Appendix A
The Keller-Liverani Perturbation Theorem

In this appendix we formulate the Keller–Liverani Perturbation Theorem from [27] in its full generality and, formally speaking, in a slightly more general form than in [27]. We also formulate all its consequences derived in [27] that we need in our manuscript, particularly in Sect. 2.4 to prove Proposition 2.4.2 which is crucial for us. We follow pretty closely the notation, formulations, and enumeration of [27] for the reader to easily compare our text with the original article [27].

We first describe the setting.

Let \((B, \| \cdot \|)\) be a Banach space. The vector space \(B\) is also equipped with a second norm \(\| \cdot \| \leq \| \cdot \|\) with respect to which \(B\) need not be complete. For any bounded linear operator \(Q : B \rightarrow B\), \(B\) understood here with the norm \(\| \cdot \|\), let

\[
\|Q\| := \sup \{|Qf| : f \in B, \|f\| \leq 1\}. \tag{KL1}
\]

Let \(\Lambda\) be a directed set having a largest element which we denote by 0. In [27] \(\Lambda = [0, +\infty)\) with the reverse order. For our applications in Sect. 2.4 \(\Lambda = \mathbb{N} \cup \{+\infty\}\), although actually it suffices to consider \(\{n, n+1, \ldots\} \cup \{+\infty\}\) where \(n \geq 0\) is large enough, with the natural order. Assume that a family \((P_\varepsilon)_{\varepsilon \in \Lambda}\) of bounded linear operators on \((B, \| \cdot \|)\) is given which enjoys the following properties.

(KL2) There are constants \(C_1, M > 0\) such that for all \(\varepsilon \in \Lambda\)

\[
|P_\varepsilon^n| \leq C_1 M^n
\]

for all \(n \geq 0\).

(KL3) There are constants \(C_2, C_3 > 0\) and \(\alpha \in (0, \min\{1, M\})\) such that for all \(\varepsilon > 0\),

\[
\|P_\varepsilon^n f\| \leq C_2 \alpha^n \|f\| + C_3 M^n |f|
\]
for all \( n \geq 0 \) and all \( f \in B \).

(KL4) If \( z \in \sigma(P_\varepsilon) \cap B(0, \alpha) \), then \( z \) is not in the residual spectrum of \( P_\varepsilon \).

(KL5) There exists a net \( \tau : \Lambda \to [0, +\infty) \) such that \( \tau(0) = 0 \), \( \tau(\Lambda \setminus \{0\}) \subseteq (0, +\infty) \)

\[
\lim_{\varepsilon \in \Lambda} \tau(\varepsilon) = 0
\]

and

\[
||| P_\varepsilon - P_0 ||| \leq \tau(\varepsilon)
\]

for all \( \varepsilon \in \Lambda \).

These are all hypotheses for the Keller–Liverani Perturbation Theorem. In order to formulate this theorem we need one more piece of notation.

For all \( \delta > 0 \) and all \( r > \alpha \) let

\[
V_{\delta,r} := \{ z \in \mathbb{C} : |z| \leq r \text{ or } \text{dist}(z, \sigma(P_0)) \leq \delta \}.
\]

The actual Keller–Liverani Perturbation Theorem from [27] is about upper bounds on the norms of resolvents \( (z - P_\varepsilon)^{-1} \) and continuity at 0 of the latter.

**Theorem A.0.1 (Keller–Liverani Perturbation Theorem)** Suppose that \( (P_\varepsilon)_{\varepsilon \in \Lambda} \) is a family of bounded linear operators on \( (B, \| \cdot \|) \) satisfying conditions (KL2)–(KL5). Fix \( \delta > 0 \) and \( r \in (\alpha, M) \) and let

\[
\eta := \frac{\log(r/\alpha)}{\log(M/\alpha)} > 0.
\]

Then there are constants \( \varepsilon_0 = \varepsilon_0(\delta, r) > 0 \), \( a = a(\varepsilon_0) > 0 \), \( b = b(\delta, r) > 0 \), \( c = c(\delta, r) > 0 \), and \( d = d(\delta, r) > 0 \) such that for every \( \varepsilon \geq \varepsilon_0 \) and all \( z \in \mathbb{C} \setminus V_{\delta,r} \), we have that

\[
\| (z - P_\varepsilon)^{-1} f \| \leq a\| f \| + b|f|
\]

(KL6)

and

\[
||| (z - P_\varepsilon)^{-1} - (z - P_0)^{-1} ||| \leq \tau^\eta(\varepsilon)(c\|(z - P_0)^{-1}\| + d\|(z - P_0)^{-1}\|^2).
\]

(KL7)

**Remark A.0.2** This remark is essential for us and corresponds to Remark 3 (and partly Remark 1) in [27]. As Keller and Liverani write in Remark 1 “In nearly all cases the two norms involved have the additional property that

(KL8) the closed unit ball of \( (B, \| \cdot \|) \) is \( | \cdot | \)-compact.”
and this yields condition (KL4) to hold. However in the case of the present paper, with \( B = \mathcal{B}_e \), \( \| \cdot \| = \| \cdot \|_e \) and \( | \cdot | = \| \cdot \|_* \), (KL8) does fail. The remedy comes from Remark 3 in [27] which we explain now.

Assume there exists a sequence of linear operators \( \pi_k : B \to B \), \( k \geq 1 \), such that

\[
\sup_k \{ \| \pi_k \| \} < +\infty, \tag{A.1}
\]

\[
\sup \{ |f - \pi_k f| : f \in B, \| f \| \leq 1 \} \leq (\alpha/M)^k. \tag{A.2}
\]

and

\[
P_\varepsilon \pi_k \text{ is a compact operator for all } k \geq 1. \tag{A.3}
\]

Then all the operators \( P_\varepsilon : B \to B \) are quasicompact with essential spectral radius \( \leq \alpha \) and in particular (KL4) holds.

We now list the selected corollaries from Theorem A.0.1 derived in [27], the ones needed to have the full proof of Proposition 2.4.2. The first one is a slightly simplified version of Remark 4 from [27].

**Corollary A.0.3** If \( \lambda \) is a simple eigenvalue of \( P_0 \) with \( |\lambda| > \alpha \) (so isolated), then for every \( \varepsilon \in \Lambda \) sufficiently close to 0, there exists a unique simple eigenvalue \( \lambda_\varepsilon \) of \( P_\varepsilon \) such that

\[
\lim_{\varepsilon \to 0} \lambda_\varepsilon = \lambda. \tag{A.4}
\]

Let \( \lambda \) be as in this corollary. Take \( \eta > 0 \) so small that

\[
\overline{B}(\lambda, \eta) \cap \sigma(P_0) = \{\lambda\}. \tag{A.5}
\]

Define for every \( \varepsilon \in \Lambda \) sufficiently close to 0:

\[
Q_\varepsilon := \frac{1}{2\pi i} \int_{\partial B(\lambda, \eta)} (z - P_\varepsilon)^{-1} \, dz. \tag{A.6}
\]

Note that \( Q_\varepsilon \) does not depend on \( \eta \) as long as (A.5) is satisfied.

As an immediate consequence of the definition of \( Q_\varepsilon \) and of item 1) of Corollary 1 from [27], we get the following.

**Corollary A.0.4** If \( \lambda \) is a simple eigenvalue of \( P_0 \) with \( |\lambda| > \alpha \) (so isolated), then

1. For every \( \varepsilon \in \Lambda \) sufficiently close to 0 the operator \( Q_\varepsilon : B \to B \) is a projector (meaning that \( Q_\varepsilon^2 = Q_\varepsilon \)) onto the one-dimensional eigenspace of the eigenvalue \( \lambda_\varepsilon \) of \( P_\varepsilon \).

2.

\[
\lim_{\varepsilon \to 0} \|Q_\varepsilon - Q_0\| = 0.
\]
Now, given \( r > \alpha \) define:
\[
\Delta_\varepsilon := \frac{1}{2\pi i} \int_{\partial B(0,r)} (z - P_\varepsilon)^{-1} \, dz. \tag{A.7}
\]

Before we deal with the next corollary we record the following, technical but crucial, consequence of formula (KL6) of Theorem A.0.1.

\[
S_{\delta,r} := \sup \left\{ \| (z - P_\varepsilon)^{-1} \| : 0 \leq \varepsilon \leq \varepsilon_0(\delta, r), z \in \mathbb{C} \setminus V_{\delta,r} \right\} < +\infty \tag{KL9}
\]

for all \( \delta > 0 \) and all \( r \in (\alpha, M) \). We shall prove the following.

**Corollary A.0.5** Let \( \lambda \) be a simple eigenvalue of \( P_0 \) with \( |\lambda| > \alpha \) (so isolated). If \( \gamma \in (\alpha, \min\{M, |\lambda|\}) \) and
\[
\sigma(P_0) \setminus \{\lambda\} \subseteq B(0, \gamma) \tag{A.8}
\]
then for every \( \varepsilon \in \Lambda \) close enough to 0, we have that

1. 
\[
P_\varepsilon = \lambda_\varepsilon Q_\varepsilon + \Delta_\varepsilon,
\]
2. 
\[
Q_\varepsilon \Delta_\varepsilon = \Delta_\varepsilon Q_\varepsilon = 0,
\]
3. there exists a constant \( C \in (0, +\infty) \) such that 
\[
\| Q_\varepsilon \| \leq C,
\]
and for every \( k \geq 0 \):
4. 
\[
\| \Delta_\varepsilon^k \| \leq C \gamma^k.
\]

**Proof** Items (1) and (2) are immediate consequences of (A.6) and (A.7) and elementary basic properties of Riesz Functional Calculus.

For the convenience of the reader we shall now provide the standard proof of item (4). Since \( \gamma \in (\alpha, \min\{M, |\lambda|\}) \), it follows from (A.8) there exists \( \hat{\gamma} \in (\alpha, \min\{M, |\lambda|, \gamma\}) \) such that \( \sigma(P_0) \setminus \{\lambda\} \subseteq B(0, \hat{\gamma}) \). Therefore there exists \( \delta > 0 \) so small that \( \partial B(0, \hat{\gamma}) \cap B(\sigma(P_0), 2\delta) = \emptyset \). Hence, formula (KL9) applies to give
\[
S_{\delta,\hat{\gamma}} < +\infty. \tag{A.9}
\]
It follows from (A.7) and the already mentioned basic properties of Riesz Functional Calculus that

$$\Delta^k := \frac{1}{2\pi i} \int_{\partial B(0,\gamma)} \gamma^k (z - P_\gamma)^{-1} \, dz$$

for every integer $k \geq 0$. Therefore, invoking (A.9), we estimate as follows:

$$\|\Delta^k\| \leq \frac{1}{2\pi} \int_{\partial B(0,\gamma)} |z|^k \|(z - P_\gamma)^{-1}\| \, |dz| = \frac{\gamma^k}{2\pi} \int_{\partial B(0,\gamma)} \|(z - P_\gamma)^{-1}\| \, |dz| \leq \gamma S_{\delta,\gamma} \gamma^k,$$

and formula (4) is proved.

Now, we shall prove item (3). It follows from (A.8) that $\overline{B}(\lambda, |\lambda| - \gamma) \cap \sigma(P_0) = \{\lambda\}$. Hence, invoking also (A.6) and (KL9), we get

$$\|Q_\varepsilon\| \leq \frac{1}{2\pi} \int_{\partial B(\lambda, (|\lambda| - \gamma)/2)} \|(z - P_\gamma)^{-1}\| \, |dz| \leq (1 - \gamma) S_{(|\lambda| - \gamma)/2, \gamma} < +\infty.$$

The proof of item (3) and, simultaneously, of entire Corollary A.0.5 is complete.
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M. Pollicott, M. Urbański, *Open Conformal Systems and Perturbations of Transfer Operators*, Lecture Notes in Mathematics 2206,  
https://doi.org/10.1007/978-3-319-72179-8
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