Relevant Work

The review paper (Floater 2015) is especially informative. It is customary to list a great number of references in a bibliography as a book finale. Only work specifically referred to in the text is included in this bibliography. Computers are so widely available with powerful search engines that a different strategy is followed here. Relatively few search words will be enumerated. These will yield far more meaningful and available references than an extensive bibliography would reveal.

Search Words

Rational finite elements, Wachspress coordinates, Rational basis functions, Barycentric coordinates, Mean value coordinates, Polar duals, Convex polygon coordinates, Polytope coordinates, Gradient bounds for polygons and polytopes, Interpolation error estimates for polygons, Convex polyhedra coordinates, Polygonal finite elements.
References


M. Bocher, *Introduction to Higher Algebra* (MacMillan, New York, 1907)

G.S. Carr, *Formulas and Theorems in Mathematics* (Chelsea, Bronx, 1970)


J. Ergatoudis, Quadrilateral elements in plane analysis. Masters thesis, University of Wales, Swansea, 1966


W. Fulton, Algebraic Curves (Benjamin, New York, 1969)


I.S. Gradshteyn, I.M. Ryzhik, Table of Integrals Series and Products, 4th edn. (Academic, Boston, 1965)

J. Hadamard, Lectures on Cauchy’s Problem in Linear Partial Differential Equations (Dover, New York, 1952)


B.M. Irons, Numerical integration applied to finite element methods, in Conf. on use of Digital Computers in Structural Eng., University of Newcastle, 1966


T. Muir, Theory of Determinants, 4 vols. (Dover, New York, 1960)


B.L. van der Waerden, Algebraische Geometrie (Springer, New York, 1939)


J. Verdina, Projective Geometry and Point Transformations (Allyn & Bacon, Rockleigh, 1971)


Index

A
Adjacent factor
ill-set polycondron, 167
nodal analysis, polypoldron
edge nodes, 155–156
face nodes, 155
interior nodes, 155
vertex nodes, 130–131, 156
polycons, 88, 92, 96
Adjoints
ill-set polycondron, 166–167
polycons, 83–84
polypoldron
exterior edge deficit points, 151–152
multiple points, 150
p-p edge deficit points, 151
surface deficit points, 150–151
positive behavior, 225–226
unique property, 152–154
Adjunct intersection points, 108–110
Algebraic geometry
associated points, 73–75
congruence theorem, 70–72
correspondence theorem, 70
Euclidean algorithm, 68
intersection of three lines, 69–70
Noether’s theorem, 68–69
ordinary multiple point, 72–73
higher degree approximation
adjacent factor, vertex node, 130–131
boundary curve, 128–129
boundary-node wedges, 129
side node q, deletion, 129–130
wedge numerators, 129
well-set rational polypols, 128
motivation, 61–62
patchwork approximation, 7
plane curve approximation
ambiguities, 66
Bezout’s theorem, 66–67
homogeneous polynomials, 65
intersection number, 64
mixed double contact, 67–68
multiplicity of component, 63
nonsingular, 63–64
Sylvester’s dialytic method, 64–65
polycons, 86
projective coordinates, 62–63, 70, 77
singularities, resolution
algebraic transform, 75–76
irregular lines, 75–77
quadratic transformation, 75, 78–79
rationality and genus zero, 79–81
Walker’s theorem, 77–78
Algebraic reticulation, 131–132

B
Barycentric coordinates, 29, 32, 215, 219, 228–229

C
Coarse mesh rebalancing, 211
Congruences, 70–72
correspondence theorem, 70
degrees of freedom, 51–52
ellipse, 48–50
Euclidean algorithm, 68
hyperbola, 48–50
Congruences (cont.)
intersection of conics, 50–51
intersection of three lines, 69–70
Noether's theorem, 68–69
ordinary multiple point, 72–73
Coordinate construction, 241

D
Deficit intersection points (DIP)
degrees of freedom, 107
EIP, 105–106
polycube, 105
wedge regularity, 107
well-set rational polypols, 107
Desargues' theorem, 137–138

E
Exterior intersection points (EIP)
DIP, 105–106
4-con of order five, 44, 46–47
patchwork approximation, 9
3-con of order five and six, 54–57

F
Finite element discretization
harmonious discretization
boundary conditions, 195
degree two approximation, 199
equilateral triangle, 202–203
harmonic wedges, 195–196
ill-set quadrilateral, 200
mosaic discretization, 197, 200–201
obtuse triangle, 204–205
optimum basis function, 196
patch conditions, 201
quadrilateral partitioning, 199
static condensation, 196, 198
Laplacian discretization
coordinates and coefficients, 189–191
degree one approximation, 189
degree two approximation, 192–193
triangulation scheme, 192
trimedian quadrature, 191
mosaic discretization
inhomogeneous problem, 187–188
isoparametric segment, 188
mosaic basis function, 186, 187
polynomial expression, 186–187
triangle network, 185
wedge equation, 185
patch test
consistent approximation, 179
diffusion equation, 178–179
linear function, 180
quadrilateral coordinates, 182–183
roundoff error, 179
stability criterion, 179
triangle coordinates, 181
quadrature formula
approximation space, 173
degree one triangle, 174
eigenvalue problem, 172–173
error component, 175
Gaussian formula, 174
segments (see Segment quadrature)
stationarity conditions, 173
wedge quadrature, 174–175
triangle averaging, 183–185

G
GADJ algorithm
polygons, 217
polyhedron
adjoint and adjacent, 236
higher order vertices, 236
planar figure, 236, 237
polar dual construction, 236
polynomial expression, 235, 237
polypoldron, 237–238
polytopes, 238–239
tesseract, 239–240
Generalized triangulation, 131

H
Harmonious discretization
boundary conditions, 195
degree two approximation, 199
equilateral triangle, 202–203
harmonic wedges, 195–196
ill-set quadrilateral, 200
mosaic discretization, 197, 200–201
obtuse triangle, 204–205
optimum basis function, 196
patch conditions, 201
quadrilateral partitioning, 199
static condensation, 196, 198
Hexahedron
degree four approximation, 141
degree k approximation, 141–142
degree three approximation, 140–141
degree two approximation, 140
planar figure, 138–139
wedge equation, 139–140
Higher degree approximation
algebraic geometry
adjacent factor, vertex node, 130–131
boundary curve, 128–129
boundary-node wedges, 129
side node q, deletion, 129–130
wedge numerators, 129
well-set rational polytopes, 128
algebraic reticulation, 131–132
data fitting, 115–116
degree three approximation, 122–125
degree two approximation
adjacent curve, 121–122
isoparametric transformation, 117
nodes, 118
quadratic variation, 122
quadrilateral, 116–117
rational basis functions, 118–119
six-node triangle, 116
3-con sides, 119–120
xy approximation, 121
intermediate approximation, 125–127
polytopes, 127–128
Higher dimensional analysis. See GADJ algorithm

I
Ill-set polycondron
nonconvex quadrilaterals
adjacent factor, 167
adjoint, 166–167
degree one approximation, 168
planar figure, 166–167
quadrilateral wedges, 167–168
wedge equation, 164–165
Irrational wedges
ill-sets, 164–168
rabbit function, 159–160
3-con planar figure, 159–160
Isoparametric coordinates
degree one approximation, 19
forbidden shapes, 17
n-con boundaries, 19
symmetry axis, 19–20
3-con and 4-con nodes, 18–19

J
Jacobian transformation, 19, 31–33, 233

L
Laplacian discretization
coordinates and coefficients, 189–191
degree one approximation, 189
degree two approximation, 192–193
triangulation scheme, 192
trimedian quadrature, 191

M
MATLAB programming, 245–279
Method of descent, 161–164
Mosaic discretization
harmonic basis function, 197, 200–201
inhomogeneous problem, 187–188
isoparametric segment, 188
mosaic basis function, 186, 187
polynomial expression, 186–187
triangle network, 185
wedge equation, 185
Multiple intersection point, 7

N
Noether’s theorem, 68–69
Nonconvex quadrilaterals
degree one approximation, 168
ill-set polycondron, 166–167
planar figure, 166–167
quadrilateral wedges, 167–168

P
Patch test
consistent approximation, 179
diffusion equation, 178–179
linear function, 180
quadrilateral coordinates, 182–183
roundoff error, 179
stability criterion, 179
triangle coordinates, 180–181
Patchwork approximation
algebraic curves, 7
algebraic geometry, 7
congruence notation, 10–11
continuity, 11–13
EIP, 9
higher-order curves, 20–21
isoparametric coordinates
degree one approximation, 19
forbidden shapes, 17
n-con boundaries, 19
symmetry axis, 19–20
3-con and 4-con nodes, 18–19
Index

Patchwork approximation (cont.)
  multiple intersection point, 7
  nodes, 9
  polycons, 8–9
  polynomials, 6–7
  rational basis functions, 11
  simple intersection point, 7
  spaces and convergence
    maximum chord length, 13
    piecewise continuity, 14
    properties, 13–14
  Ritz–Galerkin convergence analysis, 14
  3-con of order four, 15–16

wedges and pyramids
  approximation space, 3
  hat function, 4–5
  higher degree approximation, 6
  parallelogram element, 6, 10
  properties, 16–17
  rectangular element, 6
  Ritz–Galerkin type formulation, 3–4
  spline approximation, 4
  triangle basis functions, 5–6, 10

Plane curve intersection
  ambiguities, 66
  Bezout’s theorem, 66–67
  homogeneous polynomials, 65
  intersection number, 64
  mixed double contact, 67–68
  multiplicity of component, 63
  nonsingular, 63–64
  Sylvester’s dialytic method, 64–65

Plane curve intersection
  ambiguities, 66
  Bezout’s theorem, 66–67
  homogeneous polynomials, 65
  intersection number, 64
  mixed double contact, 67–68
  multiplicity of component, 63
  nonsingular, 63–64
  Sylvester’s dialytic method, 64–65

Polycondron
  ill-sets (see Ill-set polycondron)
  three-dimensional approximation
    exterior linear edge, 144
    order five, 145–146
    order seven, 147–148
    order six, 148–149
    wedge equation, 144

Polycons
  adjacent factor, 83–84
  adjoints, 88, 92
  algebraic geometry treatment, 86
  boundary curves, 85–86, 90–91
  denominator factor, 84–85
  denominator polynomial
    adjacent factors, 96
    concave sides, 97
    hyperbolic arcs, 104–105
    polynomial, 98–100
    regularity analysis, 97–98
    ring section, 103–104
    3-cons of order six, 96–97, 101–103
    well-set polycon, 100–101
  4-con of order six, 89
  multiple exterior intersection point, 87–88
  opposite factor, 83
  patchwork approximation, 8–9
  perturbation, 89–90
  properties, 93–96
  quadratic transformation, 90–92
  singularities, 92
  transverse intersection, 86–87

Polycubes
  order six, 112
  rational polypol, 110–111
  singular point, 111
  2-pol of order five, 112–113

Polygons
  adjacent factor, 216
  GADJ algorithm, 217
  quadrilateral, 34–36
  two dimensional analysis
    adjacent factor, 216
    areal coordinates, 215
    degree of approximation, 217–218
    GADJ recursion, 217–218
    generalized areal basis, 219
    singularity pattern, 216
    special adjoint, 216
    wedge function, 218–219

Polyhedron
  GADJ algorithm
    adjacent and adjacent, 236
    higher order vertices, 236
    planar figure, 236, 237
    polar dual construction, 236
    polynomial expression, 235, 237
    three-dimensional approximation, 142–144

Polyhedron (p-p)
  adjacent factor
    edge nodes, 155–156
    face nodes, 155
    interior nodes, 155
    vertex nodes, 156
  adjoint
    exterior edge deficit points, 151–152
    multiple points, 150
    p-p edge deficit points, 151
    surface deficit points, 150–151
  binomial expression, 134–135
  common component, 135
  degree k approximation, 157
  Euler’s equation, 133
  GADJ algorithm, 237–238
  geometric conditions, 133
  irreducible surface, 136
node placement, 154–155
order and vertex, 133–134
quadric-independent, 135
wedge regularity, 154
Polypols, 226–227
adjunct intersection points, 108–110
DIP
degrees of freedom, 107
EIP, 105–106
polycube, 105
wedge regularity, 107
well-set rational polypols, 107
higher degree approximation, 127–128
wedge numerators, 108–110
Polytopes, 238–239
Projective coordinates
algebraic geometry, 62–63, 70, 77
application, 29
barycentric coordinates, 29, 32
bilinear transformation, 31–33
coefficients, 30–31
Coxeter’s work, 29–30
homogeneous coordinates, 29
isoparametric coordinates, 31
Jacobian transformation, 31–33
principle of duality, 30
wedge integrals, 33–34
Rational wedges
congruences
degrees of freedom, 51–52
eclipse, 48–50
hyperbola, 48–50
intersection of conics, 50–51
4-con of order five
Bezout’s theorem, 45
coefficient matrix, 45–46
degree one approximation, 46
degrees of freedom, 44–45
denominator conic, 47
denominator curve, 44
EIP, 44, 46–47
properties, 46
pentagon, 48
polycons (see Polyccons)
polypols (see Polypols)
quadrilateral
areal coordinates, 27–28
exterior diagonal, 25–26
functions, 24
linear forms, 24–25
properties, 23–24
wedge surface, 26
sample 3-con, 56
3-con of order five and six, 53–54
circular boundary arcs, 52
construction symmetry, 53
EIP, 54–57
linearity, 55–56
three-sided elements, 57–59
3-con of order four, 43–44
circumference, 38
denominator polynomial, 38–39
homogeneous coordinates, 42
linear function, 41–42
quadrilateral wedges, 39–40
rational function, 37
reduced polynomial, 40–41
unit circle, 37–38
Quadrilateral
ill-set polycondron (see Nonconvex
quadrilaterals)
inadequacy of polynomials, 23–24
polygons, 34–36
projective coordinates
application, 29
barycentric coordinates, 29, 32
bilinear transformation, 31–33
coefficients, 30–31
Coxeter’s work, 29–30
homogeneous coordinates, 29
isoparametric coordinates, 31
Jacobian transformation, 31–33
principle of duality, 30
S
Segment quadrature
  accuracy, 174–175, 241–242
  degree two triangle, 176
  element partition, 175
  integration points, 177
  isoparametric parabola, 176–178
  median trisection points, 176
  rational wedges, 178
Simple intersection point, 7
Stencils, 229–232
  integral operation, 230–231
  modelling, 231–232
  normalization, 230
  planar diagram, 230
  Poisson kernel, 230, 231
T
Tesseract, 239–240
Tetrahedron, 136–138
Three-dimensional approximation
  hexahedron
    degree four, 141
    degree k, 142
    degree three, 140
    degree two, 140
    planar figure, 138–139
    wedge equation, 139–140
  polycondron
    exterior linear edge, 144–145
    order five, 146–147
    order seven, 148
    order six, 148–149
    wedge equation, 144
  polyhedron, 142–144
  polypoldron
    adjacency, 154–156
    adjoint, 150–154
    binomial expression, 134–135
    common component, 135
    degree k, 157
    Euler’s equation, 133
    geometric conditions, 133
    irreducible surface, 136
    node placement, 154–155
    order and vertex, 133–134
    quadric-independent, 135
    wedge regularity, 154
  tetrahedron
    Desargues’ theorem, 137–138
    planar figure, 137
    wedge equation, 136
Triangle averaging, 183–185
Triangulation, 131–132
Two-dimensional analysis
  adjoint positivity, 225–226
  mapping techniques, 232–233
  partitioning method, 229
  policons
    adjacent factors, 219–220, 222–223
    GADJ recursion, 219–220, 222
    higher degree approximation, 220
    linear pattern, 220–221
    well-set elements, 219, 224–225
  polypols, 226–227
  stencils
    integral operation, 230–231
    modelling, 231–232
    normalization, 230
    planar diagram, 230
    Poisson kernel, 230, 231
    well-set policons, 224–225
Two-level computation
  coarse mesh rebalancing, 211
  recapitulation, 207
  synthesis, 208–210
W
Walker’s theorem, 77–78