Appendix

The objective of this appendix is to prove identities (2.45), (2.46) and (2.47). Let us therefore represent the unit vector $\hat{e}_\omega$ in terms of the unit vectors linked to the Cartesian coordinate system $\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ in the form

$$\hat{e}_\omega = \alpha_x \hat{e}_x + \alpha_y \hat{e}_y + \alpha_z \hat{e}_z \quad \text{(A.1)}$$

where $\alpha_x, \alpha_y$ and $\alpha_z$ are the cosines of the angle between $\hat{e}_\omega$ and the corresponding axis directions respectively. Then the following relationship between the $\alpha$’s is established

$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1 \quad \text{(A.2)}$$

Evaluating the cross product $(\hat{e}_\omega \times X)$ by using Eq. (A.1) and $X = x \hat{e}_x + y \hat{e}_y + z \hat{e}_z$, and working out the inner product $(\hat{e}_\omega \times X) \cdot (\hat{e}_\omega \times X)$ yields

$$(\hat{e}_\omega \times X) \cdot (\hat{e}_\omega \times X) = \left(\alpha_x^2 + \alpha_z^2\right)x^2 + \left(\alpha_x^2 + \alpha_y^2\right)y^2 + \left(\alpha_x^2 + \alpha_y^2\right)z^2 - 2\alpha_x\alpha_y\alpha_z$$

Then applying the gradient operator on Eq. (A.3) one obtains

$$\frac{1}{2} \nabla[(\hat{e}_\omega \times X) \cdot (\hat{e}_\omega \times X)] = (x \hat{e}_x + y \hat{e}_y + z \hat{e}_z)$$

$$- (\alpha_x x + \alpha_y y + \alpha_z z)\left(\alpha_x \hat{e}_x + \alpha_y \hat{e}_y + \alpha_z \hat{e}_z\right) \quad \text{(A.4)}$$

It can be observed that the terms in the first brackets on the right hand side of Eq. (A.4) represent the vector $X$, the terms in the second brackets represent $(\hat{e}_\omega \cdot X)$.
and the terms in the last brackets stand for the unit vector $\hat{e}_{\alpha}$ according to Eq. (A.1). Therefore Eq. (A.4) becomes

$$\frac{1}{2} \nabla \left[ (\hat{e}_{\alpha} \times X) \cdot (\hat{e}_{\alpha} \times X) \right] = X - (\hat{e}_{\alpha} \cdot X) \hat{e}_{\alpha}$$  \hspace{1cm} (A.5)

It can be observed from Eq. (A.5) that the contribution of the term $-(\hat{e}_{\alpha} \cdot X) \hat{e}_{\alpha}$ is to cancel out the component of $X$ in the $\hat{e}_{\alpha}$ direction and to keep only the components of $X$, which are perpendicular to the axis of rotation.

To evaluate now the triple cross product $\hat{e}_{\alpha} \times (\hat{e}_{\alpha} \times X)$ we use the following vector identity, which is valid for any three vectors $A, B$ and $C$, i.e. $: A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$. Then choosing $A = B = \hat{e}_{\alpha}$ and $C = X$ yields

$$\hat{e}_{\alpha} \times (\hat{e}_{\alpha} \times X) = (\hat{e}_{\alpha} \cdot X) \hat{e}_{\alpha} - X$$  \hspace{1cm} (A.6)

By comparing Eqs. (A.6) with (A.5) we obtain

$$\hat{e}_{\alpha} \times (\hat{e}_{\alpha} \times X) = -\frac{1}{2} \nabla \left[ (\hat{e}_{\alpha} \times X) \cdot (\hat{e}_{\alpha} \times X) \right]$$  \hspace{1cm} (A.7)

Equations (A.6) and (A.7) represent the required proof of identities (46) and (47), respectively.

For proving identity (45) let us represent the unit vector $\hat{e}_g$ in terms of the unit vectors linked to the Cartesian coordinate system $\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ in the form

$$\hat{e}_g = \theta_x \hat{e}_x + \theta_y \hat{e}_y + \theta_z \hat{e}_z$$  \hspace{1cm} (A.8)

where $\theta_x, \theta_y$ and $\theta_z$ are the cosines of the angle between $\hat{e}_g$ and the corresponding axis directions respectively. Then the following relationship between the $\theta$'s is established

$$\theta_x^2 + \theta_y^2 + \theta_z^2 = 1$$  \hspace{1cm} (A.9)

Evaluating the dot product $(\hat{e}_g \cdot X)$ by using Eq. (A.8) and $X = x \hat{e}_x + y \hat{e}_y + z \hat{e}_z$ yields

$$\hat{e}_g \cdot X = \theta_x x + \theta_y y + \theta_z z$$  \hspace{1cm} (A.10)

Taking the gradient of Eq. (A.10) produces

$$\nabla (\hat{e}_g \cdot X) = \theta_x \hat{e}_x + \theta_y \hat{e}_y + \theta_z \hat{e}_z = \hat{e}_g$$  \hspace{1cm} (A.11)

Equation (A.11) represents the required proof of identity (2.45).
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