Appendix A
Time and Space Intervals Defining the Behavior of Charged Particles

The motion of charged carriers in semiconductors occurs in a medium filled with charged ions and neutral atoms. Under equilibrium conditions, any macroscopic volume is quasi-neutral. Since charges experience thermal fluctuations, the neutrality condition is satisfied on average for a certain time interval and in a certain volume. Plasma gas has similar properties. By analogy with it in semiconductors, a system of charged particles and neutral atoms is called solid-state plasma. The properties of the system can be determined by perturbing it externally and observing the process of return to a stationary state (relaxation). Such an effect may be obtained with inhomogeneous heating, injection of charge carriers, and the effect of electromagnetic field, etc. The process of returning to the initial state occurs due to internal processes inherent to the system under consideration (recombination, dissipation of energy in scattering processes, etc.). Specific relaxation time scales quite fully characterize plasma. A perturbation in time has a spatial localization scale for the perturbation associated with it. The nature of the system reaction to the external electromagnetic action will depend on the ratio of time (oscillation period $T$) and space (wavelength) parameters of this action in the investigated solid-state plasma.

A.1 Relaxation Time of Momentum and Energy

Relaxation time $\tau_p$ of momentum $\mathbf{p} = m\mathbf{v}$ characterizes the rate of change (decrease) in the momentum of the particle when perturbation is removed. According to the approximation of the relaxation time, we write

$$\frac{\partial \mathbf{p}}{\partial t} = -\frac{\mathbf{p}}{\tau_p}.$$

The solution of this equation has the form:

$$\mathbf{p}(t) = \mathbf{p}(0) \exp\left(-\frac{t}{\tau_p}\right).$$
The value of $\tau_p$ is the mean free path time. If this value is multiplied by the root-mean-square of thermal velocity $v_T$, then we obtain the mean free path of the particle $l_p$:

$$l_p \approx v_T \tau_p. \quad (A.1.1)$$

Another relaxation constant characterizes energy relaxation. When scattered on ions and neutral atoms, the particle (electron, hole) loses a small amount of energy because of the large difference in the masses of the colliding particles. Therefore, for a noticeable energy loss of the particle, it is necessary to make a large number of collisions and the energy relaxation time is much longer than the pulse relaxation time. We rewrite the equation of conservation of energy (2.4.9) when the external influence is removed and without taking into account the spatial inhomogeneity of the energy flux. In this case

$$\frac{\partial(T_e - T_0)}{\partial t} = -\frac{(T_e - T_0)}{\tau_e(T_e)}, \quad (A.1.2)$$

where $T_e$ is the actual electron temperature; $T_0$ is the equilibrium temperature, equal to the temperature of crystal lattice; and $\tau_e(T_e)$ is the energy relaxation time. The solution of this equation is

$$\Delta T(t) = T_e(t) - T_0 = \Delta T(0) \exp(-t/\tau_e).$$

It should be noted that the use of the electron temperature concept is valid only when the energy equilibrium inside the electron gas is reached quicker than the energy equilibrium in the lattice. This can only be the case when the frequency of electron-electron collisions exceeds electron-lattice collision frequency or $\tau_{ee} \ll \tau_e$. Here $\tau_{ee}$ is the characteristic duration of electron-electron collisions.

By analogy with the mean free path, we assume a certain distance at which the energy is relaxed. Let’s call this distance “the cooling length” $l_e$. When applying an external electric field, it is more logical to call this value “the heating length”. We estimate this length as follows:

$$l_e \approx v_T \tau_e. \quad (A.1.3)$$

The use of this concept is important for the analysis of current transfer features in structures with submicron dimensions.

Consider the characteristic values $\tau_p$ and $\tau_e$, as well as $l_p$ and $l_e$. It should be noted that the introduction of a single relaxation time for a variety of relaxation mechanisms, such as phonon scattering, ionized atoms, neutral particles, etc., is not rigorous. In addition, the particle energy also has a significant effect on the value of the parameters considered. Therefore, we can talk about some average (effective) values and ranges of their change. In semiconductor structures, the momentum relaxation time $\tau_p \sim 10^{-13} – 10^{-14}$ s, and energy $\tau_e \sim 10^{-11} – 10^{-12}$ s, i.e., $\tau_e$. Sometimes it is said that the “electron memory” for momentum is much shorter than the “electron memory” for energy. In other words: the relaxation process of the heated electron gas in plasma occurs in such a way that the directed carrier velocity is first lost, and then the lattice temperature and carriers are equalized.
Thus, under the conditions of elastic scattering $l_e \gg l_p$. For $T = 300$ K RMS velocity $v_T \sim 10^5$ m/s. Then $l_p \sim 10^{-8} - 10^{-9}$ m, and $l_e \sim 10^{-6} - 10^{-7}$ m. Note that the average length of heating (cooling) can be of the order of $0.1 - 1 \mu$m, which is comparable with the size of the active regions of modern microwave devices.

### A.2 Time and Length of Charge Relaxation

The work of any electronic device is associated with the creation of excess charges in a certain space. Naturally, due to electrical interaction forces, such a charge inhomogeneity will change in time. The characteristic charge relaxation parameters can be found using the Maxwell equations. This effect explains the use of the terms Maxwellian chargelaxation time and relaxation length.

Consider a medium with conductivity $\sigma$ and dielectric constant $\varepsilon$, into which a charge with volume density $q$ is placed. Calculating the divergence of the right and left parts of the first Maxwell equation (2.2.2) and the Poisson equation (2.2.4), we obtain the equation for $\rho$:

$$\frac{\partial \rho}{\partial t} = -\frac{\rho \sigma}{\varepsilon},$$  \hspace{1cm} (A.2.1)

from which

$$\rho = \rho_0 e^{-t/\tau_m},$$  \hspace{1cm} (A.2.2)

where

$$\tau_m = \varepsilon/\sigma = \varepsilon/(en\mu).$$  \hspace{1cm} (A.2.3)

is the Maxwellian relaxation time; $\rho_0$ is the charge at time $t = 0$; $n$ is the charged particle concentration; and $\mu$ is the charged particle mobility.

It follows from (A.2.2) that charge inhomogeneity will decrease (dissipate) under ordinary conditions if $\mu > 0$. Such an expected and natural reaction of the medium to an excess charge in some region is substantially complicated if not only is a small variable field, but also a sufficiently large static (heating) electric field are applied to the sample. In this case, the kinetic coefficients of current transfer (mobility, diffusion) change, and accordingly, the nature of the excess charge relaxation also changes. Figure A.1 shows a typical $A_3B_5$ semiconductor field velocity characteristic with three regions of differential carrier mobility $\mu_d = dv/dE$.

The first region is that of weak fields $E_0 < E_{th}$, where $\mu_d > 0$, the second region $E_{th} < E_0 < E_s$, where $\mu_d < 0$ and the third region $E_0 > E_s$, where $\mu_d \approx 0$.

From (A.2.1), we obtain

$$\frac{\partial \rho_m}{\partial t} = \frac{\rho_m}{e/\sigma_d},$$  \hspace{1cm} (A.2.4)
where $\rho_m$ is the charge inhomogeneity amplitude; and $\sigma_d = qn\mu_d$ is the differential conductivity. Then, by analogy with (A.2.3) we introduce the concept of differential Maxwellian relaxation time:

$$\tau_{md} = \frac{e}{en\mu_d}$$

(A.2.5)

Then (A.2.2) is rewritten in the form $\rho_m = \rho_{mo}e^{-t/\tau_d}$, where $\rho_{mo}$ is the amplitude of charge inhomogeneity at the initial moment. Hence, it follows that the amplitude of the charge inhomogeneity can decrease ($\mu_d > 0$), remain constant ($\mu_d = 0$) or even increase ($\mu_d < 0$). The correctness of this conclusion is confirmed by the existence of such an amazing physical effect as the Gunn effect. Figure A.2 shows possible variations in the amplitude of the charge inhomogeneity.

Along with the relaxation time, we introduce the concept of differential Maxwellian relaxation frequency:

$$\omega_{md} = \frac{en\mu_d}{\varepsilon}$$

(A.2.6)

Fig. A.2 The change in charge inhomogeneity amplitude for various medium parameters
If the charge inhomogeneity moves in space, then along with the Maxwellian relaxation time, the length at which this relaxation occurs is introduced. Knowledge of this parameter is especially important for the analysis of processes when carriers drift through the region of a device with small fields. An example of such a region is the quasi-neutral region of the bipolar transistor base. Carriers are transported by diffusion force with no pulling external field. We define the Maxwellian relaxation length as

$$l_m = \sqrt{D\tau_m}, \quad (A.2.7)$$

If the width of the neutral part of the base is greater than this parameter, that is, $w_b > l_m$, then the efficiency of energy extraction from such a charge will be significantly reduced.

### A.3 Period of Plasma Oscillation, Debye Length

The complexity of analyzing the interactions of particles in plasma implies the use of an integrated energy approach to these processes. Let us demonstrate the fruitfulness of this approach using the example of plasma oscillations. In semiconductors in a state of thermodynamic equilibrium with temperature $T_0$, there are free mobile charges (electrons or holes), as well as stationary charges (ionized donors and acceptors). A condition of electroneutrality is kept: $e(N_d + p - n - N_a) = 0$. It is clear that this condition is violated if the volume is small. In addition, the electroneutrality may be violated when mobile carriers change their location randomly due to their thermal velocities. When the particles are displaced relative to the equilibrium state, Coulomb forces arise, forming a force that causes a shift to the equilibrium position, as shown in Fig. A.3. Mobile carriers pass equilibrium positions by inertia on their way back, causing the appearance of a restoring force. The process acquires an oscillatory character and is called plasma oscillations.

Let us determine the period and spatial amplitude (range) of such an oscillatory process. We use Newton’s and Poisson equations for this purpose. However, within the framework of this discussion, we find these values from a comparison of particle energy expressed in terms of classical mechanics, thermodynamics and electrodynamics. It is natural to assume that the average energy of a particle, determined from thermodynamic relationships, must be equal to the kinetic energy of the particle and the energy determined by the laws of electrodynamics, that is, $W_{\text{therm}} = W_{\text{kinem}} = W_{\text{eldyn}}$, or

$$\frac{3}{2}kT_0 = \frac{mv_T^2}{2} = eU_T \quad (A.3.1)$$

where $v_T$ is the RMS thermal velocity; and $U_T$ is the characteristic thermal potential. Let us determine the spatial distribution of this thermal potential to
separate donors and electrons by a certain distance \( x \), using the Poisson equation for a one-dimensional case:

\[
- \frac{d^2 U}{dx^2} = \frac{dE}{dx} = \frac{eN_d}{\varepsilon}.
\]

Assuming that the level of doping does not vary with respect to the coordinate, i.e., \( N_d(x) = \text{const} \), we obtain the potential difference \( \Delta U \) between points at a distance \( x \): \( \Delta U = (eN_d/2\varepsilon)x^2 \), or \( x = \sqrt{2\varepsilon\Delta U/(eN_d)} \). Equating the potential difference to the value of the thermal potential \( \Delta U = U_T \), we obtain the maximum distance to which the electron shifts:

\[
L_D = \sqrt{\frac{2\varepsilon U_T}{eN_d}}.
\]  

Replacing in this expression the value of \( U_T \) in terms of energy, according to (A.3.1), we obtain:

\[
L_D = \sqrt{\frac{3\varepsilon k T_0}{e^2 N_d}}.
\]  

This quantity is called the Debye length. We note that when electrons are heated by field their energy is increased and instead of the temperature \( T_0 \) it is necessary to use \( T_e \). Figure A.4 shows the dependence of the Debye length on donor concentration for different values of \( T_e \).

Let us determine the period of plasma oscillations using relation \( T_p \approx L_D/\nu_T \) and expression (A.3.3). We obtain:

\[
T_p \approx \sqrt{\frac{e\mu}{e^2 N_d}}.
\]
In practice, the plasma angular frequency is often used:

\[ \omega_p = \frac{2\pi}{T_p} = 2\pi \sqrt{\frac{e^2 N_d}{\varepsilon m}}. \tag{A.3.5} \]

This analysis allows us to conclude that charges are periodically shifted relative to the equilibrium position. Such a shift leads to a local violation of electroneutrality. If we choose a volume larger than \(L_D^3\) as the object, then the electroneutrality will overall be fulfilled. Given this circumstance, we cannot speak of sharp boundaries in the distribution of charges in semiconductors. The boundary can be determined with an accuracy in the order of \(L_D\). It is often called the spatial scale of charge separation.

It is important to note the dependence of this parameter on the temperature of carriers and doping concentration. The effect of the external electric field leads to an increase in the energy of mobile carriers and, accordingly, to an increase in \(L_D\). This fact must be taken into account when considering the physics of current transfer in a field effect transistor, especially in modes close to the cut-off regime of the device. The obtained \(L_D\) values are in the order of 0.01–0.1 \(\mu\)m, that is comparable with the size of the current channel in field effect transistors (0.1–0.2 \(\mu\)m), even with weak heating of carriers.

With increasing heating fields, it is impossible to neglect the blurring of the channel boundaries, even from the point of view of explaining the physical principles of the device, in particular, the process of overlapping the channel in the drain region. Figure A.5 shows the typical shape of the current transistor channel and the nature of its change, taking into account the heating up of carriers. It is important to note the increase in \(L_D\) to the drain end of the gate and its comparability with the thickness of the active region \(A\). The plasma frequency in the transistor channel is in the order of \(10^{12}–10^{13}\) Hz. This means that plasma oscillates with a greater frequency than a typical microwaves signal (frequency \(10^9–10^{11}\) Hz).
Vacuum devices also use the concept of plasma frequency in the analysis of space charge forces. The occurrence of longitudinal oscillations with a certain characteristic frequency is possible in electron beam. The process leading to the displacement of carriers relative to a certain equilibrium position can be velocity modulation due to external influences. In Sect. 3.1, expression (3.1.3) was defined as follows: \( \omega_p = \sqrt{\varepsilon \rho / m e_0} \). This expression differs from (A.3.5) by a factor of \( 2\pi \). However, it should be noted that in the vacuum case we have longitudinal oscillations due to the Coulomb forces of interaction between similar charges. Therefore, it is not entirely correct to talk about plasma. In semiconductors, oscillations arise due to the forces between different charges, and the initial displacement of charges arises from the presence of thermal velocities. In this case, the concept of solid-state plasma is applied.

What is the practical need of introducing this parameter in the vacuum case? To answer this question, we estimate the value of the plasma frequency and the corresponding oscillation amplitude for the typical values of device current \( I_0 \), accelerating voltage \( U_0 \) and diameter of the beam \( D \). Figure A.6 shows the calculated concentration of electrons in the beam with parameters \( I_0 = 0.1 \, A \), \( U_0 = 3000 \, V \) for three values of beam diameter \( D = 1; 1.5; 2 \, mm \).

Comparing the concentrations of charge carriers in a vacuum device \( \sim 5 \times 10^{16} \, m^{-3} \) with a characteristic level of doping in a semiconductor device \( \sim 1 \times 10^{23} \, m^{-3} \), we see a difference of almost six orders.

**Fig. A.5** Current channel shape in FET taking into account Debye length

**Fig. A.6** Concentration of electrons in the flow of a typical vacuum device for three flow diameter values
The possibility of obtaining a high electron density in a semiconductor plasma is associated with the compensating action of a positive donor charge: the semiconductor is quasi-neutral. Note that the calculation of the plasma frequency in a vacuum gives a value in the order of $\sim 10^9$ Hz, which corresponds to the lower border of the microwave band. In this case, the characteristic distance between the peaks of the longitudinal oscillations (plasma wavelength) $\lambda_{pl}$ will be

$$\lambda_{pl} \approx v_0 T_p = \sqrt{\frac{2e}{m} U_0 \cdot T_p},$$

where $v_0$ is the drift velocity. So, we obtain $\lambda_{pl} \sim 5-20$ mm. This is comparable to the characteristic physical dimensions of the vacuum device. The analysis shows that it is important to take into account plasma oscillations in these devices, both in the time domain and in the space domain.

During the motion of charges in semiconductors, relaxation processes occur, and they are associated with the generation and recombination of charges. In order for the devices to operate in the microwave band, it is necessary to preserve the particle for at least one oscillation period $\tau_n \geq T$, or better $\tau_n \gg T$. In the space domain, the diffusion length $l_d$ should be much greater than the characteristic size of the device $l_d \gg l_{\text{device}}$.

Practical measurements of the lifetime of charge carriers in semiconductors have shown the value of $\tau_n \sim 10^{-6}-10^{-8}$ s. The spatial interval associated with this quantity, called the diffusion length $l_d$ is calculated using the relation

$$l_d = \sqrt{D \tau_n},$$

where $D$ is the diffusion coefficient.
Appendix B
Electron-Optical Systems of Microwave Devices

B.1 General Properties and Parameters of Electron Beams Used in Microwave Devices

Electron beams in electronic microwave devices are used to convert the energy of external power sources to the energy of electromagnetic waves in the microwave range. Devices must provide the greatest energy conversion efficiency, high microwave power, efficiency and gain. Hence, microwave devices use electron beams with a high current density, at which the intrinsic space charge of the beam significantly affects the nature of electron motion. Electronic beams, in the analysis of which the forces of Coulomb repulsion cannot be ignored, are conventionally called intense.

The electron beam current limited by the space charge at the cathode, is determined by the formula

\[ I = pU_0^{3/2} \]

where \( p \) is the parameter characterizing the degree of space charge influence on the beam motion, called the perveance. It is known from experience that the influence of the space charge becomes noticeable at perveance values \( p > 0.1 \mu\text{A}/\text{V}^{3/2} \). Microwave electronic devices use beams with perveances of up to \( 20 \mu\text{A}/\text{V}^{3/2} \) and even higher, which makes it necessary to take into account and compensate the forces of Coulomb repulsion.

In addition to these parameters, the “quality” of beam includes such parameters as beam phase response, the phase ellipse and beam emittance. They describe the “quality” of the beam, the degree of its structural ordering and the distribution of transverse velocities. The phase response of the beam is the set of points in the transverse phase space \( r \) and \( r' \), where \( r \) and \( r' = dr/dz \) are the radial coordinate and slope of each of the electron trajectories forming the electron beam.
Figure B.1a shows the electron trajectories in an ideally formed electron beam: the beam is uniformly compressed, reaches a minimum cross section in plane $P_2$ and expands under further action of Coulomb forces. Figure B.1b shows the phase characteristics of this beam for planes $P_1$, $P_2$ and $P_3$. Characteristic 1 corresponds to a uniformly converging beam, and the slope of trajectory $r'$ is proportional to their radial coordinate. In the minimal section plane $P_2$ the beam trajectories are parallel to axis $z$, and consequently, $r' = 0$ and phase response 2 is located on axis $r$ of the phase space. Phase response 3 corresponds to a uniformly divergent beam. The linearity of these characteristics corresponds to an ideally formed electron beam.

The aberrations of the electron gun lead to nonlinearity in the phase characteristics. An example of a nonlinear phase characteristic in the plane of the minimum beam cross section is shown in Fig. B.1b. This kind of characteristic reflects the effect of intersection of electron trajectories, since the same radial coordinate values correspond to different inclination angles of the trajectories.

The thermal spread of electron velocities leads to the fact that on the phase plane the electron beam is not represented by a line, but by some figure with a finite area. In the plane of the minimum cross-section of the beam (crossover), this figure has the form of a straight ellipse (Fig. B.2). Each radial coordinate of the beam $r$ corresponds to a set of $r'$ values. The area of the phase ellipse $A$, divided by $\pi$, is known as the beam emittance:

$$\varepsilon = A/\pi$$

Fig. B.2 A phase ellipse
For a number of applications, brightness is an important characteristic of the charged particle beam. The brightness of a continuous axially symmetric beam is given by

\[ B = \frac{I}{\pi b^2 \Omega}, \]

where \( I \) is beam current with cross section radius \( a \), and \( \Omega \) is a solid angle, which determines convergence (or divergence) of the electron beam. High requirements for the brightness of an intense electron beam are, particularly, in free-electron lasers, where it reaches values of the order of \( 10^6 \text{A/cm}^2 \text{sr} \).

Electron optical systems (EOS) of microwave devices generally include three parts. One of them initially forms the electron beam of the specified electric and geometric parameters and is called a beam formation system or, more often, an electron gun. The second is intended for the transfer of an electron beam through a channel, the extent of which considerably exceeds its transverse dimensions. This system is usually called a focusing system, although this name does not accurately reflect the functions it performs. The third part is a collector system that must ensure the reception of the spent electron flow, dissipation of the exuded heat, and in some cases the recovery of the electron beam’s residual energy.

**B.2 Systems for Electron Beam Formation**

In modern microwave electronic devices, various electron beams of different spatial configuration are used.

Devices with O-type dynamic control (klystrons, traveling-wave tubes, etc.) based on long electrons existence in the electron-field interaction space use extended electron beams with a sharply delineated boundary. In devices with quasi-static control, the use of electronic fluxes in the form of sharply outlined beams is not mandatory, but it has opened new possibilities in the development of superpower triodes and tetrodes. Thus, the formation and focusing of intense electron beams is one of the main problems solved in the development of modern microwave electronic devices.

Methods for the formation and focusing of electron beams are usually associated with the principle of controlling them. This is of major importance in those devices where the elements of the electron optical devices directly enter the design of the resonator or slow wave systems. Nevertheless, there are a number of general requirements to these systems. For a clear understanding of these requirements, we briefly consider the main types of electron optical systems used in microwave electronic devices. Let us begin this consideration with electron beam initial formation systems, or electron guns.

The main goal of the electron gun, as noted above, is to form an intense electron beam of a certain configuration with a given electron current and velocity and, if possible, with laminar electron motion.
J.R. Pierce made an important contribution to the solution of this problem when he proposed a method for the formation of rectilinear laminar electron beams with simple configurations: sheet, cylindrical and conical. Based on this method, high-efficiency electron guns were developed, and are widely known under the name of Pierce guns.

The Pierce type of electron gun (Fig. B.3) consists of a concave spherical equipotential cathode 2 with a heater 1, a focusing electrode 4 and an anode 3 with a central hole. Usually, the cathode electrode has a potential of the cathode, and is located such that its surface is kind of an extension of the cathode surface. This explains the term diode for this type of gun.

By appropriately calculating the shape of the electrodes produced by an analytical method or by mathematical modeling, an electric field configuration is created in the gun in which electrons from the entire cathode surface converge uniformly into a narrow electron beam passing through the anode hole. In particular, to obtain an axially symmetric flow with trajectories parallel to the axis of the gun, the focusing electrode slope angle should be equal to 67.5°.

The degree of convergence of electrons is characterized by the so-called coefficient of convergence (compression). There is a coefficient of convergence in current density (or cross-sectional area of the beam) termed $C_j$, equal to the ratio of the maximum current density in the electron beam to the cathode current density. The convergence coefficient $C_r$, along the radius, determined by the ratio of the cathode radius to the radius of the minimum cross section (crossover) of the beam is also used. Obviously, with a small cathode curvature $C_r \approx \sqrt{C_j}$.

As the coefficient of convergence in the beam increases, the Coulomb forces increase, preventing the beam from contracting. Consequently, the coefficient of convergence depends on the space charge in the beam being formed, which is determined by its perveance.

It should be emphasized that in Pierce diode guns, which usually operate in space charge mode, the value of perveance $p$ does not depend on the anode voltage.
and as follows from the law “power $3/2$” it is determined only by the geometric dimensions of the diode gun. Therefore, perveance, being a parameter of the electron beam, and a measure of its intensity, is simultaneously a parameter of the gun itself, that is, its design.

Using the Pierce gun, we can form converged electron beams with perveance $p \leq 1 \mu \text{A/V}$. For small values of perveance, the coefficient of convergence in current density can be 100 or more. For large values of $p$, the coefficient of convergence does not exceed several units. To obtain converging beams with a higher perveance, various modifications of Pierce guns are used.

If the initial axially symmetric Pierce gun is rotated with respect to an axis parallel to its symmetry axis, it is possible to obtain a toroidal gun capable of forming a hollow cylindrical or hollow conical beam.

Electron guns of the magnetron type also form axially symmetric hollow electron beams (Fig. B.4). In such a gun, beam formation takes place in crossed electric and magnetic fields. The gun consists of a cold cathode electrode 1 with the heater 3 and anode 2. The cathode ring between 4 and 5 cross-sections is coated with the emitting substance. The electric field is created due to the potential difference between the cathode 1 and the anode 2 electrodes. It has both radial and longitudinal components. External solenoids create the magnetic field. The magnetic induction lines of this field are directed approximately parallel to the surface of the cathode. The value of the magnetic induction exceeds the critical value (see Chap. 8). Therefore, the electrons emitted by the cathode ring do not reach the anode and create an electron cloud in the near-cathode region. Under the action of the longitudinal component of the electric field, they move in the longitudinal direction and leave the cathode-anode space, forming a hollow electron stream.

Crossed fields are also used to form sheet electron beams in M-type devices. Figure B.5 shows the design of an electron gun forming a sheet electron beam. The
gun consists of a thermal cathode 1 that generates an electron beam 2, a cold cathode 3, a control electrode 4 and an anode electrode 5, the role of which is usually played a slow wave system.

The whole system is in a homogeneous magnetic field with induction $B$. The thermo cathode and the cold cathode are under zero potential, and the anode electrode is under positive potential $U_0$.

The electrons leave the cathode and are accelerated by the electric field of the accelerating electrode $E_a = U_a/d_a$, simultaneously deviating under the action of a magnetic field. When the electron is at the top of its cycloid trajectory, its velocity \( v_y = 2E_a/B \), and it enters the interaction space. If its velocity is equal to velocity \( v_e = E_0/B \), then the electron will move along a rectilinear trajectory, since the initial conditions for rectilinear motion in the crossed fields will be satisfied. To do this, the electric field created by the control electrode must be two times less than in the interaction space, since at the top of the cycloid the electron has a velocity twice as high as the velocity of the center. The disadvantage of such a gun, also known as short optics, is the impossibility of creating rectilinear flows from long cathodes, since the conditions of rectilinear motion are satisfied only for one trajectory.

In klystrons and TWT, guns with a control electrode are used for low-voltage electron beam current modulation.

The simplest of these is the conventional Pierce electron gun, in which the focusing electrode is isolated from the cathode and used as a control electrode. However, as shown by special studies, beam control with the help of a focusing electrode voltage is ineffective, especially in guns with large perveance. So, termination of cathode current in a gun with perveance 1 $\mu$A/$\sqrt{V}$ require the application of a negative voltage to the focusing electrode $U_{\text{eth}} \approx 0.5U_0$, and in the electron gun with perveance 3.6 $\mu$A/$\sqrt{V}$—voltage $U_{\text{eth}} \approx U_0$. The decrease in control efficiency with increasing perveance is due to the increasing influence of the anode potential on the field at the cathode.

Significantly better control parameters are achieved in guns with a control grid. By choosing a grid with a small transparency, the ratio $U_{\text{eth}}/U_0$ can be reduced. However, the fraction of the beam current intercepted by the positively charged grid will also increase. This disadvantage can be eliminated if two identical grids are used instead of one and the grid closest to the cathode is connected to it. Then the interception of the beam current by a second control grid with the same positive
potentials on it, as in the previous case, can be reduced by a factor of two orders (up to 0.1%). Such guns are known as electron guns with a shadow grid.

Considering guns with a control electrode, i.e. triode guns, it should be emphasized that they allow the formation of electron beams with a higher output perveance, than the original diode guns. This is because two electrodes with adjustable potentials $U_c$ and $U_0$ not only make it possible to vary the current $I$, but also the energy of the electron beam at the electron gun output, determined by the value $U_0$. Changes in $I$ and $U_0$ can be almost independent of each other, implying two ways to adjust the perveance of the triode gun.

Triode guns which slow down the beam to increase its perveance in the interval between the first and second anodes are known as guns with longitudinal beam compression.

Figure B.6 shows one of the experimental designs of such guns, designed to form axially symmetric electron beams with a diameter of less than 4 mm. At a potential of about 7 kV in the first anode, the cathode current is approximately 1 A, which corresponds to the beam perveance of 1.5 $\mu$A/V$^{3/2}$. With a potential at the second anode equal to 1.5 kV, the beam current does not practically change, since the current transfer coefficient is 99%, but its perveance increases in magnitude, i.e. up to 15 $\mu$A/V$^{3/2}$. There are modes of operation that allow the obtaining of an exit perveance beam of up to 50 $\mu$A/V$^{3/2}$.

**B.3 Focusing (Transporting) Systems of Microwave Devices**

The forces of the Coulomb interaction that arise in the electron beam during its formation cause the beam to expand, change its configuration, which ultimately lead to current losses on the electrodes surrounding the beam. Therefore, it is possible to pass an electron beam with minimal losses through the interelectrode space of a device due to the focusing properties of the electron gun only in those cases when the length of the interelectrode space is relatively small and the electron beam has a relatively small perveance. Hence, in most cases it is necessary to use additional magnetic or electric focusing systems that ensure the conservation of transverse flow within the specified limits over its entire length.
Magnetic fields created by solenoids or permanent magnets are used to focus (transport) electron beams in electrodynamic system channels. Systems with permanent magnets are preferable, since they do not require an additional power source.

According to the nature of the axial distribution of magnetic fields, magnetic systems are divided into three main types: systems with a homogeneous field, systems with a reversible field and magnetic periodic systems. The use of reversible and periodic fields provides a significant gain in mass and dimensions of the magnetic systems.

The simplest way to limit electron beam transverse dimensions is by means of systems that create a uniform longitudinal magnetic field.

Such systems include:

- screened solenoids, which are mounted directly on the electronic device;
- systems based on permanent magnets with pole pieces made of magnetically soft material.

The use of screens and pole pieces allows an increase in the uniformity of the field in the working region and provide the required degree of screening to the gun and collector region.

An electron beam formed by an electron gun, before it enters a homogeneous longitudinal magnetic field, passes through a transition region, where along with the longitudinal component of the magnetic field there is a transverse (radial) component. Because of interaction with the field in the transition region, the beam electrons undergo rotational motion about the axis of symmetry of the magnetic system. Electron rotation in the homogeneous field region leads to the appearance of a magnetic force directed to the system axis.

The transverse motion of electrons in the homogeneous magnetic field zone is determined by the action of two forces: Coulomb interaction force $F_\rho$ and magnetic focusing force $F_\mu$. The first one is directed from the system’s axis of symmetry and defocuses the beam, and the second, directed to the system axis, must balance the action of the first. This is achieved by selecting the value of the longitudinal magnetic field. As a result, the beam will perform an equilibrium motion, retaining the original diameter over its entire length. Such an ideal beam is called a Brillouin beam.

Theoretical analysis gives the following expressions for the indicated forces acting on the boundary electrons of the beam:

\[
F_\rho = \frac{eI}{2\pi\varepsilon_0 r\sqrt{2eU_a/m}},
\]

\[
F_m = -\frac{1}{4m}e^2B_z^2r,
\]

where $I$ is the electron beam current; $U_a$ is the accelerating voltage; $r$ is the beam radius; and $B_z$ is the longitudinal component of the magnetic induction.
The equation of motion for the boundary electrons of the beam in this case has the form

\[ \ddot{z}^2 \frac{d^2 r}{dz^2} = \frac{eI}{2\pi\varepsilon_0 mr^2} + \frac{1}{4} \left( \frac{e}{m} \right)^2 B_z^2 r = 0, \tag{B.3.3} \]

where the longitudinal component of electron velocity \( \dot{z} = \sqrt{1(e/m)U_a} \).

Assuming \( d^2 r/dz^2 = 0 \), we find the balance condition of defocusing and focusing forces:

\[ B = \frac{1}{r} \sqrt{\frac{2I}{\pi(e/m)\varepsilon_0 \sqrt{2(e/m)U_a}}}. \tag{B.3.4} \]

Substituting the numerical values of the constants in this formula gives

\[ B = \frac{0.083}{r} \sqrt{\frac{I}{\sqrt{U_a}}}, \tag{B.3.5} \]

where \( B \) is Brillouin magnetic field induction, \( T \); \( r \) is the beam radius, cm; \( I \) is the electron beam current, A; and \( U_a \) is the accelerating voltage, V.

To realize a beam of particles under strict equilibrium, it is necessary to fulfill a number of conditions that are practically difficult to fulfill. For example, it is difficult to ensure the introduction of an electron beam into the region of homogeneous field with zero radial velocities and a specified radius. As a result, real beams have a pulsating boundary.

An approximate solution of (B.3.3) gives the following expression for the boundary trajectory of the electron beam:

\[ r = r_b + (r_0 - r_b) \cos \left( \sqrt{2 \frac{\omega_L}{\dot{z}}} z \right) + \frac{r_0' \dot{z}}{\sqrt{2 \omega_L}} \sin \left( \sqrt{2 \frac{\omega_L}{\dot{z}}} z \right), \]

where \( r_b \) is the equilibrium radius, the value of which is determined from formula (B.3.4); \( r_0 \) is the initial radius of flow; \( \omega_L = e/(2m)B_z \) is the parameter proportional to the induction of the magnetic field \( B_z \), known as the Larmor frequency; and \( r_0' \) is the initial boundary trajectory slope.

It follows from the solution obtained that the boundary particle, in its motion along the \( z \) axis, performs periodic oscillations with respect to the equilibrium radius \( r_b \). The amplitude \( \Delta r_m \) and the wavelength of the oscillations \( \lambda \) are determined by the expressions

\[ \Delta r_m = \left[ (r_0 - r_b)^2 + \left( \frac{r_0' \dot{z}}{\sqrt{2 \omega_L}} \right)^2 \right]^{1/2}, \]

\[ \lambda = \frac{2\pi \dot{z}}{\sqrt{2 \omega_L}}. \]
With an ideal beam input into a homogeneous field \((r_0 = r_b, r'_0 = 0)\), there are no beam boundary oscillations.

The main disadvantage of focusing (transporting) systems with a homogeneous magnetic field is the low efficiency of using a magnetic field. Indeed, in order to ensure a sufficient degree of homogeneity of the field over a section of length \(l\), the transverse dimension (diameter) of pole pieces should be of the same order. Consequently, the total volume \(V\) occupied by the field of such a system will have a value of the order of \(V \sim l^3\), while the useful working volume has the following value \(V_u = \pi r_a^2 l\), where \(r_a\) is the radius of the channel in which the electron beam propagates.

An increase in the concentration of the magnetic field in the working volume can be achieved by applying reversible and periodic focusing systems, which allow a sharp reduction in the mass and dimensions of the focusing systems.

Reverse magnetic focusing. From (B.3.2) it follows that the radial magnetic force is determined by the square of the magnetic induction and, consequently, does not depend on the direction of the magnetic field. This allows us to apply focusing systems with magnetic field reversal, which are characterized by the fact that during system focusing, the magnetic field reverses direction once or several times.

Figure B.7 shows magnetic induction distributions (B-curves) for ideal and real focusing systems with a single reverse. B-curves in Fig. B.7a correspond to the ideal reversed field, when the field instantly changes polarity and the length of the reverse is zero. If such a field could be realized, the radial motion of electrons in this field would occur in the same way as in a homogeneous magnetic field of the same intensity.

Real reverse systems have two regions: a region of homogeneous field with a length \(L_0\) and a reverse region of length \(L_p\) (Fig. B.7b). Since the magnetic induction in the region of the reverse is less than in the region of the homogeneous field \(B_{z0} < B\), then, crossing this region, the electron beam experiences perturbation. In particular, the equilibrium beam, having passed the reversal zone, begins pulsate.

This effect can be significantly reduced in a reversible system with compensating peaks, the effect of which is explained as follows. Passing the emission zone, the electrons receive some excess radial momentum directed toward the axis of

![Fig. B.7 Axial distribution of the magnetic field in ideal (a) and real (b) reverse magnetic systems](image)
symmetry, which compensates for the decrease of the magnetic focusing force in the region of the reverse. In the first approximation, the compensating peaks are selected in such a way that the average square value of the magnetic induction in the region of the reverse is equal to the induction of a homogeneous field \( B_r \):

\[
\bar{B}_r^2 = \frac{1}{L_r} \int_{L_r} B_{z0}^2 dz = B_z^2.
\]

Reverses allow a significant increase in the coefficient of magnetic field use. For a single reverse, the region with a homogeneous field has a length \( L_0 \approx L/2 \), where \( L \) is the total length of the focusing system. In this case, the field utilization factor increases approximately four-fold as compared to a system without a reverse having the same total length,

\[
K_{b1} \approx \frac{r_a^2}{L_0^2} = 4r_a^2/l^2.
\]

An even greater gain is obtained with a multiple field reverse. For a system with \( N \) reverses, we get \( K_{BN} = \frac{r_a^2}{L_0^2} = \frac{(N + 1)^2 r_a^2}{l^2} \). More efficient use of the magnetic field in reversible systems allows a substantial decrease in the mass and dimension of the focusing system, approximately \( 1/(N + 1)^2 \) times.

With a large number of reversals, the size of a single section gradually decreases, and the length of the region with the homogeneous field is reduced. The reversible system degenerates into a periodic focusing system with an axial distribution of the magnetic field close to the cosine wave.

**Periodic system with a cosine-shaped magnetic field distribution.** Among periodic focusing systems, the system with permanent magnets has the greatest practical application (Fig. B.8). The source of the magnetic field is an annular magnet 1 magnetized in the longitudinal direction. The required distribution of the field in the channel, where the electron beam passes, is formed with the help of pole pieces 2.

**Fig. B.8** Periodic magnetic system of a TWT with permanent magnets: 1 magnetic rings; 2 pole pieces
The axial distribution of the magnetic field in most of these systems is described with sufficient accuracy by the cosine law:

\[ B_{z0} = B_m \cos \frac{2\pi z}{L}, \]

where \( L \) is the system period.

The use of such a system makes it possible to sharply reduce the mass and dimensions of the focusing system as compared to a system with a homogeneous field.

Since \( B_z \) is a function of axis \( z \), the magnetic periodic system cannot provide accurate balance to the Coulomb force \( F_\rho \) and the magnetic force \( F_\mu \) throughout the entire focusing system. However, under certain conditions, it is possible to balance these forces on average over the period of the focusing system. The condition of such a balance can be written in the form

\[ \frac{1}{L} \int_0^L F_\rho \, dz = \frac{1}{L} \int_0^L F_m \, dz. \]

Assuming approximately that on the segment of axis \( z \), equal to the field length \( L \), the radius of the electronic flow varies little \((r \approx \text{const})\), taking into account the formulas (B.3.1) and (B.3.2), we get

\[ \frac{2I}{\varepsilon_0 (e/m) \sqrt{2(e/m)U_ar^2}} = \frac{1}{L} \int_0^L B_{z0}^2 \, dz. \tag{B.3.6} \]

This ratio establishes a relationship between beam parameters \( I, U_a, r \) and magnetic induction \( B_{z0} \), which balances the Coulomb and magnetic forces on average over the period of the system.

Assuming that \( B_z \) varies according to a harmonic law, from (B.3.6) we deduce

\[ B_m = \sqrt{\frac{2}{r}} \sqrt{\frac{2I}{\pi (e/m) \varepsilon_0 \sqrt{2(e/m)U_a}}} = \frac{1174}{r} \sqrt{\frac{I}{\sqrt{U_a}}}. \tag{B.3.7} \]

The quantities included in this formula have the same dimensions as the values in formula (B.3.5).
Qualitatively, the nature of electron beam motion when the Coulomb and focusing force balance is ensured on average over the period can be described as follows. In regions of small magnetic induction values, the beam expands under the action of Coulomb interaction forces. However, falling into the region of large magnetic induction values, it experiences the predominant effect of the magnetic force and begins to contract. At the end of the half-period, the beam radius approaches its initial value.

In the next half-periods the nature of beam motion is repeated. Periodic changes in beam radius with a period $k = \frac{L}{2}$, are a characteristic feature of beam motion in a magnetic periodic focusing system. These periodic changes in beam radius are called beam waviness.

If the conditions for the balance of forces are not fulfilled on average over the period, then the beam motion is more complex. Beam boundary pulsations are added to the wave ripple, depending on the ratio of the beam parameters and the magnitude of the magnetic induction.

Calculation of the beam motion generally requires solution of the differential equation for the boundary beam trajectory $B.3.3$.

\[
\frac{d^2 r}{dz^2} + \frac{eI}{2\pi\varepsilon_0 mr^2} + \frac{1}{4} \left( \frac{e}{m} \right)^2 B_m^2 r = 0.
\]

In this equation, the second term takes into account the defocusing effect of the electron beam space charge and the third term—the focusing effect of the magnetic field.

We reduce this equation to the form in which it is usually used in the periodic focusing theory. To do this, we introduce the normalized variables $R = \frac{r}{r_0}$, $Z = \frac{2\pi z}{L}$, $b(Z) = B_{30}/B_m = \cos Z$, where normalizing quantities include: $r_0$ is the initial beam radius at the entrance to the regular part of the focusing system, $L$ is the field period, and $B_m$ is the field amplitude.

The equation of boundary electron motion, written in the normalized variables, takes the form

\[
\frac{d^2 R}{dZ^2} + 2\alpha b^2(Z) - \frac{\beta}{R} = 0,
\]

or

\[
\frac{d^2 R}{dZ^2} + \alpha[1 + \cos(2Z)] R - \frac{\beta}{R} = 0,
\]

where

\[
\alpha = \frac{1}{32\pi} \left( \frac{e}{m} \right)^2 \frac{B_m^2 L^2}{z^2}
\]

is the magnetic field parameter.
\[
\beta = \frac{1}{8\pi^3\varepsilon_0 m r_0^3} e \frac{IL^2}{z^3}
\]

is the space-charge parameter.

This equation is a nonlinear differential equation with a periodic coefficient of the Mathieu type. In the general case, its solution can be obtained only by numerical methods. The analysis of numerical solutions yields the following main results:

- solution characters depend on the value of the magnetic field parameter: they can be either limited or increasing;
- for values \( x \leq 0.4 \) (the first region of stable solutions) solutions are bounded and periodic, which corresponds to the boundary trajectory of the electron beam with periodically changing radial coordinate \( R(Z) \); With condition \( x = \beta \) the amplitude of the ripple of the boundary trajectory \( \Delta R_m \) is minimum, the pulsation form is determined by a simple harmonic law:

\[
\Delta R = \Delta R_m \cos 2Z = \Delta R_m \cos (4\pi z/L)
\]

When the condition \( x = \beta \) is satisfied, the force caused by the magnetic field balances the space-charge force on the average over the period of the field \( L \). From the equality \( x = \beta \), a formula can be obtained for determining the optimum amplitude of the magnetic induction, at which quasi-equilibrium motion of the electron beam is ensured. It coincides with formula (B.3.7).

Comparing formulas (B.3.6) and (B.3.7), we find \( B_m = \sqrt{2}B \). In fact, this means that the RMS value of the magnetic induction during the field period \( L \), is numerically equal to the equilibrium value of the magnetic induction of a homogeneous field \( B_{ms} = B_z \).

\[
B_{ms}^2 = \frac{1}{L} \int_0^L B_m^2 \cos^2 \left( \frac{2\pi z}{L} \right) dz = \frac{B_m^2}{2}
\]

The magnetic periodic system presented in Fig. B.8, is located outside the vacuum envelope of the device. In this version, it is widely used in traveling wave tubes with a spiral slow wave system.

In a TWT with a coupled cavity slow wave system, the focusing system is integrated into the slow wave system (Fig. B.9). The ring magnets 1 are located outside the vacuum envelope, and the pole pieces 2 enter the vacuum envelope, while simultaneously being the walls of the cavities. In order to ensure high conductivity of the walls, the surfaces of the pole pieces are covered with a thin layer of copper.

In magnetic periodic systems, barium ferrites, neodymium-iron-boron, and samarium-cobalt are used as magnetically hard materials (permanent magnets).
The pole pieces are made of soft magnetic material, for example, of technically pure iron (Armco iron).

### B.4 Electron Beam Energy Recovery Systems

In microwave vacuum devices, the electron beam is an instrument (intermediary) used to convert the energy of a power source into microwave energy. The efficiency of energy conversion is characterized by an electronic efficiency. This parameter is defined as the ratio of the power transmitted by the electron beam to the electrodynamic system field $P_e$, to the total power of the electron beam $P_0 = U_aI_0$:

$$\eta_e = \frac{P_e}{P_0}.$$

The power that is stored in the spent electron beam is called residual power $P_{rez}$

$$P_{rez} = P_0 - P_e = P_0(1 - \eta_e).$$

It is possible to return a part of this power to the device power source and thereby increase the efficiency of energy conversion. The process of returning the power (energy) of the spent electron beam to a power source is called the recuperation of the electron beam energy. This process can be realized by decelerating the spent electron flow in the region of the electron collector. If all the electrons in the beam have the same velocities, they can be braked to a stop. As a result, the electronic efficiency of the device will be equal to 100%.

In real beams there is a scatter of electrons in terms of energy. It can be caused by a number of factors: thermal energy spread, the result of collisions of fast electrons (the Boers effect) and the result of the interaction of electrons with the electrodynamic system field (EDS). The last of these is of primary importance for microwave devices. As a result of interaction with alternating electric fields of the EDS, the electrons entering the collector can have a wide energy spectrum, less or more than $eU_a$. 

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**Fig. B.9** Integrated periodic magnetic system: 1 ring magnets, 2 pole pieces
Recuperation of the electron beam energy in a single-stage collector. A single-stage collector is shown schematically in Fig. B.10. The first electrode of the collector 1 has a potential equal to the potential of the electrodynamic system (EDS), which is assumed to be zero, \( U_1 = U_{es} = 0 \). A negative potential is applied to the second electrode \( U_2 = -\alpha U_a \), where \( \alpha < 1 \) is the numerical coefficient, and \( U_a \) is the anode voltage of the electron gun. It is assumed that the anode potential \( U_a \) is equal to potential EDS: \( U_A = U_{es} = 0 \), and the cathode potential \( U_c \) is negative and equal to \( U_c = -U_a \). If the value of the coefficient is one, then the potential of the second collector electrode is equal to the cathode potential: \( U_2 = U_c = -U_a \). Difference in potentials of the first and second electrodes in an electric field creates a decelerating field.

The electrons entering into the collector have a spectrum of velocities. As the electrons move toward the second electrode, the electron velocity decreases. Slow electrons can be completely retarded and their motion direction can be inversed. Fast ones can overcome the braking field and land on the second electrode of the collector.

Efficiency of recuperation is defined as the ratio of the power returned to the power source \( P_{rec} \), to the residual beam power \( P_{res} \):

\[
\eta_r = \frac{P_{rec}}{P_{res}}.
\]

Recuperation efficiency of a single-stage collector is usually small.

Better results are obtained from two-stage collectors, the construction of which is shown in Fig. B.11. The first electrode of the collector 1 is simultaneously a manifold body. The second 2 and the third 3 electrodes are fixed inside this case with ring isolators 4.

The first electrode has a zero potential, and negative potentials are applied to the second and third electrodes \( U_2 = -\alpha_2 U_a, U_3 = -\alpha_3 U_a \), where \( \alpha_2 \) and \( \alpha_3 \) are numerical coefficients, \( \alpha_2 < \alpha_3 < 1 \). By the appropriate choice of the coefficients \( \alpha_2 \) and \( \alpha_3 \), the efficiency of recuperation can be increased.
and \( z_3 \), the process of energy recuperation can be optimized and the maximum value of recuperation efficiency \( \eta_c \) can be obtained. It is obvious that in the two-stage collector it is possible to obtain a recuperation efficiency greater than in the single-stage collector.

In the practical implementation of collectors with energy recovery, it is necessary to take into account secondary electrons that arise as a result of bombardment of the collector electrode surface. Under the action of the electric fields existing in the collector, these electrons are accelerated and move along complex trajectories in the direction of increasing potential, i.e. towards the inlet to the collector.

They can be deposited on collector electrodes having a higher potential than the electrode potential from which they are emitted, or penetrate into the channel of the electrodynamic system. The acceleration of these electrons consumes the energy of the power source and thereby reduces recuperation efficiency. In addition, as a result of the bombardment of the collector by secondary electrons, its thermal load increases. The penetration of secondary electrons into the channel of the collector can promote self-excitation of the device.

The effect of secondary electrons can be reduced by applying anti-emission coatings to the receiving surface of electrodes and by selecting the geometry of the electrodes.

**Multi-stage collectors with soft electrons landing.** The problem of secondary electrons was most radically solved in multi-stage collectors, which have been called collectors with a soft electron deposit.

One of the variants of such collectors is the Kosmal collector, shown in Fig. B.12. The collector contains four stages, which are formed by five electrodes. The first electrode has zero potential \( U_1 = 0 \), negative potentials are given to the others, respectively \( U_2 = -0.55U_a, U_3 = -0.75U_a, U_4 = -0.95U_a, U_5 = -U_a \). The fifth electrode with a potential close to the cathode potential has an axial projection, which is called the reflector needle.

The configuration of the electrodes and the distribution of their potentials are chosen in such a way that the electrons deviate from the axis of the collector and, as a result of the braking process, are deposited on the rear surfaces of the electrodes.

![Fig. B.12 Multi-stage collector design. 1–5 Electrodes with indicated potentials](image-url)
In this case, the secondary electrons knocked out by them fall into the retarding electric field and the main mass returns to the electrode.

For collectors containing 4–5 steps, the efficiency of the collector can reach 60–80% depending on the type of device (klystron, TWT, etc.) and its operating mode.
Appendix C
Electrodynamic Systems of Microwave Electron Devices

C.1 Cavity Resonators

C.1.1 Elements of the General Theory of Cavity Resonators

In the absence of external currents, the electromagnetic field in the cavity resonator (CR) is described by the Helmholtz equation:

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0,$$

where $\mathbf{E}$ is electric field intensity, $k = \omega / c$—wave number in free space, $\omega$—angular frequency of field oscillations. This equation has an infinite countable set of solutions, each of them having its own eigen wave number $k_j$ (eigen frequency $\omega_j = c/k_j$) and eigen function $E_j$. The $k_j$ and $E_j$ together determine the cavity eigen mode. At each mode, the cavity is characterized by the following basic parameters:

1. The complex eigen angular frequency of the $j$-th oscillation mode $\omega_{0j} = \omega_{0j}' + j\omega_{0j}''$, which is determined by the formula

$$\omega_{0j}^2 = c^2 \frac{\int_V \varepsilon^{-1} \left| \nabla \times \mathbf{H}_j \right|^2 dV + j\omega_{0j} \oint_S \left( \mathbf{E}_j \times \mathbf{H}_j^* \right) dS}{\int_V \mu \left| \mathbf{H}_j \right|^2 dV}.$$

The second term in the numerator of this expression expresses the power dissipated in the cavity and radiated from it. If this power is zero and there is no energy loss in the medium filling the resonator ($\varepsilon'' = \mu'' = 0$), the eigen frequency is real, otherwise it is complex. The real part of the angular frequency determines the
frequency of the oscillations mode, and the imaginary part determines the rate of their damping:

\[ E_j(t) = E_j(0)e^{-\omega_0't}e^{j\omega_0't}, \quad H_j(t) = H_j(0)e^{-\omega_0't}e^{j\omega_0't}. \]

2. Internal Q-factor

\[ Q_{0j} = \frac{\omega_0'}{2\omega_0''}. \]

3. Wave (characteristic) impedance

\[ \rho_j = \frac{|U_{ej}|^2}{2\omega_0'W_j}, \quad (C.1.2) \]

where \( U_{ej} = -\int_1^2 E_j dl \) is the equivalent voltage, defined as a linear integral on the given line \( l \) between points 1 and 2, located on the surface of the resonator, \( W_j \) is the energy stored in the resonator at a given mode of oscillation.

In electronics, the integration path is usually chosen as the charged particle trajectory, so that equivalent voltage determines the intensity of the cavity electric field effect on the charged particle.

4. Equivalent impedance and equivalent admittance

\[ R_{ej} = \rho_j Q_{0j}, \quad G_{ej} = \frac{1}{R_{ej}}. \]

An important parameter of the resonator is frequency separation, that is, the difference between the frequencies of the working and neighboring oscillation modes. Frequency separation is characterized by the parameter

\[ \delta f = \frac{|f_0 - f_1|}{f_0} \]

where \( f_0 \) is the eigen frequency of the working mode; and \( f_1 \) is the eigen frequency of the closest mode to the working one.

All these parameters can be measured or calculated with the help of special computer programs (Ansis HFSS, CST Microwave Studio, RFS, etc.).

C.1.2 Types and Structures of Cavity Resonators

The large frequency band in which microwave systems operate gives rise to a wide variety of types and structures of cavity resonators that are used in these devices. In microwave electronics, the cavities are mainly used for providing interaction of their electromagnetic field with beams of charged particles.

Cavity resonators (CRs) are divided into closed and open types. Closed cavity examples have the closed envelope impenetrable to the electromagnetic field, while
open cavity examples have no such envelope. To increase the efficiency of interaction, the electric field of the closed cavities should be concentrated in the zone where a charged particles beam passes through the cavity. Consequently, the magnetic field is concentrated in other parts of the resonator. These parts can be attributed to capacity and inductance, that is, such resonators are well described by equivalent circuits. They are called resonators with quasi-lumped parameters.

One of the most common quasi-lumped cavities is the re-entrant (toroidal) cavity, the scheme of which is shown in Fig. C.1. It includes a cylinder 1 closed by covers with bushes 2. In the bushes, there are axial holes, forming a transit canal 3. The space between the bushes forms a high-frequency gap in which the electric field of the working mode of oscillations is concentrated. Holes can be tightened by metal grids (grid gap) or be without grids (gridless gap).

Consider an electric field in the gap. For the longitudinal component of the electric field of an azimuthally homogeneous mode in the cylindrical coordinate system, (C.1.1) assumes the form

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E_z}{\partial r} \right) + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0.$$  \hspace{1cm} (C.1.3)

In the gridded gap, the electric field can be assumed to be homogeneous with a sufficient degree of accuracy, that is $\partial^2 E_z / \partial z^2 = 0$. Equation (C.1.2) in this case turns into the Bessel equation, the solution of which has the form

$$E_z = AJ_0(kr),$$  \hspace{1cm} (C.1.4)

where $J_0(x)$ is the Bessel function of the first kind of zero order. The graphs of the $E_z$ distribution along the radius for various $ka$ values is shown in Fig. C.2. We can see that for $ka = 1$, field distribution is rather homogeneous.

Electric field distribution in symmetry planes of the re-entrant cavity, obtained with the help of a simulation program, are shown in Fig. C.3. As can be seen, the maximum value of the longitudinal component of the electric field placed near the bushes ends.

Figure C.1 shows the $H$-shaped cavity. However, one of its symmetrical halves (the $U$-shaped cavity) can also be successfully used (under other equal conditions, the $H$-form cavity has the advantage of a higher wave resistance).
Cavities with gridless gaps are used mainly in high-power devices, in which grids do not withstand the thermal load that arises from the electron bombardment. A magnetic field is concentrated in the peripheral part of the resonator. An electric field in a gridless gap depends on coordinate \( z \). Hence, all components should be conserved in (C.1.2). To solve this equation, we represent the field strength as the product of two functions:

\[
E_z(r, z) = \psi(r)\zeta(z).
\]

Substituting in (C.1.2), after simple transformations we obtain

\[
\frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} + k_c^2 \psi = 0, \quad (C.1.5)
\]

\[
\zeta'' + \beta_c^2 \zeta = 0, \quad (C.1.6)
\]
where with dashes denoting the differentiation with respect to coordinate \( z \) and \( \beta_e^2 + k_c^2 = k^2 \).

The solution of the equation C.1.6 is

\[
\zeta(z) = A \cos \beta_e z + B \sin \beta_e z, \quad \beta_e = \omega/v_e.
\]

Admissible electron transition angle in gap \( \beta_e d = 2\pi/3 \) (coupling factor 0.825). From this \( \beta_e = \omega/v_e = 2\pi/(3d) \), where \( v_e \) is the electron velocity in the resonator gap, and \( d \) is the length of the gap. Since electron velocity is always less than the speed of light in a vacuum, \( \beta_e > k \) and \( k_c^2 < 0 \). Consequently, \( k \) is an imaginary number: \( k_c = j\gamma \), where \( \gamma \) is the real number. Considering that, (C.1.5) can be rewritten:

\[
\frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} - \gamma^2 \psi = 0. \tag{C.1.7}
\]

The solution of this equation with a finite value on the cavity axis, is written in the form of a modified Bessel function of the first kind of zero order:

\[
\psi(r) = AI_0(\gamma r).
\]

Thus, the distribution of the electric field in the transverse cross-section of the transit canal is described by the formula

\[
E_z(r) = E_{z0} I_0(\gamma r)/I_0(\gamma a), \tag{C.1.8}
\]

where \( E_{z0} \) is the longitudinal component value of the electric field intensity at the boundary of the transit canal. The graph of the function \( E_z(z) \) is shown in Fig. C.4 for different values of \( \gamma a \). As can be seen, on the axis of the transit canal the field intensity is less than on its boundary. The non-uniformity of the field leads to a non-uniform interaction of electrons with the field of the resonator.

**Fig. C.4** Distribution of the electric field along the radius in a resonator with a gridless gap.
In order to reduce the negative effects of this effect, the transit-channel radius is usually chosen from the condition $\gamma a \approx 0.9$, which corresponds to the ratio $E_z(0)/E_z(a) \approx 0.8$.

An analogue of the re-entrant cavity is the brick cavity with bushes, designed to interact with a sheet electron beam (Fig. C.5). The resonator consists of a parallelepiped volume with lamellae 3. The transit canal 1 and gap 2 have a rectangular shape. The distance between lamellae and side walls is approximately $\lambda_c/4$, where $\lambda_c$ is the cutoff wavelength for the rectangular waveguide. Hence, magnetic wall boundary conditions exist at the ends of the lamellae. This provides homogeneous field distribution along lamellae.

Consider the field distribution in the transit canal of the cavity. Assuming that the field is homogeneous along the $y$-axis, we write the Helmholtz equation:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0.$$ 

Performing transformations analogous to those used to analyze the field in the re-entrant cavity, we obtain an expression for the function describing the distribution of the field in the cross section of the transit channel:

$$\psi(r) = A \cosh(\gamma r).$$

The graphs of this function for different values of $\gamma a$ are shown in Fig. C.6. Usually the ratio $E_z(0)/E_z(a)$ should not be greater than 1.25. Hence, the condition $\gamma a \leq 0.7$ should be satisfied. As can be seen, this is a more stringent condition than for a cylindrical channel.

Figure C.7 shows the distribution of the electric field in the H-shaped resonator along the $x$, $y$, $z$ axes calculated using the simulation program. As can be seen, the distribution of the field along the $x$ axis is close to uniform.

To interact with hollow electron fluxes, cavities with an annular gap are used. The scheme of such a cavity and the distribution of the field in it are shown in Fig. C.8. Ring cavities (Fig. C.9) are used for the same purposes. These resonators can also be used in multi-beam devices.
Fig. C.6  Field distribution in the gap of a gridless cavity

Fig. C.7  The calculated field distribution in the gap of the flat H-shaped resonator: a along the x-axis; b along the z-axis; c along the y-axis

Fig. C.8  A re-entrant cavity with an annular gap and the distribution of the electric field in it
The desire to increase the wave impedance of the resonator has led to the appearance of multi-gap resonators. Let us imagine that, using the coupling elements, $N$ single-gap resonators are combined into one so that the electron beam sequentially penetrates all the gaps. The distance between the gaps is chosen such that the electron beam passes each of them in the same phase of the field. It is obvious that in this case the energy stored in the resonator increases by a factor of $N$, and the total voltage acting on the electron beam also increases by a factor of $N$. But, since in the formula for the wave resistances the voltage is squared, the wave resistance increases by a factor of $N^2$ compared with its value for a single-gap resonator. In fact, the gain is less, since part of the energy is stored additionally in the communication elements, but it is still significant.

The most commonly used is a two-gap resonator based on a coaxial or strip line. The line is shorted at one end and contains a hole for the electron beam to pass on the other end. The same holes are also made in the bushes on the outer case of the resonator (Fig. C.10). The length of the line is close to a quarter of the wavelength. The electric fields in the gaps are directed in opposite directions, so the transition angle between the centers of the gaps should be $180^\circ$. The wave impedance of such resonators can reach 200 $\Omega$.

Resonators with a large number of gaps are formed from the short-circuited section of the slow wave system (SWS). Usually, a SWS of coupled cavity type, a comb-like SWS or an interdigital SWS are used. Section C.3 a contains a more detailed description of the properties of these cavities.
In multi-barrel devices, special resonators are used, the designs of which can be quite complex. One such design, developed at the Saratov Technical University and intended for a four-barrel multi-beam klystron is shown in Fig. C.11. The two-gap resonator consists of a cylindrical body 1 with a rod 2, on which radial rods with blocks of passing tubes 3 are located. The inductive loop 4 is intended to couple the resonator to the external transmission line. Unfortunately, the complexity of the design does not allow the use of similar resonators in the short-wave part of the microwave band.

Open waveguide resonators (OWR) are used in gyro-resonance devices. Such a resonator consists of a circular inhomogeneous waveguide segment 1 (Fig. C.12a), one end of which is connected to a canal 2 serving to inject an electron beam, and the other end is connected via a diaphragm 3 to a waveguide 4 through which energy is output. This waveguide serves as a collector for waste electrons. The shape of the resonator, i.e., the dependence of the radius of waveguide 1 on coordinate $z$, is chosen such as to obtain the optimum field distribution along the length of the resonator, ensuring the most effective interaction with the electron beam (Fig C.12b). Diaphragm 3 provides the necessary degree of resonator coupling with the waveguide and determines its loaded Q-factor. In powerful devices, it may be absent.

Since in gyro-resonance devices, electrons transfer their rotational motion energy to the field, OWR uses oscillations of the form $\text{TE}_{mnp}$, without longitudinal
Fig. C.11  Two-gap resonator for a multi-barrel klystron

Fig. C.12  Waveguide open cavity. a shape, b field distribution along axis

Fig. C.13  Equivalent circuit of a resonator near the eigen frequency: a parallel, b series
electric field components. The longitudinal index $p$ is usually chosen as equal to one, and because cavity length $L \gg \lambda$, the eigen frequency of the resonator is approximately equal to the cutoff frequency of the waveguide averaged over its length. The second index determining the distribution of the field along the radius is chosen such as to provide maximum value of the azimuthal component of the electric field in the region of the electron beam. The first index determines the number of variations of the field along the azimuth. Often, it is chosen to be zero, since oscillations of the form $H_{0np}$ have the greatest quality factor.

**C.1.3 Excitation of Cavity Resonators**

Near the eigen frequency, the resonator can be represented by an equivalent circuit in the form of a parallel (Fig. C.14a) or sequential (Fig. C.14b) LC oscillatory circuit, the parameters of which are determined by the electrodynamics parameters of the resonator for a given mode of oscillation:

$$L_e = \frac{\rho}{\omega_0'}, \quad C_e = \frac{1}{(\omega' \rho)}.$$  

For a parallel circuit

$$G_e = \frac{1}{(Q_0 \rho)},$$

for a series circuit

$$R_e = \rho/Q_0$$

The input admittance of a parallel equivalent circuit near the eigen frequency:

$$Y_{in} = G_{in} + jB_{in} = G_e + j[\omega C_e - 1/(\omega L_e)] = G_e(1 + j\xi),$$

![Fig. C.14](image-url)  

**Fig. C.14** Dependence of admittance (a) and impedance (b) on frequency of the parallel equivalent resonator circuit
where

$$\xi = Q_0 \left( \frac{\omega}{\omega_0} - \frac{\omega'_0}{\omega} \right) \approx 2Q_0(\omega - \omega'_0)$$

is the generalized detuning. The overall impedance of this circuit: $Z_{in} = R_{in} + jX_{in} = 1/Y_{in}$. The graphs of these dependencies are shown in Fig. C.14.

As we see, the active conductivity near the resonance remains constant, and the reactive conductivity varies linearly. The greatest value of active impedance $R_{in} = 1/G_e$ is observed at resonance ($\xi = 0$). The active impedance of the resonator decreases by a factor of two at $\xi = \pm 1$, which determines the bandwidth of the resonator.

Analogous, the input resistance of a serial circuit $Z_{in} = R_{in} + jX_{in} = R_e + j[\omega L_e - 1/(\omega C_e)] = R_e(1 + j\xi)$.

The graphs of dependencies $R_{in}$ and $X_{in}$ are similar to the graphs of Fig. C.15 substituting $Y$ with $Z$.

If the resonator is connected to an external circuit, part of the oscillation energy goes into the load of this circuit (or is radiated into the free space). In this case, an external and loaded $Q$-factor of the resonator are introduced:

$$Q_{out} = \frac{W}{\omega'_0 P_{out}}, \quad Q_l = \frac{W}{\omega'_0 (P_{out} + P_0)},$$

where $P_0$ and $P_{out}$ are power dissipated in the resonator and its external circuit. Obviously the ratio is valid

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_{out}}$$

**Fig. C.15** Cavity loaded with external transmission line through the coupling element (a). Equivalent circuit recalculated in the primary circuit (b)
Various circuits are used to connect resonators to external transmission lines—inductive loops, pins and holes in the walls of the resonator. Sometimes the tip of the pin is connected to some element of the resonator structure. Such a connection is called conductive. The equivalent circuit of any coupling element can be represented as an ideal transformer, in series or in parallel with the primary winding of which (connected to the resonator) reactivity of the coupling element is connected. Loops and pins are used, as a rule, to couple with the coaxial transmission line and holes (diaphragm)—with waveguides. In open resonators so-called diffraction coupling is also used, where part of the resonator energy excites the wave in the free space through the hole in the mirror.

Consider the excitation of the resonator from an external source, using an equivalent circuit of the resonator in the form of a parallel LC circuit. Most interesting is the dependence of the voltage on the capacitor (on the resonator gap) on the excitation frequency. The entire equivalent circuit is shown in Fig. C.15a. It consists of a generator 1, a transmission line 2 with wave impedance \( Z_0 \), a coupling element consisting of an ideal transformer 3 with a transformer coefficient \( n \), of the coupling element intrinsic reactivity 4 and the resonator 5 with parameters \( L_0, C_0, G_0 \). The gap of the resonator corresponds to the capacitor \( C_0 \). In parallel, electronic conductivity is connected \( Y_e = G_e + B_e \) (see Sect. 4.2.4). With small changes in frequency, the active part of the electron conductivity can be considered independent of frequency, and the reactive part, proportional to it. In this case, it is possible to simplify the equivalent resonator circuit by entering parameters \( G = G_0 + G_e, C = C_0 + B_e/\omega \). We note that if amplitude and phase conditions are satisfied at a certain frequency \( G_0 + G_e \leq 0, B_0 + B_e = 0 \), the resonator is self-excited at this frequency (if there is no coupling with the load).

After recalculating the circuit parameters in the primary circuit of the transformer (Fig. C.15b), the input resistance of the coupling element is determined by the formula

![Fig. C.16 Equivalent circuit of a resonator excited by electron beam (a). Circuit with the load recalculated in the primary circuit (b)](image)
\begin{equation}
Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}} = X + \frac{1}{Y_{0}} = X + \frac{R'_{e}}{1 + j\xi} = X + \frac{R'_{e}(1 - j\xi)}{1 + \xi^{2}},
\end{equation}

where $R'_{e} = \rho'Q = n^{2}R_{e}$. To fully transfer power from the generator to the load, it is necessary that

\begin{equation}
R_{\text{in}} = \frac{n^{2}R_{e}}{1 + \xi^{2}} = Z_{0},
\end{equation}

\begin{equation}
X_{\text{in}} = X - \xi \frac{n^{2}R_{e}}{1 + \xi^{2}} = 0.
\end{equation}

From these equations, we obtain a generalized detuning necessary for matching the resonator with the generator:

\begin{equation}
\xi_{\text{opt}} = X/Z_{0}.
\end{equation}

The voltage on the resonator gap:

\begin{equation}
U = I_{2}Z = I_{2}Re \frac{1 - j\xi}{1 + \xi^{2}} = I_{2} \frac{Re}{1 + \xi^{2}} e^{j\phi},
\end{equation}

where $\phi = \arctan(1/\xi)$. Since $I_{2} = I_{1}/n$ and $I_{1} = \sqrt{2P/Z_{0}}$ for the gap voltage modulus we obtain the expression

\begin{equation}
|U| = \frac{Re}{\sqrt{Z_{0}} \sqrt{1 + \xi^{2}}} = \sqrt{2PR_{e}} \frac{ReZ_{0}}{Z_{0}^{2} + X^{2}},
\end{equation}

where $P$ is the excitation power. At the same power supply, the voltage on the gap increases as the equivalent resonator impedance increases and as the reactive resistance of the coupling element decreases.

A bunched electron beam can excite the oscillations in the resonator. In this case, the equivalent circuit of the device has the form shown in Fig. C.16a. The electron beam creates an induced current in the resonator circuit $I_{u}$. The effect of the electron beam is also taken into account by electronic conductivity $Y_{e}$, parallel to the gap capacity $C_{0}$. A transmission line with a wave resistance is connected with the resonator $Z_{n}$. The same scheme with TL parameters recalculated in the resonator circuit is shown in Fig. C.16b.

The load power is given by

\begin{equation}
P_{l} = \frac{1}{2} |U|^{2} G'_{g}
\end{equation}

where $G'_{g} = n^{2}/Z_{g}$. Ignoring the reactive resistance of the coupler, we find the voltage on the load:
\[
U = \frac{I}{G_0 + G_e + G_g' + j[\omega C_e - 1/(\omega L_e) + B_e]}.
\]

Obviously, the maximum voltage corresponds to the frequency at which the imaginary part of the denominator of the last formula is zero:

\[
P_{l_{\text{max}}} = \frac{|I_H|^2}{2G_g'} \left( 1 + \frac{G_0 + G_e}{G_g'} \right)^2.
\]

The output power increases when the coupling of the resonator with the load decreases (transformation coefficient \(n\)). However, power growth stops when the voltage reaches value \(U_0\). Further reduction of the load leads to the appearance of the rejection effect, as a result of which the induced current begins to decrease.

**Filter systems.** To expand the device bandwidth coupled resonators are often used in the output circuit of the klystron. The active cavity (excited by an electronic current) is coupled with a passive resonator located in the output transmission line. As a rule, a passive resonator has the form of a waveguide segment, bounded by two inductive apertures or rods.

Figure C.17 shows one of the variants of coupled cavities. The toroidal resonator 1 through diaphragm 2 is connected to a rectangular waveguide 3. Usually a non-standard waveguide with a reduced size of a narrow wall is used. A waveguide segment located between inductive irises 4 forms the passive resonator. The waveguide 5 is connected to a vacuum window (not shown in the figure). The distance from the active resonator to the passive resonator must be an integer number of half-waves, such that the transformation ratio of this segment is equal to one.

Figure C.18 shows frequency response characteristics of a single output resonator (curve 1) and an output resonator with a passive resonator (curve 2). As can be seen, the use of a passive resonator allows a significant increase in the bandwidth of the output system due to a certain decrease in the amplitude of the output voltage.

Fig. C.17  Output circuit with passive cavity
C.2 Slow Wave Systems

C.2.1 Characteristics and Parameters of SWS

Slow wave systems (SWS) are used in O-type TWT, twistrons, M-type devices, orostrons and some other microwave devices. Short-circuited or closed in ring SWS segments form multi-gap cavities, used in klystrons with distributed interaction and magnetrons.

The purpose of SWSs is to slow down the phase velocity of the wave propagating in it $n_p$ times compared to the speed of light in vacuum. This allows a long-term interaction of the SWS field with the electron beam, because the wave in SWS and the electrons in the beam move with almost identical velocities. SWS is a transmission line (TL), which coincides with itself during displacement (translation) by a distance that is a multiple of a certain minimum distance, called the SWS period $D$ (periodic TL). There exist one-dimensional SWS in which alignment is possible for translation in only one direction, and two-dimensional periodic SWS, in which alignment occurs upon translation in two independent directions. The most common are one-dimensional SWS, which are considered in this appendix.

In SWS, as in any transmission lines, the existence of different wave modes is possible. Based on Floquet’s theorem, the electromagnetic field of each mode can be represented as a superposition of spatial harmonics:

$$E(z) = \sum_{p=-\infty}^{\infty} E_p e^{-\gamma z} e^{-j \beta_p z}, \quad H(z) = \sum_{p=-\infty}^{\infty} H_p e^{-\gamma z} e^{-j \beta_p z},$$

where $p$ is the harmonic number; $E_p, H_p$ are complex amplitudes of its electric and magnetic fields intensities; $\beta_p = (\varphi + 2\pi p)/D$ is the harmonic phase constant; $-\pi \leq \varphi \leq \pi$ is the phase shift on the SWS period; and $\gamma$ is the attenuation constant, common to all harmonics. The phase velocity of the harmonic is given by

$$v_p = \frac{\omega}{\beta_p} = \frac{\omega D}{\varphi + 2\pi p}. \tag{C.2.1}$$
Group velocity

\[
\nu_{gp} = \left( \frac{d\beta_p}{d\omega} \right)^{-1} = D \left( \frac{d\varphi}{d\omega} \right)^{-1},
\]

obviously is common for all harmonics.

Amplitudes of spatial harmonics are defined as the coefficients of the Fourier series:

\[
\begin{align*}
E_p &= \frac{1}{D} \int_{-D/2}^{D/2} E(z) e^{j\beta_p z} dz, \\
H_p &= \frac{1}{D} \int_{-D/2}^{D/2} H(z) e^{j\beta_p z} dz.
\end{align*}
\]

Usually, harmonics with small absolute numbers \(|p|\) have the largest amplitude. The spatial harmonic having the greatest phase velocity is called the fundamental harmonic.

The dispersion characteristic of a SWS is constructed as the dependence of the phase velocity deceleration \(n_p = c/\nu_p\) on wavelength in free space. Another form of the dispersion characteristic is the dependence of the wave number in free space on the phase constant. The dispersion characteristic allows the determination of the type of spatial harmonic dispersion: (normal positive, anomalous positive or anomalous negative), phase and group velocities.

In addition to dispersion, an important characteristic of SWS is the coupling impedance of the spatial harmonic, defined by formula

\[
R_{cp} = \frac{|E_{zp}|^2}{2\beta_p^2 P},
\]

where \(E_{zp}\) is the amplitude of the electric field intensity of the \(p\)-th spatial harmonic; and \(P\) is the power transmitted by this mode. Coupling impedance depends on the spatial harmonic number and on the frequency.

In the millimeter band, the attenuation constant of SWS \(\alpha\), is important and can significantly affect the parameters of the device.

### C.2.2 Main Types of SWS

**Helical SWS.** A helical SWS was used in the first TWT created by Kompfner, and is still widely used in these devices. The simplest helical SWS consists of copper or molybdenum wire, coiled into a spiral and placed in a metal shell (screen) (Fig. C.19a). The spiral conductor can have a rectangular cross-section (ribbon spiral) 1 (Fig. C.19b). The spiral inside the screen 2 is supported by ceramic support rods 3. In SWS with a circular conductor, the amplitudes of higher spatial
harmonics ($|p| > 0$) are insignificant compared to the fundamental ($p = 0$). In SWS with a ribbon spiral, the amplitudes of the higher spatial harmonics with $p = \pm 1$ are comparable with the fundamental harmonic, which makes it possible to use such SWS in the BWO.

Figure C.19c shows a SWS with two spirals with counter-coiling and Fig. C.19d shows a ring-rod SWS type, which can be considered as a kind of helical SWS with counter-coiling. This type of SWS is characterized by a rigid construction and is used in high-power TWTs. A SWS with a double helix is distinguished by strong dispersion and a narrower bandwidth in comparison with a conventional helical SWS.

In the roughest approximation, we can assume that the wave in a helical SWS propagates along the wire at the speed of light. Then the deceleration of the phase velocity (slowing factor) of the basic spatial harmonic can be found as the ratio of the turn length to the period:

$$n_{p0} = \sqrt{\left(\frac{2\pi a}{D}\right)^2 + 1} = \sqrt{1 + \left(\frac{2\pi a}{D}\right)^2} = \frac{1}{\sin \xi},$$

where $\xi = \arctg[D/(2\pi a)]$ is the winding angle of the spiral. As can be seen, this quantity is independent of frequency, that is, there is no dispersion in the helical SWS. In fact, the dispersion of the fundamental spatial harmonic in a helix SWS is observed, but in a very wide frequency band it is sufficiently small (Fig. C.20). To control the shape of the dispersion characteristic, radial metal fins are connected to the screen (Fig. C.21). By adjusting the height and number of fins, it is possible to control the shape of the dispersion characteristic.

The coupling impedance of the fundamental harmonic in a helical SWS has a value in the order of 100 $\Omega$ and depends, comparatively weakly, on the frequency. Supporting rods and radial fins (if any) strongly influence the value of coupling impedance. To increase $R_c$, it is necessary to choose a rod material with a low dielectric constant.

A helical SWS has a low heat dissipating capacity. Therefore, it cannot be used in powerful devices. To improve the thermal characteristics, the rods are sometimes made of ceramics based on beryllium oxide, which has good thermal conductivity. Sometimes metal rods with diamond endings are used to make thermal contact with
the spiral. Such holders simultaneously correct the dispersion characteristic of the SWS.

**Coupled cavity chain type of SWS (CCC-SWS).** They have a rigid structure providing good heat dissipation. Therefore, they are used in high-power O-type TWT. SWS design is shown in Fig. C.22. The circular waveguide 1 is divided by diaphragms 2 into separate resonators. Slots 4 serve to couple the resonators with each other. The number of slots in the diaphragm, their dimensions, and the angle of rotation relative to adjacent slots can vary. Usually, one slot is used, rotated at an angle of 180° with respect to neighboring slots, as shown in Fig. C.22.

Dispersion of the fundamental spatial harmonic CCC-SWS ($p = 0$) is negative, therefore in TWT, the negative first harmonic is used as the working one ($p = -1$) which has positive dispersion. Figure C.23 shows dispersion characteristics and coupling impedance of the minus one spatial harmonic of a typical CCC-SWS in the two lowest-frequency pass bands, resonator and slot. These pass bands are so named because the boundary frequency of one of them practically coincides with the eigen frequency of the resonator without the coupling slots, and the boundary frequency of another pass band (in this case, a higher one) coincides with the eigen frequency of the slot.
The relative width of the resonant bandwidth is 36%, the coupling impedance in this band varies from 0.2 to 10 Ω. By choosing the geometric parameters of the SWS, it is possible to connect the resonator and slotted bandwidths, creating one wide band.

**COMB SWS** are used in TWTO of the millimeter band due to their simple and rigid design. The use of them is particularly effective in devices with a sheet electronic beam. Designs of some varieties of comb SWS are shown in Fig. C.24. Figure C.25a shows a simple open comb with grooves of rectangular shape. The electron flow is passed between the ends of the ridges and the screen. Dispersion

---

**Fig. C.22** Coupled cavity SWS. 1 round waveguide; 2 diaphragm; 3 bushes with drift holes, 4 slots

**Fig. C.23** Dispersion characteristic of a CCC-type SWS in two bandwidths
characteristics of the comb for different ratios of the transit canal height $d$ to the groove depth $h$ are shown in Fig. C.25.

Dispersion increases with increasing ratio $d/h$, however, this also increases the coupling impedance. In electronic devices, metal walls close the comb on its sides. The walls are nearly a quarter of the wavelength from the ridges. Such a screen has little effect on the dispersion characteristic of the comb. Comb grooves can have different shapes and sizes. More about this is written in the section on multi-cavity magnetrons.

Figure C.25b shows an interdigital comb SWS. In such a system, the field of each wave mode can be represented as the sum of two sets of spatial harmonics, called symmetric and anti-symmetric components. If for one of the components the phase shift for the SWS period is equal to $\varphi$, then for the other it is equal to $\pi - \varphi$. The electric field of the symmetric component is symmetric with respect to the sliding plane symmetry SWS, and the electric field of the antisymmetric component is antisymmetric and equals zero in the plane of symmetry. Therefore, the electron beam passing through the transit canal in the symmetry plane interacts only with the symmetric component of the total field.
The dispersion characteristic of the zero spatial harmonic of the interdigital SWS is shown in Fig. C.26. The symmetric component (curve 1) has a negative dispersion, which makes it possible to use this SWS in wide-band BWO.

**Stub SWS** are used in type-O TWT and BWO, as well as in M-type devices (mitrons and magnetrons). Figure C.27 shows the most common types of rod SWS: ladder SWS with a ledge (Fig. C.27a) and groove (Fig. C.27b), interdigital stubs
SWS (Fig. C.27c) and meander (Fig. C.27d). The dispersion characteristics of ladder SWS are shown in Fig. C.28. As can be seen, they have a strong dispersion, but a simple design allows them to be used in TWT of the millimeter band.

The interdigital SWS have a dispersion characteristic similar to the SWS counter-type combs (Fig. C.27). They are used in the BWO. The SWS dispersion characteristic of the meander type is shown in Fig. C.29. The symmetrical component of this SWS has a small dispersion at $\varphi / \pi < 0.3\pi$, which allows these systems to be used in broadband TWTs. Coupling impedance of the symmetric component of the main spatial harmonic in the rod SWS amounts to tens of Ohms.

Note that the pin structure of this type of SWS can be located on a dielectric substrate (also made in the form of a printed circuit board). Such systems provide a greater deceleration with decreasing coupling impedance.

### C.3 SWS-Based Resonators

The SWS segments can be used as resonators. There are two types of resonators - linear, made of the rectilinear SWS segment, short-circuited at both ends, and annular ones made of a SWS segment, closed in a ring.
The resonance condition in the linear resonator is the addition of a wave reflected twice from the ends, with the original wave. This condition is written as follows:

$$2\varphi N = 2\pi m + 2\pi,$$

where \( N \) is the number of periods of the SWS; \( m = 0, 1, \ldots \) is the integer number, the last term on the right-hand side takes into account that when reflected from an ideally conducting wall, the phase of the wave changes by 180°. From (A.3.34) it follows that the phase shift on the SWS period can take fixed values

$$\varphi_m = \frac{\pi m}{N}.$$

Each value of \( m \) corresponds to a particular resonator mode of oscillation, which has its own eigen frequency \( \omega_m \). Since the phase shifts lay in the range \( 0 \leq \varphi \leq \pi \), in this type of resonator there can exist \( N + 1 \) modes corresponding to \( m = 0, 1, \ldots, N \). The first mode \((m = 0)\) of oscillation is usually called the zero mode, and the last is called the \( \pi \)-mode. These types are, as a rule, operable.

It should be noted that, since for \( \pi \)-type oscillations the longitudinal components of the electric field in adjacent periods have opposite directions, it is necessary to use boundary conditions of a magnetic wall type on the ends of the resonator. Since this is impossible, the \( \pi \)-type oscillations in the resonators are not excited, and the last type is with \( m = N - 1 \). It is not difficult to see that a field of this type also has an opposite direction in neighboring periods. However, the amplitude of the field varies from gap to gap, taking the maximum value in the middle of the resonator (Fig. C.30). The dashed curve in this figure shows the envelope of the field amplitude in the gaps, and the solid curve shows the amplitude of the longitudinal component of the electric field in the gaps. To correct the distribution of the field, SWS periods are made unequal.

Calculating the wave impedance of a multi-gap resonator according to the formula (C.1.1) can produce a result of zero, as, for example, for a two-gap resonator with identical gaps operating on the \( \pi \)-mode.

**Fig. C.30** The distribution of the longitudinal component of the electric field in the gaps of the multi-gap resonator on \( \pi \)-mode.
It should be taken in mind that in microwave electronics, wave impedance characterizes the degree of the resonator field effect on a charged particle. In order for this effect to be maximum, the velocity of the particle must be chosen such that each gap it transits is in the same field phase. Then the action of all the gaps on the particle is summed. Therefore, in formula (C.1.1) the equivalent voltage must be calculated from formula

\[ U_e = \int_0^L |E_z(z)| \, dz, \]

taking the absolute value of the field strength in each gap.

In ring resonators based on SWS, resonance is observed if the wave, passing through the resonator, coincides in phase with the original wave, i.e.,

\[ \varphi N = 2\pi m, \quad m = 0, 1, \ldots, N/2. \]

Hence, the values of the phase shift angle corresponding to different modes of oscillations in the ring resonator:

\[ \varphi_m = 2\pi m/N. \]

Each mode has its own eigen frequency, which can be determined from the dispersion characteristic of the SWS. As a working type, the \( \pi \)-type (\( m = N/2 \)) is often used.

**C.4 Vacuum Windows**

Energy input and output from the vacuum device must be carried out through vacuum-tight transitions, which are called vacuum windows. The design of vacuum windows depends on the type of transmission lines into which they are integrated and the level of transmitted power.

Low-power coaxial vacuum windows consist of a section of a coaxial transmission line into which a ceramic insulator is soldered (Fig. C.31). The diameter of the inner conductor at the location of the insulator is reduced such that the wave impedance of the line remains constant.

Waveguide windows consist of a segment of a rectangular waveguide 1 (Fig. C.32), into which the metal diaphragm 2 is soldered. The diaphragm is made of metal, the thermal expansion coefficient (TEC) of which is close to the TEC of ceramic.

The ceramic insulator 3 is soldered into the diaphragm 2. The dimensions of the aperture in the diaphragm are selected such that the window has almost zero reflection coefficient at the operating frequency of the device. The formula for
determining ceramic window dimensions \( a', b' \) is obtained using equality of wave impedances of the waveguide section filled by air and ceramic:

\[
\frac{b'}{b} = \frac{a'}{a} \sqrt{\frac{\varepsilon_r}{\mu_r}} \sqrt{1 - \left( \frac{\lambda}{2a'} \right)^2} \frac{1}{\varepsilon_r \mu_r} \left[ \sqrt{1 - \left( \frac{\lambda}{2a} \right)^2} \right]^{-1},
\]

where \( a, b \) are the empty waveguide cross-section dimensions, \( \varepsilon_r, \mu_r \) — relative permittivity and permeability of the ceramic.

To output energy from high power devices, can windows are used (Fig. C.33). They consist of a circular waveguide segment 2 built into a rectangular waveguide 1. A wave of \( H_{10} \) mode in a rectangular waveguide excites a \( H_{11} \) mode in a circular waveguide. At the other end of the circular waveguide, the \( H_{11} \) mode is converted back to an \( H_{10} \) wave type in a rectangular waveguide. The length of the circular waveguide is chosen as equal to half the wavelength. If the resonator formed by a circular waveguide segment is weakly coupled to rectangular waveguides, an almost standing wave is established in it, and in the middle plane, where the vacuum window 4 is located, the electric field intensity decreases almost to zero, which sharply reduces losses in ceramics and heat release in it. Conditions for the occurrence of electrical breakdown on the surface of ceramics are also hindered. However, the bandwidth of such a window is very small, so the coupling with the
waveguides is done sufficiently strong and the field in the ceramic region is attenuated 1.5–2 times compared to the traveling wave field in a circular waveguide of the same cross section. Nevertheless, such windows can transmit large power, up to several megawatts. If necessary, the external surface of the circular waveguide is forced-cooled with air or liquid. Ceramic based on Al₂O₃, BeO, quartz and synthetic diamond is used as a material.

It should be noted that the dielectric sheet itself is a resonator, with resonant frequencies that can lie in the operating frequency band of the device. Therefore, the parameters of the sheet (thickness, radius, dielectric permittivity) should be chosen so that there are no parasite resonances in the operating frequency band of the device.

The windows with conical insulators (Fig. C.34) can have even greater impulse power, since they have a longer path for surface breakdown. However, the heat dissipation path from ceramics in such windows is longer than in planar ones, so it is inappropriate to use them to pass large average power. In modern high-power devices, conical windows are rarely used.

Modern window structures can transmit power of up to 10 MW in pulsed mode, and hundreds of kW in continuous mode in the three-centimeter wavelength band. The operating bandwidth is usually 10–15%, reaching up to 50% in special designs. Nevertheless, in high-power microwave devices, the power transmitted by the window is sometimes insufficient, and several power outputs must be made so that the output power is shared between them.

**Fig. C.33** A can waveguide window: 1 rectangular waveguide; 2 circular waveguide; 3 diaphragm; 4 ceramic window

**Fig. C.34** A can waveguide window with conical insulator: 1 rectangular waveguide; 2 circular waveguide; 3 diaphragm; 4 ceramic window
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