Appendix A: Dyadics

In theoretical physics, linear operators in vector spaces are usually defined in terms of tensors and their corresponding matrices. Dyadics provide a convenient alternative in the three-dimensional space. It is possible to establish a one-to-one correspondence between tensor and dyadic spaces. The main advantage of the dyadic formalism is that dyadics are built up from vectors which often have a clear physical meaning. Using vector algebra, many powerful theorems of dyadic algebra can be established [1], and some of them have no simple equivalent in tensor algebra. Another advantage is that this formalism is coordinate independent.

Definitions and dyadic algebra

Dyad is a pair of vectors: \( ab \) (which is not the same as \( ba \)). Dyadic is a linear combination (introduced formally on this stage) of dyads:

\[
\overline{A} = ab + cd + ef + \ldots
\]

Multiplication by a scalar \( \alpha \) is defined as

\[
\alpha \overline{A} = (\alpha a)b = a(\alpha b)
\]

(17)

Addition satisfies

\[
ab + cb = (a + c)b, \quad ab + ac = a(b + c)
\]

(18)

Scalar multiplication by a vector is defined by:

\[
\overline{A} \cdot \mathbf{x} = (ab + cd + ef + \ldots) \cdot \mathbf{x} = a(b \cdot \mathbf{x}) + c(d \cdot \mathbf{x}) + e(f \cdot \mathbf{x}) + \ldots
\]

(19)

\[
\mathbf{x} \cdot \overline{A} = (\mathbf{x} \cdot a)b + (\mathbf{x} \cdot c)d + (\mathbf{x} \cdot e)f + \ldots
\]

(20)

This operation defines a linear operator acting on vectors:

\[
g = \overline{A} \cdot \mathbf{x}
\]

(21)
Vector multiplication by a vector is defined through
\[
\overrightarrow{B} = \overrightarrow{A} \times \overrightarrow{z} = (ab + cd + ef + \ldots) \times \overrightarrow{z} = a(b \times \overrightarrow{z}) + c(d \times \overrightarrow{z}) + e(f \times \overrightarrow{z}) + \ldots
\]
(22)
\[
\overrightarrow{C} = \overrightarrow{z} \times \overrightarrow{A} = \overrightarrow{z} \times (ab + cd + ef + \ldots) = (z \times a)b + (z \times c)d + (z \times e)f + \ldots
\]
(23)
Transposition simply changes the order of vectors in pairs:
\[
\overrightarrow{A}^T = (ab + cd + ef + \ldots)^T = ba + dc + fe + \ldots
\]
(24)
In a given basis, every dyadic can be uniquely expressed in terms of pairs of basis vectors \(e_i\) \((i = 1, 2, 3)\):
\[
\overrightarrow{A} = A^{ij}e_i e_j = A^{11}e_1 e_1 + A^{12}e_1 e_2 + \cdots + A^{33}e_3 e_3
\]
(25)
Nine coefficients \(A^{ij}\) form the matrix of the dyadic in this coordinate frame. It can be shown that coefficients \(A^{ij}\) transform as components of a second-rank tensor. This way we establish correspondence between tensors, matrices, and dyadics.

**The unit dyadic. Symmetric and antisymmetric dyadics**

By definition, the unit dyadic corresponds to the identity operator:
\[
\overrightarrow{I} \cdot \overrightarrow{a} = a
\]
(26)
for all vectors \(\overrightarrow{a}\). For example, in Cartesian coordinates
\[
\overrightarrow{I} = x_0 x_0 + y_0 y_0 + z_0 z_0
\]
(27)
From this representation it is obvious that the unit dyadic is symmetric: \(\overrightarrow{I}^T = \overrightarrow{I}\), thus, for any \(\overrightarrow{a}\) we have \(\overrightarrow{a} \cdot \overrightarrow{I} = \overrightarrow{I} \cdot \overrightarrow{a} = a\).

Dyadics for which \(\overrightarrow{A} = -\overrightarrow{A}\) are called antisymmetric dyadics. Arbitrary dyadic can be uniquely decomposed into symmetric and antisymmetric parts:
\[
\overrightarrow{A} = \frac{1}{2}(\overrightarrow{A} + \overrightarrow{A}^T) + \frac{1}{2}(\overrightarrow{A} - \overrightarrow{A}^T) = \overrightarrow{A}_s + \overrightarrow{A}_a
\]
(28)
Furthermore, any antisymmetric dyadic can be written as a vector product of a vector and the unit dyadic:
\[
\overrightarrow{A}_a = \overrightarrow{v} \times \overrightarrow{I}
\]
(29)
You might note that the basic theory of dyadics develops in parallel with the tensor or matrix algebra.
References

Appendix B: Reciprocity theorem

Suppose that there are two source currents $J_1$ and $J_2$ in a medium described by complex tensor parameters $\varepsilon(r)$, $\mu(r)$ which generate electromagnetic fields $E_1$, $H_1$ and $E_2$, $H_2$, respectively. Because of the linearity of the Maxwell equations we can write equations for these two fields separately:

\[
\nabla \times E_1 = -j\omega \mu \cdot H_1 \quad (30)
\]
\[
\nabla \times H_1 = j\omega \varepsilon \cdot E_1 + J_1 \quad (31)
\]
\[
\nabla \times E_2 = -j\omega \mu \cdot H_2 \quad (32)
\]
\[
\nabla \times H_2 = j\omega \varepsilon \cdot E_2 + J_2 \quad (33)
\]

Let us multiply the equations in the first set by the field vectors from the second set:

\[
H_2 \cdot \nabla \times E_1 = -j\omega H_2 \cdot \mu \cdot H_1 \quad (34)
\]
\[
E_2 \cdot \nabla \times H_1 = j\omega E_2 \cdot \varepsilon \cdot E_1 + E_2 \cdot J_1 \quad (35)
\]

Similarly,

\[
H_1 \cdot \nabla \times E_2 = -j\omega H_1 \cdot \mu \cdot H_2 \quad (36)
\]
\[
E_1 \cdot \nabla \times H_2 = j\omega E_1 \cdot \varepsilon \cdot E_2 + E_1 \cdot J_2 \quad (37)
\]

Next, we subtract (37) from (35):

\[
E_2 \cdot \nabla \times H_1 - E_1 \cdot \nabla \times H_2 = j\omega (E_2 \cdot \varepsilon \cdot E_1 - E_1 \cdot \varepsilon \cdot E_2) + E_2 \cdot J_1 - E_1 \cdot J_2 \quad (38)
\]

An important step here: If $\varepsilon$ is a symmetric matrix, that is, it equals to its transpose, $\varepsilon = \varepsilon^T$, then $E_2 \cdot \varepsilon \cdot E_1 - E_1 \cdot \varepsilon \cdot E_2 = 0$ in the right-hand side. Consider symmetric matrices $\varepsilon$ and $\mu$. Subtracting (36) from (34) we get, using the symmetry of $\mu$,

\[
H_2 \cdot \nabla \times E_1 - H_1 \cdot \nabla \times E_2 = 0 \quad (39)
\]

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Next, we sum up (38) and (39) which yields

\[ \mathbf{E}_2 \cdot \nabla \times \mathbf{H}_1 - \mathbf{E}_1 \cdot \nabla \times \mathbf{H}_2 + \mathbf{H}_2 \cdot \nabla \times \mathbf{E}_1 - \mathbf{H}_1 \cdot \nabla \times \mathbf{E}_2 = \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2 \]  

(40)

Note that the underlined terms together give

\[ \mathbf{E}_2 \cdot \nabla \times \mathbf{H}_1 - \mathbf{H}_1 \cdot \nabla \times \mathbf{E}_2 = \nabla \cdot (\mathbf{H}_1 \times \mathbf{E}_2) \]  

(41)

Also, the other two terms combine as

\[ \mathbf{H}_2 \cdot \nabla \times \mathbf{E}_1 - \mathbf{E}_1 \cdot \nabla \times \mathbf{H}_2 = \nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2) \]  

(42)

Finally we integrate (40) over an arbitrary volume \( V \) bounded by surface \( S \). The result is, after applying the Gauss theorem,

\[ \int_S (\mathbf{H}_1 \times \mathbf{E}_2 + \mathbf{E}_1 \times \mathbf{H}_2) \cdot d\mathbf{S} = \int_V (\mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2) \, dV \]  

(43)

This relation is called the Lorentz lemma.

Consider the limiting case when the integration volume is the whole space. Then, assuming even negligible losses which are always present, we conclude that the surface integral vanishes, and the result is the reciprocity theorem

\[ \int_V (\mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2) \, dV = 0 \]  

(44)

Note that the only condition for its validity (besides the Maxwell equations) is the symmetry of matrices \( \mathbf{\sigma} \) and \( \mathbf{\mu} \). Otherwise, the system can be lossy or lossless, homogeneous or inhomogeneous.
Appendix C: Description of Matlab programs

The modified Marcatili’s and Goell’d methods for calculation of dispersion characteristics of rectangular open dielectric waveguides with uniaxial cores with the axis parallel to the waveguide axis are provided on disk. The algorithms, described in detail in Chapter 5, Sections 5.3.1 and 5.3.2, have been realized in Matlab programming language.

In the corresponding directories there are the files named

marcatili.m
Goell.m

that are the main files calling some functions. Comments inside these files explain how to modify the input data for specific waveguides and frequency ranges. Programs have been checked to be working with Mat-Lab versions 5.3 and 6.5.
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