General Conclusion

In this book, we introduced a special class of fuzzy numbers, fuzzy dual numbers which are at the crossroad between classical fuzzy numbers and dual numbers in an attempt to take benefits from the properties of both of them.

Thus, fuzzy dual calculus has been developed in such a way that the interpretation of their dual part as an uncertainty level remains valid through the basic operations with these numbers. Then it has been shown how the induced pseudo norm allows to set different partial orders between fuzzy dual numbers.

This enabled a rather simple definition of fuzzy feasible sets through fuzzy dual constraints. To display the generality of the proposed approach, the case of fuzzy dual LMI’s has been considered since many mathematical programming problems adopt the LMI representation for their feasible sets.

The application of the fuzzy dual formalism to estimate trip matrices under uncertainty has been developed and a more general approach for estimation, involving a fuzzy dual entropy, has been displayed.

Then, fuzzy dual programming problems with either uncertain parameters or variables have been considered. Although only the linear case has been considered in this study, it appears that the proposed approach to treat uncertainty leads for the solution of a fuzzy dual programming problem to the consideration of a finite collection of classical mathematical programming problems. When considering dynamic programming, it appears that fuzzy dual dynamic programming provides a feasible numerical way to tackle sequential decision problems under uncertainty.

It can be considered that the approach developed in this book provides a way to tackle a large class of analysis and optimization problems under uncertainty with a manageable trade-off between the accuracy of the representation of uncertainty and the resulting computer burden.
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