

Index

A

Actuators, 314
Anti-resonant, 18, 124, 150–153, 252, 254

B

Bar, 2, 3, 7, 8, 13–16, 19, 20, 28, 31, 33, 58, 83, 87, 89, 90, 92, 93, 108–111, 113–118, 183, 184, 187–191, 193, 194, 196–212, 272, 274, 275, 279, 280, 303, 340, 356, 371, 377, 378
Beam, 33, 58, 76, 83, 272, 274, 278–280, 291–293, 295, 300, 301
 curved, 371, 377, 378
 Euler–Bernoulli, 1, 30, 225, 377
 HR-FE, 156–159
 multi-span, 20, 119, 160, 161, 167–171
 overhang, 19, 170–175, 177–180
 Rayleigh, 7, 377
 three-span overhang, 256, 257, 262, 263, 265, 266
 Timoshenko, 377
 two-span overhang, 256–258, 262, 265, 266, 268, 269
Binet–Cauchy identity, 35, 45, 51
Block Lanczos algorithm, 153
Boundary conditions, 90–92, 94, 96, 101, 104–106, 111, 119, 121, 124, 125, 127, 130, 133, 134, 142, 152, 173, 184–186, 188, 191, 197, 198, 204, 212, 216–221, 224, 226, 227, 230, 232, 234, 235, 237–240, 243, 245, 247, 251–253, 256, 262
Boundary constraints, 250

C

Characteristic equation, 252, 337
Characteristics of nodes, 6, 15, 18
Chebyshev functions, 64, 65, 74
Chebyshev vectors, 78, 79
Classical derivatives, 332, 340
Classical solution, 25, 330, 332–334, 351, 352, 380
Compatibility, 15, 31, 203, 204
Compatibility conditions, 107, 108
Compatible, 378
Conjugate bar, 205, 207
Conjugate beam, 119, 129–131, 137, 215, 232, 237, 238, 264, 265
Conjugate system, 7, 98, 102, 103
Continuity conditions, 274, 275, 285, 319–321
Continuous model, 156
Continuous object, 78, 83
Continuous system, 71, 72, 75–77, 82, 84, 87–89, 91, 92, 116, 120, 154, 155, 157, 158, 196, 203, 204, 212, 215, 217, 224, 248, 252, 255, 269, 271, 272, 281, 283, 296, 303, 307, 308, 347
 of a bar, 14–16, 206
 of a beam, 12, 19
Control, 25
 force, 314, 315
 matrix, 315
 vibration, 271, 313, 314, 318
Controllability, 314

D

Differential operator, 217, 224, 239
Dimension reduction, 313, 314, 318

- Discrete model, 8, 12, 87, 90, 115, 116, 175, 279, 280
 physical, 120
- Discrete system, 3, 5–8, 12, 16, 19–22, 24, 25, 57, 71, 76–79, 82–84, 87–90, 117, 118, 129, 138–142, 147, 154, 155, 167, 169, 170, 188, 203, 209, 212, 221, 237, 279, 281, 296, 299, 303, 306–308, 312
- Displacement, 72, 76, 79, 85, 88, 95, 96, 98, 101, 106, 108–110, 113–116, 124, 128–130, 133, 135, 136, 138, 141–143, 147, 151, 155, 158, 160, 168, 171, 173, 178, 179, 183, 187, 198, 199, 204, 207, 208, 219, 229, 235, 237, 238, 240, 245, 248, 253, 258, 260, 261, 264, 266
 admissible, 335, 384
 comparison, 356, 358
 generalized, 28, 274, 279–282, 285, 296, 306, 312–315
- Distinctiveness of natural frequencies, 6, 14, 17, 231, 237–239, 263
- E**
- Eigenfunction, 22–24, 58, 61, 63, 70, 85, 185, 217, 218, 240, 242, 253, 263, 267, 268, 344–346
- Eigenpair, 41, 53, 56, 57, 63, 64, 69, 95, 117, 250, 336, 344, 387
- Eigenvalue, 8, 11, 22–24, 40–43, 49–58, 60, 61, 63, 70, 79, 80, 82, 83, 85, 94, 106, 108, 113, 127, 169–171, 185, 186, 217, 218, 250, 262, 263, 273, 276, 278, 282, 284, 286, 298, 299, 303–305, 307, 312, 324, 330, 336, 344–346, 382, 387
- Eigenvector, 40–43, 49, 50, 52–56, 79, 80, 101, 141, 151, 169–171, 298
- Elastic foundation, 7, 15, 16, 89, 90, 117, 118, 184, 187, 188
- Elastic solid, 272, 274, 280, 311
- Existence of solutions
 in Elasticity, 3, 25, 327, 328
 in structural theories, 2, 4, 9, 12, 13, 28, 327–329
 of static deformation, 1, 31
 of static deformation and vibrational modes, 327, 328, 349, 352, 356, 364, 379
- Existence theorem
 for modal solutions in Elasticity, 351
 for modal solutions in structural theory, 363
 for the solution of static deformation in Elasticity, 350
 for the solution of static deformation in structural theory, 362
- F**
- Family of Chebyshev functions, 64, 65, 74
- Finite difference, 7, 8, 16, 19, 119–122, 124, 125, 127–130, 142, 147, 148, 153, 154, 160, 161, 166, 167, 190, 193, 212, 221, 222, 226, 248, 303
- Finite difference model, 83, 92, 108, 109, 118
- Finite element, 119, 153, 155, 156, 158, 159, 193, 212, 280, 281
 method, 16, 383
 model, 109–111, 113, 114
 system, 7, 20, 21, 76
- Flexibility coefficients, 83, 84, 116
- Forced vibration, 4, 25, 109, 151, 209, 253, 271, 312, 313
- Frame
 planar, 291, 292
 rigid, 291
- G**
- Generalized derivative, 332, 339–341, 383
- Generalized matrix eigenvalue problem, 279, 282, 283, 299, 307, 308
- Generalized solution, 27, 30, 331, 332, 339–341, 343–345, 348, 351, 362, 363, 379–383
- Green's function, 5, 7, 31, 57, 58, 71, 76, 82, 83, 185–188, 191–193, 196, 215, 218–222, 224–227, 262, 266
 harmonic dynamic, 373–375
 static, 374, 376
- H**
- Hermite shape function, 7, 158
- I**
- Integral equation, 3, 57, 58, 60, 63, 64, 69, 70, 192, 193, 195, 225, 240, 242, 262, 263, 266, 328
- Interlacement
 of a bar, 14
 of a beam, 17
 of natural frequencies, 96, 149–151, 209, 211, 252–254
 of nodes, 6, 14, 17, 101, 243
- Inverse problems in vibration, 2
- K**
- Kernel
 compound, 60–63, 70
 iterated, 61, 62
 Kellogg, 70, 83–85

- oscillatory, 5–7, 57–60, 64, 69–72, 75, 76, 79, 82, 83, 85, 193, 195, 215, 225, 226, 231, 239, 240, 242, 257, 258, 262, 263, 265, 266
- symmetric, 57, 58
- Kinetic energy, 111
 - coefficient, 27, 28, 359, 361
 - magnitude, 27, 28, 349, 353
- M**
- Matrix
 - complex, 297
 - compound, 50, 51, 53
 - conjugate, 287
 - differential operator, 285
 - flexibility, 5–7, 71, 76, 78, 79, 81–84, 157
 - flexibility matrix, 116, 221, 223, 226
 - Gram determinant of the matrix, 385, 386
 - identity, 276, 277, 286, 287, 297, 304, 316
 - Jacobian, 37, 38, 40–42, 48, 49, 94–96, 98
 - mass, 20, 21, 46, 79, 82, 83, 90, 116, 124, 151, 155, 158, 159, 162, 281, 297, 303, 314
 - minor of the matrix, 35
 - nonnegative, 34–37, 43, 44, 46–49, 59, 79, 81, 114
 - orthogonal, 281, 304
 - oscillatory, 5, 43, 44, 46–49, 53, 56, 58–60, 78, 82–84, 113, 114, 117, 177
 - positive, 34, 35, 43, 48, 53–55
 - row-permutation, 287
 - sign-oscillatory, 46, 47, 55, 56, 83, 95, 101, 126–128, 164, 168, 169
 - sign-reverse, 36, 46, 55, 57, 125, 126, 164, 167, 176
 - stiffness, 16, 22, 24, 79, 83, 90, 95, 96, 126, 127, 154, 156–158, 161, 162, 164, 166–169, 174–177, 221, 222, 226, 281, 303, 314, 367
 - totally nonnegative, 34–37, 43, 44, 46–49, 59, 79, 81
 - totally positive, 34, 35, 43, 45, 48, 53–55
 - transformation, 276
 - tridiagonal matrix, 94
 - unitary, 287
- Membrane, 328, 340, 372, 377, 381, 383
- Minimization problem, 333, 384, 387
- Modal equation, 88, 94, 100, 103–107, 114, 119, 122, 123, 125, 129, 130, 132, 135–137, 139, 140, 142, 147, 148, 161, 162, 164, 167, 172, 173, 184, 197, 205, 207, 217, 218, 225, 234, 237, 241–243, 249, 264, 273, 285–287, 289, 290, 296, 297, 306, 308
- Modal solution, 209
 - in Elasticity, 28
 - in the structural theory, 30
- Modal superposition, 81, 151
- Mode, 82, 142, 147, 148, 203–205, 212
 - independent, 8, 15, 18, 107, 215, 245
 - normal, 1
 - rigid-body, 107
 - vibrational, 1–4, 8–13, 21, 24–26, 31, 271, 272, 278, 279
- Models in structural theories, 1, 31, 329, 364, 365
- Mode shape, 33, 71, 183, 196, 198–200, 202–206, 210, 212, 336, 345–347, 352, 353, 363, 373, 382
 - antisymmetric, 276–279, 283, 311
 - complex, 293, 297
 - components, 286, 297, 299, 306
 - deformation, 101
 - displacement, 14, 17–20, 94, 95, 98–101, 104–106, 108, 115, 130, 140, 141, 178, 240–244, 256, 263, 264
 - of the angle of rotation, 17, 130, 179, 239, 267, 268
 - of the bending moment, 17, 130–132, 135, 172, 178, 180, 238, 264, 266, 268
 - of the displacement, 17, 95, 98–101, 106, 139, 245, 268
 - of the entire system, 277
 - of the shear force, 17, 130, 180, 234, 268, 269
 - of the spring deformation, 98, 99, 102, 104
 - of the spring force, 98, 102, 103
 - strain, 8, 14–16, 199, 206, 208
 - symmetric, 277–279, 283
- Movable points, 116, 185, 196
- N**
- Necessary condition, 7, 79, 135, 142, 159, 199, 204, 232
- Nodal
 - displacement, 129, 156, 159
 - distribution, 2
 - force, 154, 155, 159
 - line, 4, 11, 12
 - properties, 2
 - surface, 4
- Node, 197–200, 207, 208
 - count, 6, 10, 14, 15, 17–20, 95, 170, 206, 245, 264, 269
 - movable, 76
 - null anti-node, 19, 20, 64, 66, 68, 69, 178, 259, 261, 263, 264, 267
- Normality, 210

Number of independent modes, 8, 15, 18, 197, 203, 215, 245
 Number of sign reversals, 5, 10, 14, 38–40, 53, 74, 77–79, 81, 99, 100, 102, 114, 128, 133, 134, 138, 140, 141, 154, 179, 180, 199, 206, 229–236, 241, 243, 258–261, 266

O

Observability, 314

Operator

bi-Laplacian, 335
 boundary condition, 273, 284
 differential, 274, 277, 330, 334, 335, 341, 348, 353, 372
 differential operator in the structural theory or Elasticity, 330
 differential operator of boundary conditions, 330
 elasticity, 26, 27, 29–31, 348–351, 353, 358, 362–364, 369, 380, 387
 elasticity differential, 273
 embedding, 344, 345, 348, 350, 385, 387
 energy embedding, 27, 29–31, 328, 349, 350, 353, 357–359, 362, 363, 365, 369, 380, 387
 inertia, 330
 inertia differential, 273, 284
 Laplace, 335, 381
 linear, 309
 positive, 332, 333, 336, 354
 positive definite, 332, 338, 341, 344, 347, 384
 self-adjoint, 332
 structural theory, 28, 29, 353, 357, 358, 362, 364, 365, 380
 symmetric, 332
 Orthogonality, 106, 210, 324
 Orthogonality condition, 345–347
 Orthogonal set, 28, 30, 344, 351, 363
 Orthogonal system, 58
 Oscillatory matrix, 195
 Oscillatory properties
 in static deformation, 5–7, 71, 72, 76, 77, 79, 83, 96, 153–155, 159, 196, 215, 226, 257
 in vibration, 5–7, 12, 16, 18, 21, 71, 79, 82, 83, 93, 108, 114, 117, 128, 154, 155, 158, 159, 193, 197, 207, 208, 231, 232, 237
 Oscillatory system, 5

P

Perron's theorem, 49, 50, 53, 60, 61, 70

Plate

Mindlin, 4, 12, 30, 31, 376–378
 rectangular, 284
 thick, 1
 thin, 1, 30, 31, 272, 274

Positive vector, 50

Principle

D'Alembert's principle of inertial forces, 192
 extremum principle in variational methods, 2
 Hamilton's Principle, 337, 338
 Hellinger–Reissner Principle, 7, 20, 156
 of minimum potential energy, 7, 21, 158, 333, 334
 of virtual work, 224
 principle of supposition, 192
 superposition principle, 261
 variational principle, 250, 329, 338

Q

Qualitative properties, 33, 57

in static deformation, 318, 320, 322, 323
 modal, 2, 11, 12, 108, 109, 115, 117, 119, 125, 129, 136, 160, 169, 170, 172, 205, 215, 232, 255, 264, 272, 276, 278, 279, 283, 286, 296, 299–301, 306, 308, 309, 312, 313

Qualitative theories, 1–4, 13

R

Rayleigh quotient, 331, 336, 344, 385, 387

Recurrence relation, 37, 40

Rigid-body motion, 6, 27, 127, 129, 131, 133, 136, 138, 216, 218, 237, 240, 243, 350, 351

Rigid link, 272, 306

Ritz method, 13

applicability of the Ritz method, 387
 convergence of the Ritz method, 346, 385, 387

S

Sensors, 314, 315

Sequence of Markov functions, 64, 67

Sequentiality of nodes, 6, 14, 17, 20

Shaft, 16, 87, 89, 90, 117, 118, 377

Shell, 324

- membrane, 1
 - membrane theory of shells, 377
 - moment theory of shells, 376–378
 - nonmoment, 1
 - revolving, 25
 - shallow, 328
 - shells with transverse shear deformation, 377
 - thin, 31, 352, 353, 365, 369
 - Sign oscillatory, 226
 - Singularity, 365, 375–377
 - Singular point
 - removable or non-removable, 249
 - Solutions
 - in Elasticity, 3, 9, 25–28, 328, 329, 350, 351
 - in Solid Mechanics, 327
 - in structural theories, 2, 4, 9, 12, 13, 28, 330
 - of static deformation, 1, 2, 4, 9, 31, 328–330, 345, 348, 349, 353, 365
 - of vibrational modes, 24, 327, 328, 348, 353, 362, 365, 379, 383
 - two levels of solutions, 330
 - Space
 - Hilbert, 27–29, 327–329, 331, 338, 340, 341, 349, 354, 356–358, 384
 - Sobolev, 2, 328, 329, 340, 341
 - square-integrable, 27, 356
 - with the kinetic energy norm, 27–30, 349–351, 357, 359, 363
 - with the strain energy norm, 27, 28, 30, 349–351, 357, 358, 363, 387
 - Spring, 276, 279, 285, 301, 303
 - Spring–mass system, 16, 89, 90, 93–108, 111, 118
 - Static deformation, 158, 271, 272, 318–324, 327, 333, 335, 337, 343, 345, 351–353, 362, 379–383
 - Strain energy, 84, 85, 111, 231, 257, 258, 266
 - magnitude of the strain energy, 27, 28, 349, 354
 - String, 2, 4, 7, 8, 15, 16, 87–92, 117, 118, 184, 196, 204, 279, 377
 - Structures
 - assembly, 1, 9, 28, 31, 327–329, 352, 353, 356, 365, 377–379
 - axisymmetric, 8, 25, 271, 308, 323
 - chain, 24, 271, 301, 303–305, 322, 324
 - composite, 1, 327, 329, 353, 371
 - linearly periodic, 22, 24, 271, 300, 301
 - repetitive, 4, 8, 11, 13, 21, 25, 271, 272, 312, 313, 318, 324
 - rotationally periodic, 11, 24, 271, 283, 320
 - symmetric, 21, 271, 272, 318
 - Sturm–Liouville, 239
 - operator, 184
 - system, 15, 89, 183–186, 192, 193, 196, 207, 208
 - Sturm sequence, 38, 39, 42
 - Substructures, 271–276, 280, 284–286, 292, 293, 296, 299–302, 305, 306, 312, 318–323
 - Sufficient condition, 6, 35, 41, 44, 46, 48, 71, 74, 75, 147, 148, 203, 206, 212, 217, 245, 248
 - Superposition, 15, 260
 - Sylvester’s identity, 36
 - System input, 314
- T**
- Three properties and two links, 5, 6
- U**
- U*-line, 5, 41, 42, 56, 57, 76, 96, 101, 115, 129, 154, 169–172, 178–180
 - Under-constrained beam, 129, 215, 232, 237
 - Under-constrained system, 7, 111, 117
- V**
- Validity of models, 1, 9, 30, 329, 364, 372
 - Valid structural model, 31, 378, 379
 - Vibrational modes, 329, 348, 379
- W**
- Well-constrained beam, 125, 158, 159, 215, 219, 226, 231, 232, 244
 - Well-constrained spring–mass system, 96
 - Well-constrained system, 6, 71, 82, 83, 155, 197, 198
- Z**
- Zeros, 15, 19, 20, 64–68, 74, 169, 173, 178, 180, 196, 207, 229, 232, 240, 242, 264, 269