in quarter \( t \)
\( b_{st} \): demand at plant \( s \) in quarter \( t \)
\( y_{s}^{L_{\text{max}}} \): capacity of inventory of plant \( s \)
\( M \): sufficiently big number

Top-level

\( a^{T} \) = \( \{ x_{sk}, \delta_{s} | \forall s, k \} \)
\( C^{T} \) = \( C^{TT} + C^{TB} \)
\( C^{TT} = \sum_{s=1}^{\bar{s}} \sum_{k=1}^{k} c_{sk} x_{sk} + \sum_{s=1}^{\bar{s}} c_{s}^{\text{invest}} \delta_{s} \)
\( C^{TB} = \sum_{s=1}^{\bar{s}} \sum_{t=1}^{20} (h_{s} \bar{y}_{st} + k_{s} \Delta K_{st}^{*} + c_{st}^{p} \bar{v}_{st}^{*}) \)
\( A^{T} \) = \[ \begin{cases} 
\sum_{s=1}^{\bar{s}} x_{sk} = \sum_{t=1}^{20} d_{kt}^{c} & \forall k \\
\sum_{k=1}^{k} x_{sk} \leq M \delta_{s} & \forall s \\
x_{sk} \geq 0, \delta_{s} \in \{0, 1\} & \forall s, k 
\end{cases} \]
\( IN(a^{T}) \) = \[ \begin{cases} 
\sum_{k=1}^{k} \frac{x_{sk}}{d_{kt}^{c}} & \forall t, s 
\end{cases} \]

Anticipated base-level

A linear program is set up for every plant \( s = 1, \ldots, \bar{s} \).

\[ \hat{a}_{s}^{B} = (\hat{y}_{st}^{L}, \hat{y}_{st}, \hat{\Delta K}_{st} | \forall t = 1, \ldots, 20) \forall s \]
\[ \hat{C}_{s}^{B} = \sum_{t=1}^{20} (h_{s} \hat{y}_{st}^{L} + k_{s} \hat{\Delta K}_{st} + c_{st}^{p} \hat{v}_{st}) \forall s \]
Exercise 4: Determination of Transfer Prices

a) Verbal characterization

The top-level maximizes the overall gain of the company and determines an optimal transfer price $\lambda$. It reactively considers the base-level in using the solution of the production model.

The determined transfer price serves as an instruction and consequently influences the optimal production of the subsidiaries.

b) Formulating the DDM model

\textit{Indices}

\begin{align*}
  v : & \quad \text{subsidiary, } v = 1, \ldots, V \\
  i : & \quad \text{product, } i = 1, \ldots, n_v
\end{align*}

\textit{Decision variables}

$\lambda$: transfer price of the common resource

$x_{vi}$: production volume of product $i$ at subsidiary $v$

\textit{Data and parameters}

$k_{vi}$: variable out of pocket cost of product $i$ at subsidiary $v$

$a_{vi}$: consumption rate of product $i$ of the common resource at subsidiary $v$
Exercise 4

\( b_{vi} \): consumption rate of product \( i \) of the local resource at subsidiary \( v \)

\( d_{vi} \): demand of product \( i \) at subsidiary \( v \)

\( M \): capacity of the common resource

\( m_v \): capacity of the local resource at subsidiary \( v \)

**Top-level**

**Action:**

\[ a^T = \lambda \]

**Top-criterion:**

\[ C^T = \sum_{v=1}^{V} C_v^{TB} \]

\( C^{TT} \) is not used

\[ C_v^{TB} = \sum_{i=1}^{n_v} (p_{vi} \hat{x}_{vi}^* - k_{vi} \hat{x}_{vi}^*) \]

**Top-decision field:**

\[ A^T = A_{AF}^T = \{(I), (II)\} \]

(1) \[ \sum_{v=1}^{V} \sum_{i=1}^{n_v} a_{vi} \hat{x}_{vi}^* \leq M \]

(II) \( \hat{x}_{vi}^* \geq 0 \quad \forall v = 1, \ldots , V; i = 1, \ldots , n_v \)

**Instruction**

\[ IN = IN(a^T) = a^T = \lambda \]

**Anticipated and real base-level**

For every subsidiary \( v, v = 1, \ldots , V; \) there is an anticipated base-model:

**Base-action:**

\[ \hat{a}_v^B = (\hat{x}_{vi}) \quad \forall i = 1, \ldots , n_v \]

**Base-criterion:**

\[ \hat{C}_{v_IN}^B = \sum_{i=1}^{n_v} (p_{vi} - k_{vi} - a_{vi} \lambda) \hat{x}_{vi} \]
Decision field:

\[ \hat{A}_v^B = ((i), (ii), (iii)) \]

(i) \[ \sum_{i=1}^{n_v} b_{vi} \hat{x}_{vi} \leq m_v \]

(ii) \[ \hat{x}_{vi} \leq d_{vi} \quad i = 1, \ldots, n_v \]

(iii) \[ \hat{x}_{vi} \geq 0 \quad i = 1, \ldots, n_v \]

\[ AF_v(IN) = \arg \max_{\hat{a}_v^B \in A_v^B} \hat{C}_{v,IN}(\hat{a}_v^B) \]

**Anticipation function**

\[ AF(IN) = (AF_1(IN), \ldots, AF_V(IN)) \]

c) A **team situation** is characterized by a non-antagonistic relationship between the participants of a DDM system, in particular, \( C^{TB} = \hat{C}^B^* \). This is fulfilled since, according to b), the top-down-criteria

\[ C_v^{TB} = \sum_{i=1}^{n_v} (p_{vi} - k_{vi}) \hat{x}_{vi}^*, \quad v = 1, \ldots, V \]

coincide with the (optimal) base-criteria before the control \( \lambda \) is exerted:

\[ \hat{C}_{v,IN}^B = \sum_{i=1}^{n_v} (p_{vi} - k_{vi} - a_{vi} \lambda) \hat{x}_{vi}^* = C_v^{TB} \quad \text{if} \ \lambda = 0 \]

d) \( \lambda \) **can be determined** using search methods.

e) **Only forecasts are known**

At the point in time \( t_0 \) when the allocation of overhead cost is done, demand is not known, only forecasts \( \hat{d}_{vi} \) are available. The anticipated base-level provides optimal production quantities as anticipation function being based on demand forecasts. At the point in time when the production decision is being made \( (t_1) \), demand is known. The production model is solved using realized demand \( d_{vi} \).
Exercise 5: Capacity Adaptation

a) Prophetic solution

Indices:
i : product \( i = 1,2 \)
f : level of experience \( f = 1,2 \)
r : scenario \( r = 1, \ldots, 4 \)

Decision variables:
\( x_f \) : manpower capacity with experience \( f \) [hours]
\( y_i \) : production quantity of product \( i \) [units]

Data and parameters:
\( c_f \) : personnel cost of employees having experience \( f \) [\$ per hour]
\( a_{if} \) : consumption rate of product \( i \) manufactured by personnel having experience \( f \) [hours per unit]
\( p_i \) : price of item \( i = 1,2 \) [EUR per unit]
\( k_i \) : variable cost of product \( i = 1,2 \) [EUR per unit]
\( d_{ir} \) : demand for product \( i = 1,2 \) in scenario \( r = 1, \ldots, 4 \)

The prophetic solution knows at \( t_0 \) which scenario \( r = r' \) is realized, i.e., \( d_{ir} = d_{ir'} =: d_i \ (i = 1,2) \).

Optimization problem:

\[
g = -c_1 x_1 - c_2 x_2 + (p_1 - k_1) y_1 + (p_2 - k_2) y_2 \longrightarrow \max
\]

subject to
\[
\begin{align*}
a_{11} y_1 + a_{21} y_2 & \leq x_1 \\
a_{12} y_1 + a_{22} y_2 & \leq x_2 \\
0,8 \cdot d_1 & \leq y_1 \leq d_1 \\
0,8 \cdot d_2 & \leq y_2 \leq d_2 \\
x_1, x_2, y_1, y_2 & \geq 0
\end{align*}
\]

b) Stochastic program

\[
a^T := (x_1, x_2)
\]
\[ C^T^* = \max_{(x_1,x_2) \in A^T} \left( C^{TT}(x_1,x_2) + \sum_r w_r \max_{(\hat{y}_1,\hat{y}_2) \in \hat{A}^{B}_{r,IN}} C^B(\hat{y}_1,\hat{y}_2) \right) \]

\[ C^{TT} = -c_1 x_1 - c_2 x_2 \]

\[ C^{TB} = C^B(\hat{y}_1^*,\hat{y}_2^*) = (p_1 - k_1)\hat{y}_1^* + (p_2 - k_2)\hat{y}_2^* \]

\[ A^T = \begin{pmatrix} x_1 \leq a_{11} \max d_{1r} + a_{21} \max d_{2r} \\ x_2 \leq a_{12} \max d_{1r} + a_{22} \max d_{2r} \\ x_1, x_2 \geq 0 \end{pmatrix} \]

\[ \hat{A}^B_{r,IN} = \begin{pmatrix} a_{11} \hat{y}_1 + a_{21} \hat{y}_2 \leq x_1 \\ a_{12} \hat{y}_1 + a_{22} \hat{y}_2 \leq x_2 \\ 0.8d_{1r} \leq \hat{y}_1 \leq d_{1r} \\ 0.8d_{2r} \leq \hat{y}_2 \leq d_{2r} \\ \hat{y}_1, \hat{y}_2 \geq 0 \end{pmatrix} \quad \forall r \]

**Base-level**

Realization of \( r = r' \)

\[ a^B = (y_1, y_2), IN^* = (x_1^*, x_2^*) \]

\[ C^{B^*} = \max_{(y_1,y_2) \in \hat{A}^B_{r',IN^*}} \left( (p_1 - k_1)y_1 + (p_2 - k_2)y_2 \right) \]

\[ \hat{A}^B_{r',IN^*} = \begin{pmatrix} a_{11} y_1 + a_{21} y_2 \leq x_1^* \\ a_{12} y_1 + a_{22} y_2 \leq x_2^* \\ 0.8d_{1r'} \leq y_1 \leq d_{1r'} \\ 0.8d_{2r'} \leq y_2 \leq d_{2r'} \\ y_1, y_2 \geq 0 \end{pmatrix} \]

**c) Mean demand**

\[ \bar{d}_i = \sum_r w_r d_{ir} \quad (i = 1, 2) \]

\[ C^{T^*} = \max_{(x_1,x_2) \in A^T} \left( C^{TT}(x_1,x_2) + \max_{(\hat{y}_1,\hat{y}_2) \in \hat{A}^B_{IN}} \hat{C}^B(\hat{y}_1,\hat{y}_2) \right) \]
\[A^T = \begin{cases} x_1 \leq a_{11}\bar{d}_1 + a_{21}\bar{d}_2 \\
 x_2 \leq a_{12}\bar{d}_1 + a_{22}\bar{d}_2 \\
 x_1, x_2 \geq 0 \end{cases}\]
\[\hat{A}_{fin}^B = \begin{pmatrix} a_{11}\hat{y}_1 + a_{21}\hat{y}_2 \leq x_1 \\
 a_{12}\hat{y}_1 + a_{22}\hat{y}_2 \leq x_2 \\
 0.8\bar{d}_1 \leq \hat{y}_1 \leq \bar{d}_1 \\
 0.8\bar{d}_2 \leq \hat{y}_2 \leq \bar{d}_2 \\
 \hat{y}_1, \hat{y}_2 \geq 0 \end{pmatrix}\]

Notation and base-level as under b)

d) Solutions obtained using an LP-solver

| \(C^B_r\) & \(r\) | \(15\hat{y}_1 + 12\hat{y}_2\) |
|---|---|---|
| \(C^B_1\) | \(30, 50\) | \(108.00\) |
| \(C^B_2\) | \(138.00\) |
| \(C^B_3\) | \(156.00\) |
| \(C^B_4\) | \(165.60\) |
| \(\tilde{C}^T(30, 50)\) | \(-2 \cdot 30 - 1 \cdot 50 + E(C^B_r)\) | \(31.90\) |

| \(C^B_r\) & \(r\) | \(15\hat{y}_1 + 12\hat{y}_2\) |
|---|---|---|
| \(C^B_1\) | \(34, 45\) | \(108.00\) |
| \(C^B_2\) | \(138.00\) |
| \(C^B_3\) | \(148.80\) |
| \(C^B_4\) | \(169.20\) |
| \(\tilde{C}^T(34, 45)\) | \(-2 \cdot 34 - 1 \cdot 45 + E(C^B_r)\) | \(28.00\) |

| \(C^B_r\) & \(r\) | \(15\hat{y}_1 + 12\hat{y}_2\) |
|---|---|---|
| \(C^B_1\) | \(25, 40\) | \(108.00\) |
| \(C^B_2\) | \(133.00\) |
| \(C^B_3\) | \(136.80\) |
| \(C^B_4\) | \text{Infeasible} |
| \(\tilde{C}^T(25, 40)\) | \text{Infeasible} |

Optimal solution: \(x^* = (30, 50), \tilde{C}^{T*} = 31.90\)
e) **Expected demand:** $d_1 = 5, d_2 = 6$

\[
C^B = \max_{(\hat{y}_1, \hat{y}_2) \in \hat{A}_{30,50}} (15\hat{y}_1 + 12\hat{y}_2) = 108.00
\]

\[
C^T(30,50) = -2 \cdot 30 - 1 \cdot 50 + 108 = -2
\]

\[
C^B = \max_{(\hat{y}_1, \hat{y}_2) \in \hat{A}_{34,45}} (15\hat{y}_1 + 12\hat{y}_2) = 147.00
\]

\[
C^T(34,45) = -2 \cdot 34 - 1 \cdot 45 + 147 = 34
\]

\[
C^B = \max_{(\hat{y}_1, \hat{y}_2) \in \hat{A}_{25,40}} (15\hat{y}_1 + 12\hat{y}_2) = 137.00
\]

\[
C^T(25,40) = -2 \cdot 25 - 1 \cdot 40 + 137 = 47
\]

Optimal solution: $x^* = (25, 40), C^{T^*} = 47$

**Exercise 6: Quality Management**

a) **Description as PA problem**

Principal: management
Agent: quality inspector
Delegated task: checking of the manufactured component
Information status: hidden action.
b) Formulation of DDM model

\[ a^{T*} = \arg \max_{a^T \in AT} E\{C^{TB}(AF(a^T))|I^T\} \]

\[ AF(a^T) = \arg \max_{\hat{a}^B \in A^B} E\{\hat{C}^B(\hat{a}^B))|I^B\} \]

\[ = \arg \max_{a^B \in A^B} E\{C^B(a^B))|I^B\} \]

\[ a^{B*} = \arg \max_{a^B \in A^B} E\{C_{AF}^B(a^B))|I^B\} \]

\[ a^{T} = \Phi(P) := F + f \cdot P \]

\[ P: \quad \text{profit} \]

\[ a^{B}: \quad \text{care of the quality control, i.e., spent time} \]

\[ C^{TB} = P - \Phi(P) = (1 - f) \cdot P - F \]

\[ C^B = u^B(\Phi(G(a^B)), a^B) = \Phi(P(a^B)) - V(a^B) \]

\[ V(a^B): \quad \text{disutility} \]

\[ A^B(\Phi) := \{a^B : E\{C^B\} \geq AN^B\} \]

\[ AF = \hat{a}^{B*}(\Phi) \]

\[ \text{anticipated optimal effort of the agent} \]

\[ I^T: \quad \text{hidden action situation} \]

\[ I^B: \quad \text{principal’s and agent’s information about the agent’s environment} \]

\[ \text{c) Determination of the profit share } f \]

\[ a^B: \quad \text{care of the quality control which guarantees an expected utility of at least 7000 EUR, i.e.,} \]

\[ \hat{C}^B(a^B = 1) \geq 7000 \quad \text{or} \quad \hat{C}^B(a^B = 2) \geq 7000 \]

\[ \text{In case the agent chooses a low effort } (a^B = 1): \]

\[ \frac{1}{3}(5500 + 15000f) + \frac{1}{3}(5500 + 20000f) + \frac{1}{3}(5500 + 25000f) - 500 \geq 7000 \]

\[ 5500 + f \left( \frac{1}{3} - \frac{1}{3} - \frac{1}{3} \right) \geq 7000 \]
Solutions to the Exercises

\[20000 f \geq 7000 + 500 - 5500 = 2000\]
\[f \geq 0.1\]

In case the agent chooses a high effort \((a^B = 2)\):
\[
\frac{1}{3} (5500 + 25000 f) + \frac{1}{3} (5500 + 31000 f) + \frac{1}{3} (5500 + 34000 f) - 3000 \geq 7000
\]
\[5500 + f \left( \frac{1}{3} 25000 + \frac{1}{3} 31000 + \frac{1}{3} 34000 \right) - 3000 \geq 7000\]
\[30000 f \geq 7000 + 3000 - 5500 = 4500\]
\[f \geq 0.15\]

Hence, the participation condition of the agent results in
\[
a^B = \begin{cases} 
1 & \text{if } 0.1 \leq f < 0.15 \\
2 & \text{if } f \geq 0.15 
\end{cases}
\]

Clearly, the company will offer the smallest possible \(f\), i.e., for \(a^B = 1\), \(f = 0.1\), and for \(a^B = 2\), \(f = 0.15\).

c2) First let us consider the case that the quality inspector is to choose the low level of effort, i.e.,
\[
a^{B^*} = \arg \max_{a^B \in \{1,2\}} E\{C^B(a^B)\} = 1,
\]
i.e.,
\[
E\{C^B(a^B = 1)\} > E\{C^B(a^B = 2)\}.
\]

The range of \(f\) for which this condition is fulfilled is given by
\[
\frac{1}{3} (5500 + 15000 f) + \frac{1}{3} (5500 + 20000 f) + \frac{1}{3} (5500 + 25000 f) - 500 > \]
\[
\frac{1}{3} (5500 + 25000 f) + \frac{1}{3} (5500 + 31000 f) + \frac{1}{3} (5500 + 34000 f) - 3000
\]
\[f < 0.25\]

I.e., the quality inspector chooses a low effort if \(f < 0.25\). Otherwise he chooses a high effort. If \(f < 0.1\), the quality inspector is not willing to sign a contract at all. Hence, the anticipation function is:
\[
\hat{a}^{B^*}(a^T) = \hat{a}^{B^*}(f) = \arg \max_{a^B \in \{1,2\}} E\{C^B(a^B(a^T))\}
\]
\[
= \begin{cases} 
1 & \text{if } 0.1 \leq f < 0.25 \\
2 & \text{if } f \geq 0.25 
\end{cases}
\]
(Remark: If \( E\{C^B(a^B = 1)\} = E\{C^B(a^B = 2)\} \) (i.e., \( f = 0.25 \)), PA theory assumes that the agent chooses the high effort.)

**c3)** The criterion of the top-level (management) for determining the optimal \( f = f^* \) is

\[
\max_{f \in [0,1]} \bar{C}^{TB} = \max_{f \in [0,1]} E\{(1-f) \cdot P(AF(f)) - F\}
\]

The quality inspector will show a low effort, if \( f \in [0.1,0.25] \). In this case management maximizes the expected net profit by

\[
\max_{f \in [0.1, 0.25]} \frac{1}{3} \left( (1-f)15000 - 5500 \right) + \frac{1}{3} \left( (1-f)20000 - 5500 \right) + \frac{1}{3} \left( (1-f)25000 - 5500 \right) =
\]

\[
= \max_{f \in [0.1, 0.25]} (1-f)20000 - 5500 = 18000 - 5500 = 12500 \text{ if } f^*_1 = 0.1
\]

Analogously, the quality inspector will show a high effort, if \( f \in [0.25, 1] \):

\[
\max_{f \in [0.25, 1]} \frac{1}{3} \left( (1-f)25000 - 5500 \right) + \frac{1}{3} \left( (1-f)31000 - 5500 \right) + \frac{1}{3} \left( (1-f)34000 - 5500 \right) =
\]

\[
= \max_{f \in [0.25, 1]} (1-f)30000 - 5500 = 22500 - 5500 = 17000 \text{ for } f^*_2 = 0.25
\]

Hence, management obtains maximal net profit by offering \( f^* = 0.25 = 25\% \).

**Exercise 7: Hierarchical Production Planning**

**a) Characterization of DDM system**

The medium-term planning level is defining the top-level deciding on monthly capacity and on the target stock at the end of the first macro period. Operational planning is represented by the base-level and determines the production plan and the setup of final products of the second stage.
**Information status:** At the point in time $t_0$ when medium-term planning takes place, only forecasts of aggregated demand are available. When short-term planning is actually performed (at time $t_1$), demand is revealed.

**Type of DDM system:** It is an organizational DDM system (more precisely, it is a decision time hierarchy) since there is a (weak) information asymmetry between the two levels.

**b) Formulation of DDM model**

**Indices**

$k$: product group $k = 1, \ldots, \bar{k}, \bar{k} = 3$

$j$: final product $j \in N_k, N_k$: set of product indices comprising product group $k$

$t$: macro period (months) $t = 1, \ldots, T, T = 12$

$\tau$: micro period (weeks) $\tau = 1, \ldots, \bar{\tau}, \bar{\tau} = 4$

**Decision variables**

$X_{kt}^T$: amount produced of product group $k$ in macro period $t$

$X_{kt}^L$: amount of stock of product group $k$ at the end of macro period $t$

$\Delta K_{\tau}$: capacity adaptation in micro period $\tau$

$x_{j\tau}^T$: amount of product $j$ produced at the end of micro period $\tau$

$x_{j\tau}^L$: amount of stock of product $j$ in micro period $\tau$

$\delta_{j\tau}$: setup indicator (product) $j$ in micro period $\tau$

**Data and parameters**

$K^T$: production capacity of the central system in every macro period $t$

$\hat{D}_{kt}^T$: aggregate forecast of demand of product group $k$ in macro period $t$

$v_k^T$: capacity consumption rate of product group $k$ of the central system

$\tilde{a}_{k}^T$: aggregate consumption rate of product group $k$

$\bar{h}_k^T$: aggregate inventory holding cost of product group $k$

$K$: manpower capacity

$c$: cost of additional manpower capacity

$d_{j\tau}^T$: demand of product $j$ in micro period $\tau$

$a_{j\tau}^T$: consumption rate of product $j$

$h_j^T$: inventory holding cost of product $j$
$s_j^B$: setup time of product $j$
$c_j^B$: setup cost of product $j$
$x_{j0}^L$: initial inventory of product $j$
$M$: sufficiently big number

**Top-level**

$$a^T = \{(X_{kt}, X_{kt}^L) : \forall t, k\}$$

$$C^T = C^{TT} = \sum_{t=1}^{12} \sum_{k=1}^{3} h_k^T X_{kt}^L$$

$$A^T = \begin{cases} 
X_{kt}^L = X_{k,t-1}^L + X_{kt} - \hat{D}_k^T & \forall k, t \\
X_{k0}^L = X_{k12}^L & \forall k \\
\sum_{k=1}^{3} v_k^T \cdot X_{kt} \leq K^T & \forall t \\
\sum_{k=1}^{3} \bar{a}_k^T \cdot X_{kt} \leq K & \forall t \\
X_{kt}, X_{kt}^L \geq 0 & \forall k, t 
\end{cases}$$

$I_{t_0}^T$: describes aggregate forecast of demand at $t_0$

$IN = \{X_{kt}^L : \forall k\}$

**Base-level**

$$a^B = \{(x_{jt}, x_{jt}^L, \delta_{jt}) : \forall j, t\}$$

$$C^B = \sum_{k=1}^{3} \sum_{j \in N_k} \sum_{\tau=1}^{4} (h_j^B x_{j\tau}^L + s_j^B \delta_{j\tau} + c\Delta K_\tau)$$

$$A_{IN}^B = \begin{cases} 
x_{j\tau}^L = x_{j,\tau-1}^L + x_{j\tau} - a_j^B & j \in N_k \forall k \forall \tau \\
x_{j0}^L = x_{j0}^L & j \in N_k \forall k \\
\sum_{k=1}^{3} \sum_{j \in N_k} (a_j^B x_{j\tau} + s_j^B \delta_{j\tau}) \leq \frac{K}{4} + \Delta K_\tau & \forall \tau \\
x_{j\tau} \leq M \delta_{j\tau} & j \in N_k \forall k \\
\sum_{j \in N_k} x_{j4}^L = X_{k1}^L & \forall k \\
\delta_{j,\tau} \in \{0, 1\} & j \in N_k \forall k \forall \tau \\
x_{j\tau}, x_{j\tau}^L \geq 0 & j \in N_k \forall k \forall \tau 
\end{cases}$$
IB: describes realized demand of product items in $t_1$

c) Aggregations

$$\tilde{a}_k = \frac{\sum_{\tau=1}^{4} \sum_{j \in N_k} (a_j^B \hat{d}_{j\tau}^B + s_j^B)}{\sum_{\tau=1}^{4} \sum_{j \in N_k} \hat{d}_{j\tau}^B} \quad \forall k$$

$$\tilde{h}_k = \frac{\sum_{\tau=1}^{4} \sum_{j \in N_k} h_j \hat{d}_{j\tau}^B}{\sum_{\tau=1}^{4} \sum_{j \in N_k} \hat{d}_{j\tau}^B} \quad \forall k$$

Exercise 8: Integrative Hierarchical Production Planning

a) Formulation of DDM model

Indices

$k$: product group $k = 1, \ldots, K$

$j$: product $j \in N_k$

$t$: aggregate period (quarter) $t = 1, \ldots, 4$

$\tau$: detailed period (month) $\tau = 1, \ldots, 12$

Decision variables

$X_{kt}$: amount of product group $k$ in macro period $t$

$X_{kt}^L$: amount of stock of product group $k$ in macro period $t$

$P_t$: provided capacity in macro period $t$

$P_t^+$: capacity expansion indicator

$P_t^-$: capacity reduction indicator

$x_{j\tau}$: amount of production of product $j$ in micro period $\tau$

$x_{j\tau}^L$: amount of stock of product $j$ in micro period $\tau$

$\delta_{j\tau}$: setup indicator of product $j$ in micro period $\tau$

$\Delta K_{k\tau}$: capacity adaptation in micro period $\tau$

Data and parameters

$\hat{D}_{kt}$: aggregate demand forecast

$\hat{d}_{j\tau}$: detailed demand forecast
\( P'_0 \): initial capacity \\
\( P \): fixed amount of capacity to be expanded or reduced \\
\( c^K \): cost of capacity \\
\( c^+ \): cost of capacity expansion \\
\( c^- \): cost of capacity reduction \\
\( A_k \): aggregate consumption rate of product group \( k \) \\
\( H_k \): aggregate inventory holding cost of product group \( k \) \\
\( h_j \): inventory holding cost of product \( j \) \\
\( a_j \): consumption rate of product \( j \) \\
\( c^s_j \): (sequence-independent) setup cost of product \( j \) \\
\( s_j \): (sequence-independent) setup time of product \( j \) \\
\( c^c \): cost of additional capacity (of the base-level) \\
\( p_{k\tau} \): amount of capacity being available for product group \( k \) in micro period \( \tau \) \\
\( x'_{j0} \): initial stock of product \( j \) \\
\( M(t) \): set of months in quarter \( t \) \\
|\( |t| \)|: number of months per quarter 

**Top-level**

\[
a^T = \left\{ (X_{kt}, X^L_{kt}, P_t, P^+, P^-) \right\} \quad \forall k, t
\]

\[
C^{T*} = \min_{a^T \in A^T_{AF}} \left\{ \sum_{t=1}^{4} c^+ \cdot P^+_t + c^- \cdot P^-_t + c^K \cdot P_t + \sum_{t=1}^{4} \sum_{k=1}^{K} H_k \cdot X^L_{kt} \right\}
\]

\[
A^{T}_{AF} = \left\{ \begin{array}{l}
P_t = P_{t-1} + P \cdot P^+_t - P \cdot P^-_t \\
P_0 = P'_0 \\
X^L_{kt} = X^L_{k,t-1} + X_{kt} - \hat{D}_{kt} \\
X^L_{k0} = X^L_{k4} \\
\sum_{k=1}^{K} A_k \cdot X_{kt} \leq P_t \\
X^L_{kt}, X_{kt}, P_t \geq 0 \\
P^+_t, P^-_t \in (0, 1)
\end{array} \right\} \quad \forall k
\]

\[
IN : \left\{ p_{k\tau} = \frac{1}{3} P_t \cdot \frac{A_k \cdot X_{kt}}{\sum_{k=1}^{K} A_k X_{kt}}, \tau \in M(t), \forall k, t \right\}
\]
Anticipated base-level (perfect anticipation)

\[ \hat{a}_k^B = (\hat{x}_{j,I}, \hat{x}_{j,F}, \hat{\Delta}k_{r}, \hat{\delta}_{j,I}) , \; j \in N_k, r \in M(t), \; \forall t, k \]

\[ \hat{C}_k^B = \sum_{\tau=1}^{12} \left( \sum_{j \in N_k} (h_j \hat{x}_{j,I} + c_j \hat{\delta}_{j,I}) + c^* \cdot \hat{\Delta}K_{k_{r}} \right) ; \; k = 1, \ldots, K \]

\[ \hat{A}_{k,IN}^B = \begin{cases} \hat{x}_{j,I} = \hat{x}_{j,I-1} + \hat{\delta}_{j,I} & \forall j, \tau \\ x_{j,0} = x_{j,0}^L & \forall j \\ \sum_{j \in N_k} a_j \hat{x}_{j,I} + s_j \hat{\delta}_{j,I} \leq p_{k_{r}} + \hat{\Delta}K_{k_{r}} & \forall \tau \\ \hat{x}_{j,I} \leq M \hat{\delta}_{j,I} & \forall j, \tau \\ \hat{x}_{j,F}, \hat{x}_{j,I}, \hat{\Delta}K_{k_{r}} \geq 0, \; \hat{\delta}_{j,I} \in \{0,1\} & \forall j, \tau \end{cases} \]

b1) Non-reactive anticipation of the base-level

\[ A_k = \frac{\sum_{\tau=1}^{12} \sum_{j \in N_k} (a_j \cdot \hat{\delta}_{j,I} + s_j)}{\sum_{\tau=1}^{12} \sum_{j \in N_k} \hat{\delta}_{j,I}} \; \forall k \]

\[ H_k = \frac{\sum_{\tau=1}^{12} \sum_{j \in N_k} (h_j \cdot \hat{\delta}_{j,I})}{\sum_{\tau=1}^{12} \sum_{j \in N_k} \hat{\delta}_{j,I}} \cdot |t| \; \forall k \]

b2) Perfect anticipation of the base-level (at an intermediate step)

\[ A_k = \frac{\sum_{\tau=1}^{12} \sum_{j \in N_k} (a_j \cdot \hat{x}_{j,I}^* + s_j \cdot \hat{\delta}_{j,I}^*)}{\sum_{\tau=1}^{12} \sum_{j \in N_k} \hat{x}_{j,I}^*} \; \forall k \]

\[ H_k = \frac{\sum_{\tau=1}^{12} \sum_{j \in N_k} (h_j \cdot \hat{x}_{j,I}^*)}{\sum_{\tau=1}^{12} \sum_{j \in N_k} \hat{x}_{j,I}^*} \cdot |t| \; \forall k \]
Exercise 9: Supply Chain

a) Sketch

```
Customer

External Demand d_t

Sold Amount v_t

Dealer

Inventory x^L_t

Ordered Amount b_t

Supplied Amount y_t

Manufacturer

Produced Amount z_t

Inventory z^L_t
```

b) Formulation of DDM model

*Decision variables:*

- \( b_t \): ordered amount of the product in week \( t \) (\( t = 1, \ldots, 52 \)) (top-level)
- \( v_t \): sold amount of product in week \( t \) (top-level)
- \( \delta^+_t \): amount of the product that is not delivered in \( t \)
- \( \delta^-_t \): delivered amount exceeding the ordered amount in \( t \)
$z_t$: produced amount of the product in week $t$ (base-level)
$y_t$: supplied amount of the product in week $t$ (base-level)

*State variables:*

$x_t^L$: inventory of the product at the dealer at the end of week $t$ ($t = 1, \ldots, 52$)
$z_t^L$: inventory of the product at the manufacturer at the end of week $t$ ($t = 1, \ldots, 52$)

*Data and parameters:*

$d_t$: demand in week $t$ [units]
$h$: inventory holding cost of the dealer [$ per unit and week]
$f$: shortage cost of the dealer [$ per unit and week]
$q$: purchase price [$ per unit]
$p$: selling price [$ per unit]
$k$: inventory holding cost of the manufacturer [$ per unit and week]
$c$: variable production cost of the manufacturer [$ per unit]
$K$: penalty cost of the manufacturer [$ per unit and week]
$z_t^{r_{max}}$: estimated production capacity of the manufacturer [units per week]
$x_0^L$: initial inventory

*Model of the dealer (top-level):*

Top-equation:

$$a^T^* = \arg \max_{a^T \in A_T} \{ C^{TT}(a^T) + C^{TB}(AF(IN)) \}$$

Action:

$$a^T = (b_t, v_t)_{t=1, \ldots, 52}$$

Instruction:

$$IN(a^T) = \{ b_t : t = 1, \ldots, 52 \}$$

Private criterion:

$$C^{TT}(a^T) = \sum_{t=1}^{52} (p \cdot v_t - h \cdot x_t^L - f \cdot (d_t - v_t))$$
Top-down criterion:

\[ C^{TB}(AF(IN)) = \sum_{t=1}^{52} \left( -q \cdot \hat{y}_t^* + K \cdot (\hat{\delta}_t^* + \hat{\delta}_t^-) \right) \]

Decision field:

\[ A^T_{AF} = \{ a^T : (i), \ldots, (iv) \} \]

(i) \( v_t \leq d_t \quad t = 1, \ldots, 52 \) (sales constraint)

(ii) \( x_t^L = x_{t-1}^L + \hat{y}_t^* - v_t \quad t = 1, \ldots, 52 \) (inventory balance)

(iii) \( x_0^L = x_0^L' \) (initial inventory)

(iv) \( b_t, v_t, x_t^L \geq 0 \quad t = 1, \ldots, 52 \) (non-negativity constraints)

Anticipated base-model

Action:

\[ \hat{a}^B = (\hat{y}_t)_{t=1, \ldots, 52} \]

Criterion:

\[ \hat{C}^B_K(\hat{a}^B) = \sum_{t=1}^{52} \left( q \cdot \hat{y}_t - k \cdot \hat{z}_t^L - K \cdot (\hat{\delta}_t^+ + \hat{\delta}_t^-) - c \cdot \hat{z}_t \right) \]

\[ AF(IN) = G^* = \arg \max_{\hat{a}^B \in \hat{A}_{IN}^B} \hat{C}^B(\hat{a}^B) \]

Decision field:

\[ \hat{A}_{IN}^B = \{ \hat{a}^B : (i), \ldots, (vii) \} \]

(i) \( \hat{z}_t^L = \hat{z}_{t-1}^L + \hat{z}_t - \hat{y}_t \quad t = 1, \ldots, 52 \) (inventory balance)

(ii) \( \hat{z}_t \leq \hat{z}_{t}^{\text{max}} \quad t = 1, \ldots, 52 \) (capacity constraint)

(iii) \( \hat{y}_{t+1} + \hat{\delta}_{t+1}^+ - \hat{\delta}_{t+1}^- = b_{t+1} + \hat{\delta}_t^+ - \hat{\delta}_t^- \quad t = 1, \ldots, 51 \) (order-delivery relationship)

(iv) \( \hat{y}_1 + \hat{\delta}_1^+ - \hat{\delta}_1^- = b_1 \) (order-delivery relationship (first period))

(v) \( \sum_{t=1}^{52} \hat{y}_t \leq \sum_{t=1}^{52} b_t \) (total order constraint)

(vi) \( \hat{z}_0^L = 0 \) (initial inventory)

(vii) \( \hat{y}_t, \hat{z}_t, \hat{z}_t^L, \hat{\delta}_t^+, \hat{\delta}_t^- \geq 0 \quad t = 1, \ldots, 52 \) (non-negativity constraints)
c) The top-level anticipates the base-level by its supplied quantity \( \hat{y}_t \). The influence of the base-decision on the top-level is taken into account by the purchase cost and the penalty cost term of the top-down criterion. However, there is an influence on the top-decision field (inventory balance equation) as well.

d) The top-level influences the base-level through the base-criterion \( \hat{C}_B^T \) (penalty) and the ordered quantity.

e) A non-reactive anticipation \( \hat{y}_t^* \) could be accomplished in estimating the deviation \( \delta_t \): \( \hat{y}_t^* \) (non-reactive) := \( b_t + \bar{\delta}_t \) with \( \bar{\delta}_t \) being an appropriate estimate.

f) For large penalty cost \( K \) in relation to inventory cost \( h \), the manufacturer will increase stocks to avoid deviations from the dealer's orders.
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