References

Index

A
Absolute, 68
Absolute between $V$ and $M$, 68
Absoluteness of well-foundedness, 69
Accumulation point, 4
Ackermann’s set theory, 20
AD, 280
$a \Delta b$, 87
AD$_R$, 301
$\aleph_\alpha$, 34
Almost disjoint, 7
Amenable, 73
$A^\alpha_{M^\beta}$, 250
Antichain, 95
maximal, 95
$A^\beta_M$, 237
Approximation property, 122
AST, 20
At most countable, 3
Axiom
of choice, 12
of extensionality, 9
of foundation, 9, 30
of infinity, 10
of pairing, 10
of power set, 10
of replacement, 11
of separation, 11
of union, 10
Axiom of Determinacy, 280
Axiom of Real Determinacy, 301

B
Baire Category Theorem, 6
Baire space, 127
Banach-Mazur game, 287

Base, 288
Bernstein property, 155
$\Delta^2_1$, 63
BG, 18
BGC, 19
Bijective, 14

C
Cantor normal form, 30
Cantor space, 127
Cantor’s Discontinuum, 8
Cantor–Bendixson
Theorem of, 5
Cantor–Bendixson rank, 144
Cantor–Schröder–Bernstein
Theorem of, 2
Cardinal, 33
$\alpha$-Erdös, 230
$\alpha$-strong, 58
$\lambda$-supercompact, 59
(strongly) Mahlo, 46
(strongly) inaccessible, 46
ineffable, 50
limit, 33
measurable, 53
regular, 36
Reinhardt, 52
remarkable, 232
singular, 36
strong, 58
strong limit, 46
strong up to $\delta$ with respect to $A$, 227
subcompact, 61
subtle, 90
successor, 33
supercompact, 59
threadable, 277
weakly compact, 50
weakly inaccessible, 46
weakly Mahlo, 46
with the tree property, 50
Woodin, 227
Cardinal successor, 33
Cardinality, 33
Cartesian product, 13
CH, 36
Closed, 40
\(\delta\), 40
in \(\Theta^{<\kappa}\), 45
Closure, 128
Club
\(\delta\)-in \(\alpha\), 40
in \(\Theta^{<\kappa}\), 45
in \(\alpha\), 40
Club filter, 41
Cofinal, 36, 239
Cofinality, 36
Cohen forcing, 94
at \(\kappa\), 109
Collection principle, 30
Compatible, 93
Comprehension
\(\Sigma_0\), 73
axiom for \(\varphi\), 19
schema, 19
Condensation
full, 81
local, 81
Condensation lemma, 80
Condensation point, 5
Cone
\(S^-\), 288
Turing, 288
Constructible universe, 74
Continuum Hypothesis, 3, 36
Continuum Problem, 3
Countable, 3
Countable chain condition, 105
Covering game, 286
Critical point, 50
crit(\(\pi\)), 50

D
Decides
\(p - \varphi(\tau_1, \ldots, \tau_k)\), 98
\(\Delta\)-Lemma, 106
\(\Delta^+\), 68
\(\Delta\)-system, 106

Dense, 93
below \(\rho\), 93
Dependent choice, 31
Determinacy
\(\text{OD}_{\delta}\), 291
\(\diamond\), 82
\(\diamond_\kappa\), 82
\(\diamond_\kappa(R)\), 82
\(\diamond^+\), 90
\(\diamond^+(R)\), 90
\(\diamond^\kappa\), 83
\(\diamond^\kappa(R)\), 83
\(\diamond^\kappa\), 83
\(\diamond^\kappa(\mathcal{R})\), 275
Distance, 127
Domain, 13
Downward absolute, 68

E
Elementary embedding, 50
non-trivial, 50
Embedding normal form, 303
\(\kappa\)-closed, 303
additivity, 307
Extender, 210
\(\lambda\)-closed, 217
\(\lambda\)-complete, 219
continuum–complete, 220
countably complete, 220
derived from \(\pi, E_\pi\), 211
long, 210
short, 210
strength of an, 216
Extension of embeddings lemma
downward, 240
general downward, 253
general upward, 253
upward, 243
Extensional, 28

F
Filter, 40, 93
\(<\mu\)-closed, 40
Fréchet, 41
normal, 41
nowhere dense, 180
Ramsey, 180
rapid, 165
selective, 180
uniform, 180
weakly normal, 59
Index

Finite, 3
Forces, 97
Forcing
  Mathias, 181
  Namba, 276
  notion of, 93
  Prikry, 188
  supercompact tree Prikry, 233
Forcing conditions, 93
Formula
  \( \Pi_n \), 67
  \( \Sigma_0 \), 67
  \( \Sigma_n \), 67
  \( P^+ \), 42
Fullness, 101, 123
Function
  rudimentary (in \( E \)), 70
  simple, 72, 296
  uniformizing, 139

G
Gödel pairing function, 35
GCH, 36
Generalized Continuum Hypothesis, 36
Generic
  \( \mathcal{P} \)-over \( M \), 94
  \( \mathcal{P} \), 93
Generic extension, 96
Good embedding, 243

H
Hausdorff formula, 38
HC, 38
Height of a tree \( T \), 46
Hereditarily in, 86
Hereditarily ordinal definable from \(-\), 86
Hereditarily smaller than \( \kappa \), 38
HF, 38
\( H_\kappa \), 38
\( h_M \), 193
\( h_M^{\kappa+1} \cdot \mathcal{P} \), 252
HOD, 86
HOD\(), 86
Homomorphism, 113
dense, 113

I
Ill–founded, 26
Inaccessible to the reals
  \( \omega_1 \) is \(-\), 141
Incompatible, 93

Induction principle, 23, 26
Inductive, 10, 23
Ineffable, 50
Injective, 14
Inner model, 51
Interpretation
  \( G \)-of \( \tau \), 96
Invariant
  \( S \), 288
  Turing–, 288
Isomorphic, 15
Isomorphism, 15
Iterability
  of a ppm \( \mathcal{M} \), 197
Iterable by \( U \) and its images, 184
Iteration
  linear – of \( V \) of length \( \gamma \) given by \( U \), 183
  of a ppm \( \mathcal{M} \) of length \( \alpha \), 197
  putative – of a ppm \( \mathcal{M} \) of length \( \alpha \), 196
  putative linear – of \( V \) of length \( \gamma \) given
  by \( U \), 183

J
\( J_\alpha \), 74
\( J_\alpha [E] \), 73
\( J \)-structure, 79
\( J \)-structure
  acceptable, 235

K
\( \kappa \)-chain condition, 105
\( \kappa \)-Knaster, 105
KP, 90
Kripke–Platek set theory, 90
Kurepa’s Hypothesis, 90

L
\( L \), 74
\( \lambda^{<\kappa} \), 37
\( L [E] \), 73
Lebesgue measurable, 148
Length (of a well–ordering), 29
\(<\kappa\)-closed, 109
\(<\kappa\)-distributive, 109
\(<\mathcal{M}, 65
Level of \( s \) in \( T \), 46
\( \lfloor \nu_T (s) \rfloor \), 46
Levy collapse, 111
Limit point, 40
\( \text{Łoś Theorem}, 54, 195, 213 \)
M
Measure, 53
Mitchell order, 65, 233
$M^{n,p}$, 250
Monotone enumeration, 29
Mostowski collapse, 28
Mouse
$x$, 198
$M^p$, 237
Mutual generics, 119

N
Names
$P$, 96
$n$-completion, 254
$n$-embedding, 259
Nice name, 108
Norm on $A \subset \omega^\omega$, 145
Null set, 7, 147
Number
natural, 23
ordinal, 24

O
OD, 86
OD$, 86
$\omega$-complete, 197
$\omega^1$, 91
Open, 95
$x \oplus y$, 133
Order
atomless, 95
linear, 14
partial, 14, 93
separable, 115
Order type, 29
Order-preserving, 15
Ordered pair, 13
Ordinal, 24
$\mathbf{z}$-admissible, 91
limit, 25
successor, 25
Ordinal definable from $-$, 86
Opt, 29
Outer measure, 147

P
P-point, 180
Parameters
good, 237, 251
very good, 238, 251

PD, 307
Perfect subset property, 142
Pigeonhole Principle, 33
$\pi^M_U$, 57
$P_M$, 237
$P''_M$, 251
Polish space, 127
Positive sets, 42
Power set, 2
$\mathcal{P}(X)$, 2, 10
Premouse
$x$, 194
Prikry sequence, 189
Product, 118
Product Lemma, 118
Projection of T, 128
Projective Determinacy, 307
Projectum
$\Sigma_1$, 236
$n$th, 250

Q
Q-point, 180
Q-formula, 238

R
Range, 13
Rank, 28, 30
Rank initial segment of $V$, 28
$\text{rk}_R(x)$, 28
$\| x \| _R$, 28
Real
Cohen over $M$, 152
dominating, 181
generic over $M$, 153
random over $M$, 152
unbounded, 181
Recursion theorem, 26
Reduct, 237
$n$th, 250
Reduction property, 141
Reflection Principle, 89
Regressive, 42
Relation, 13
set-like, 26
Reshaped subset of $\omega_1$, 163
Restriction, 14
$\rho_n(M)$, 250
$\rho_1(M)$, 236
Rigid, 202
$R_M$, 238
$R''_M$, 251
$\mathbf{r} \Sigma_{n+1}$ elementary, 254
rud$_E$ closed, 72
Rudimentary relation, 71
Russell’s Antinomy, 12

S
$S_{\omega}[E]$, 75
Scale on $A \subseteq \omega^\omega$, 145
$\text{SCH}$, 39
Sequence, 29
Set
  admissible, 91
determined, 280
Set of reals
  $F_\alpha$, 128
  $G_\alpha$, 128
  $\Delta^1_1(x)$, 137
  $\Pi^1_1(x)$, 137
  $\Sigma^1_1(x)$, 137
  $\Sigma^1_{n+1}(x)$, 137
  $\alpha$–Souslin, 131
  $\Pi^1_n$, 132
  $\sim^1_n$, 132
  $\sim^1_{n+1}$, 132
  $\Sigma^1_n$, 132
  $\Sigma^1_{n+1}$, 132
  $\kappa$–universally Baire, 149
  $\Delta^1_n$, 135
  analytic, 132
  basic open, 127
  Borel, 128
closed, 3, 127
coanalytic, 132
  complete coanalytic, 134
  $\delta$–homogeneously Souslin, 323
  $\delta$–weakly homogeneously Souslin, 323
dense, 4
determined, 280
  homogeneously Souslin, 323
meager, 6, 148
nowhere dense, 6, 148
  of first category, 6
  of second category, 6
open, 3, 127
perfect, 4
projective, 132
small, 174
Solovay over $M$, 153
universal $\Sigma^1_1$, 133
universally Baire, 149
  weakly homogeneously Souslin, 324
Shift map, 186, 198

Shoenfield tree, 131
$\sigma$–algebra, 128
Silver indiscernibles for $L[x]$, 209
Singular Cardinal Hypothesis, 39
Skolem function
  $\Sigma_1$–, 193
Solid, 258
  1–, 258
  $n$–, 258
Solidity witness, 256
Solovay game, 289
Solovay sequence, 301
  length of, 302
Sound, 253
  $n$–, 253
$\square^+_\kappa$, 269
$\square^\Box_\kappa$, 269
Standard code, 237
$n^{	ext{ith}}$, 250
Standard parameter, 255
  $n$th, 254
Standard reduce
  $n$th, 254
Standard witness, 257
Stationary, 41
Stationary set
  reflecting, 278
Steel forcing, 295
Stem
  of a Prikry condition, 188
Strategy, 279
Subset, 9
  proper, 9
Support, 105, 110
Surjective, 14
Symmetric difference, 87

T
$\text{TC}([x])$, 27
$||R||$, 28
Transitive, 23
Transitive closure, 27
Transitive collapse, 28
Tree, 46
  $\kappa$–, 47
  $\kappa$–Aronszajn, 47
  $\kappa$–Kurepa, 47
  $\kappa$–Souslin, 47
on $\omega \times \alpha$, 128
  on $X$, 127
  perfect, 127
Turing reducible, 288
$2^{<\kappa}$, 37
U
ult($V; U$), 57
Ultrafilter, 40
Ultrapower
  $\Sigma_0^-$, 194, 216
  $r\Sigma_{n+1}^-$, 262
Ultrapower embedding, 57
Ultrapower map
  $\Sigma_0^-$, 195, 216
  $r\Sigma_{n+1}^-$, 262
Ultrapower of $M$ by $U$, 57
Unbounded
  in $[\theta]^\kappa$, 45
  in $\gamma$, 40
Uncountable, 3
Upward absolute, 68

V
$V_\alpha$, 28
Ville’s Lemma, 91

W
Weakly $r\Sigma_{n+1}$ elementary, 254
Well-founded, 26
Well-founded part, 26
Well-ordering, 15
Wfp($B$), 26
Windus tree, 304
Winning strategy, 280

X
$x^\#$, 202

Z
$Z$, 12
$ZC$, 12
$0^\#$, 202
$ZF$, 12
$ZFC$, 12
$ZFC^-$, 12
$ZFC^{-\infty}$, 12
Zorn’s Lemma, 16