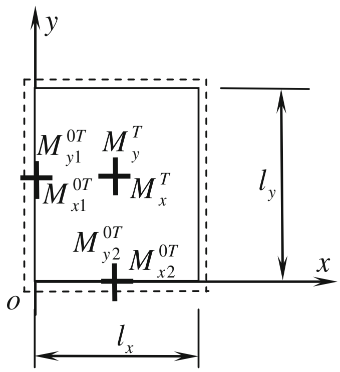
Appendix A
Thermal Bending Calculation Coefficient Tables

See Tables A.1, A.2, A.3, A.4, A.5, A.6, A.7, A.8, A.9, A.10, A.11 and A.12.

X. Cheng, *Thermal Elastic Mechanics Problems of Concrete Rectangular Thin Plate*, Springer Tracts in Civil Engineering,
DOI 10.1007/978-981-10-4472-4

Table A.1 Thermal bending calculation coefficient of four edges simply supported under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \ M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T, \ M_{y1}^{0T} = k_{y1} M^T$$

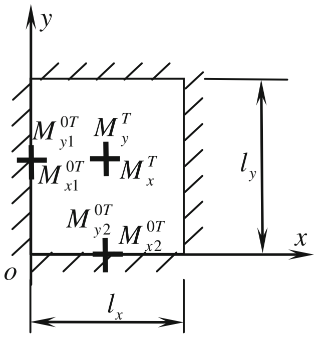
$$M_{x2}^{0T} = k_{x2} M^T, \ M_{y2}^{0T} = k_{y2} M^T$$

$$w(x, y) = f \frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_{x2}	k_{y2}	k_x	k_y	f
0.50	0.0000	0.8333	0.8333	0.0000	0.0915	0.7419	0.1139
0.55	0.0000	0.8333	0.8333	0.0000	0.1215	0.7118	0.1102
0.60	0.0000	0.8333	0.8333	0.0000	0.1537	0.6796	0.1063
0.65	0.0000	0.8333	0.8333	0.0000	0.1874	0.6460	0.1022
0.70	0.0000	0.8333	0.8333	0.0000	0.2216	0.6117	0.0980
0.75	0.0000	0.8333	0.8333	0.0000	0.2561	0.5772	0.0937
0.80	0.0000	0.8333	0.8333	0.0000	0.2902	0.5431	0.0895
0.85	0.0000	0.8333	0.8333	0.0000	0.3235	0.5098	0.0854
0.90	0.0000	0.8333	0.8333	0.0000	0.3559	0.4775	0.0813
0.95	0.0000	0.8333	0.8333	0.0000	0.3870	0.4464	0.0774
1.00	0.0000	0.8333	0.8333	0.0000	0.4167	0.4167	0.0737
1.10	0.0000	0.8333	0.8333	0.0000	0.4717	0.3616	0.0666
1.20	0.0000	0.8333	0.8333	0.0000	0.5209	0.3125	0.0602
1.30	0.0000	0.8333	0.8333	0.0000	0.5640	0.2693	0.0545
1.40	0.0000	0.8333	0.8333	0.0000	0.6018	0.2315	0.0494
1.50	0.0000	0.8333	0.8333	0.0000	0.6346	0.1987	0.0448
1.60	0.0000	0.8333	0.8333	0.0000	0.6629	0.1704	0.0407
1.70	0.0000	0.8333	0.8333	0.0000	0.6873	0.1460	0.0371
1.80	0.0000	0.8333	0.8333	0.0000	0.7083	0.1250	0.0339
1.90	0.0000	0.8333	0.8333	0.0000	0.7264	0.1070	0.0310
2.00	0.0000	0.8333	0.8333	0.0000	0.7419	0.0915	0.0285

Table A.2 Thermal bending calculation coefficient of four edges clamped under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T, \quad M_{y1}^{0T} = k_{y1} M^T$$

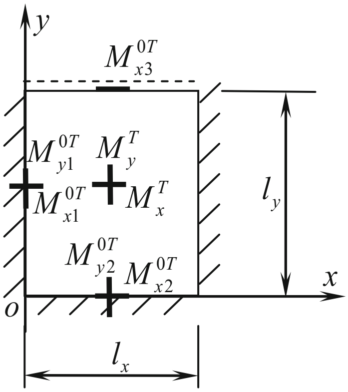
$$M_{x2}^{0T} = k_{x2} M^T, \quad M_{y2}^{0T} = k_{y2} M^T$$

$$w(x, y) = f \frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_{x2}	k_{y2}	k_x	k_y	f
0.50	1.0000	0.8333	0.8333	1.0000	1.0000	1.0005	0.0000
0.55	1.0000	0.8333	0.8333	1.0000	1.0000	1.0006	0.0000
0.60	1.0000	0.8333	0.8333	1.0000	1.0000	1.0007	0.0000
0.65	1.0000	0.8333	0.8333	1.0000	1.0000	1.0008	0.0000
0.70	1.0000	0.8333	0.8333	1.0000	1.0000	1.0010	0.0000
0.75	1.0000	0.8333	0.8333	1.0000	1.0000	1.0010	0.0000
0.80	1.0000	0.8333	0.8333	1.0000	1.0000	1.0013	0.0000
0.85	0.0000	0.8333	0.8333	0.0000	1.0000	1.0014	0.0000
0.90	0.0000	0.8333	0.8333	0.0000	1.0000	1.0014	0.0000
0.95	0.0000	0.8333	0.8333	0.0000	1.0000	1.0016	0.0000
1.00	0.0000	0.8333	0.8333	0.0000	1.0000	1.0018	0.0000

Table A.3 Thermal bending calculation coefficient of three edges clamped and one edge simply supported under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T, \quad M_{y1}^{0T} = k_{y1} M^T$$

$$M_{x2}^{0T} = k_{x2} M^T, \quad M_{y2}^{0T} = k_{y2} M^T$$

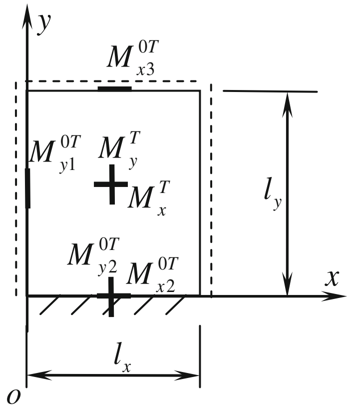
$$M_{x3}^{0T} = k_{x3} M^T$$

$$w(x, y) = f \frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_{x2}	k_{y2}	k_{x3}	k_x	k_y	f
0.50	1.0000	0.8333	0.8333	1.0000	0.8333	0.4319	1.0000	0.0087
0.55	1.0000	0.8333	0.8333	1.0000	0.8333	0.4189	0.9866	0.0105
0.60	1.0000	0.8333	0.8333	1.0000	0.8333	0.3956	0.9809	0.0121
0.65	1.0000	0.8333	0.8333	1.0000	0.8333	0.3708	0.9984	0.0136
0.70	1.0000	0.8333	0.8333	1.0000	0.8333	0.3458	0.9877	0.0148
0.75	1.0000	0.8333	0.8333	1.0000	0.8333	0.3219	0.9750	0.0159
0.80	1.0000	0.8333	0.8333	1.0000	0.8333	0.3996	0.9609	0.0167
0.85	1.0000	0.8333	0.8333	1.0000	0.8333	0.2796	0.9456	0.0174
0.90	1.0000	0.8333	0.8333	1.0000	0.8333	0.2620	0.9294	0.0179
0.95	1.0000	0.8333	0.8333	1.0000	0.8333	0.2471	0.9125	0.0182
1.00	1.0000	0.8333	0.8333	1.0000	0.8333	0.2348	0.8951	0.0184
1.10	1.0000	0.8333	0.8333	1.0000	0.8333	0.2177	0.8603	0.0185
1.20	1.0000	0.8333	0.8333	1.0000	0.8333	0.2095	0.8258	0.0182
1.30	1.0000	0.8333	0.8333	1.0000	0.8333	0.2084	0.7930	0.0177
1.40	1.0000	0.8333	0.8333	1.0000	0.8333	0.2127	0.7621	0.0170
1.50	1.0000	0.8333	0.8333	1.0000	0.8333	0.2211	0.7335	0.0162
1.60	1.0000	0.8333	0.8333	1.0000	0.8333	0.2322	0.7075	0.0153
1.70	1.0000	0.8333	0.8333	1.0000	0.8333	0.2450	0.6839	0.0145
1.80	1.0000	0.8333	0.8333	1.0000	0.8333	0.2589	0.6626	0.0136
1.90	1.0000	0.8333	0.8333	1.0000	0.8333	0.2731	0.6435	0.0128
2.00	1.0000	0.8333	0.8333	1.0000	0.8333	0.2874	0.6264	0.0121

Table A.4 Thermal bending calculation coefficient of one edge clamped and three edges simply supported under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

$$M_{y1}^{0T} = k_{y1} M^T$$

$$M_{x2}^{0T} = k_{x2} M^T, \quad M_{y2}^{0T} = k_{y2} M^T$$

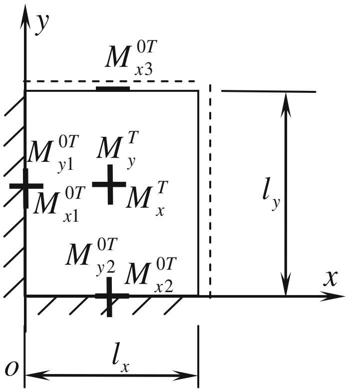
$$M_{x3}^{0T} = k_{x3} M^T$$

$$w(x, y) = f\frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{x2}	k_{y2}	k_{x3}	k_x	k_y	f
0.50	0.8333	0.8333	1.0000	0.8333	0.1720	0.7253	0.1052
0.55	0.8333	0.8333	1.0000	0.8333	0.2196	0.6988	0.0997
0.60	0.8333	0.8333	1.0000	0.8333	0.2683	0.6277	0.0942
0.65	0.8333	0.8333	1.0000	0.8333	0.3169	0.6476	0.0886
0.70	0.8333	0.8333	1.0000	0.8333	0.3645	0.7357	0.0831
0.75	0.8333	0.8333	1.0000	0.8333	0.4105	0.6022	0.0778
0.80	0.8333	0.8333	1.0000	0.8333	0.4543	0.5823	0.0728
0.85	0.8333	0.8333	1.0000	0.8333	0.4955	0.5643	0.3072
0.90	0.8333	0.8333	1.0000	0.8333	0.5343	0.5482	0.3385
0.95	0.8333	0.8333	1.0000	0.8333	0.5702	0.5340	0.3687
1.00	0.8333	0.8333	1.0000	0.8333	0.6034	0.5216	0.3979
1.10	0.8333	0.8333	1.0000	0.8333	0.6621	0.5014	0.0481
1.20	0.8333	0.8333	1.0000	0.8333	0.7113	0.4867	0.0421
1.30	0.8333	0.8333	1.0000	0.8333	0.7518	0.4764	0.0368
1.40	0.8333	0.8333	1.0000	0.8333	0.7851	0.4695	0.0324
1.50	0.8333	0.8333	1.0000	0.8333	0.8124	0.4652	0.0286
1.60	0.8333	0.8333	1.0000	0.8333	0.8344	0.4630	0.0254
1.70	0.8333	0.8333	1.0000	0.8333	0.8523	0.4623	0.0226
1.80	0.8333	0.8333	1.0000	0.8333	0.8666	0.4625	0.0202
1.90	0.8333	0.8333	1.0000	0.8333	0.8783	0.4636	0.0182
2.00	0.8333	0.8333	1.0000	0.8333	0.8875	0.4651	0.0164

Table A.5 Thermal bending calculation coefficient of two adjacent edges clamped and two edges simply supported under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T, \quad M_{y1}^{0T} = k_{y1} M^T$$

$$M_{x2}^{0T} = k_{x2} M^T, \quad M_{y2}^{0T} = k_{y2} M^T$$

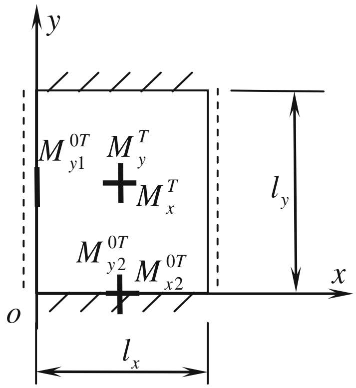
$$M_{x3}^{0T} = k_{x3} M^T$$

$$w(x, y) = f\frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_{x2}	k_{y2}	k_{x3}	k_x	k_y	f
0.50	1.0000	0.8333	0.8333	1.0000	0.8333	0.5457	0.8710	0.0569
0.55	1.0000	0.8333	0.8333	1.0000	0.8333	0.5607	0.8560	0.0551
0.60	1.0000	0.8333	0.8333	1.0000	0.8333	0.5769	0.8398	0.0531
0.65	1.0000	0.8333	0.8333	1.0000	0.8333	0.5936	0.8231	0.0511
0.70	1.0000	0.8333	0.8333	1.0000	0.8333	0.6108	0.8059	0.0490
0.75	1.0000	0.8333	0.8333	1.0000	0.8333	0.6280	0.7887	0.0469
0.80	1.0000	0.8333	0.8333	1.0000	0.8333	0.6451	0.7716	0.0448
0.85	1.0000	0.8333	0.8333	1.0000	0.8333	0.6617	0.7550	0.0427
0.90	1.0000	0.8333	0.8333	1.0000	0.8333	0.6779	0.7388	0.0407
0.95	1.0000	0.8333	0.8333	1.0000	0.8333	0.6934	0.7233	0.0387
1.00	1.0000	0.8333	0.8333	1.0000	0.8333	0.7083	0.7084	0.0368
1.10	1.0000	0.8333	0.8333	1.0000	0.8333	0.7358	0.6809	0.0333
1.20	1.0000	0.8333	0.8333	1.0000	0.8333	0.7604	0.6563	0.0301
1.30	1.0000	0.8333	0.8333	1.0000	0.8333	0.7820	0.6347	0.0272
1.40	1.0000	0.8333	0.8333	1.0000	0.8333	0.8009	0.6158	0.0247
1.50	1.0000	0.8333	0.8333	1.0000	0.8333	0.8173	0.5994	0.0224
1.60	1.0000	0.8333	0.8333	1.0000	0.8333	0.8315	0.5852	0.0204
1.70	1.0000	0.8333	0.8333	1.0000	0.8333	0.8436	0.5731	0.0186
1.80	1.0000	0.8333	0.8333	1.0000	0.8333	0.8541	0.5626	0.0169
1.90	1.0000	0.8333	0.8333	1.0000	0.8333	0.8631	0.5536	0.0155
2.00	1.0000	0.8333	0.8333	1.0000	0.8333	0.8709	0.5458	0.0142

Table A.6 Thermal bending calculation coefficient of two opposite edges clamped and two edges simply supported under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

$$M_{y1}^{0T} = k_{y1} M^T$$

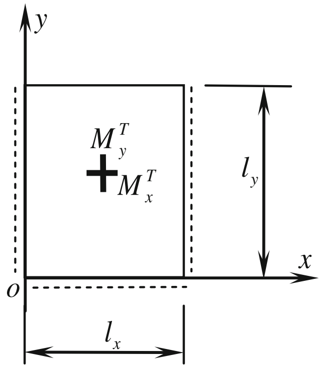
$$M_{x2}^{0T} = k_{x2} M^T, \quad M_{y2}^{0T} = k_{y2} M^T$$

$$w(x, y) = f \frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{y1}	k_{x2}	k_{y2}	k_x	k_y	f
0.50	0.8333	0.8333	1.0000	0.2528	0.7088	0.0965
0.55	0.8333	0.8333	1.0000	0.3176	0.6857	0.0893
0.60	0.8333	0.8333	1.0000	0.3828	0.6658	0.0821
0.65	0.8333	0.8333	1.0000	0.4464	0.6494	0.0750
0.70	0.8333	0.8333	1.0000	0.5073	0.6365	0.0683
0.75	0.8333	0.8333	1.0000	0.5648	0.6273	0.0620
0.80	0.8333	0.8333	1.0000	0.6183	0.6214	0.0561
0.85	0.8333	0.8333	1.0000	0.6675	0.6188	0.0506
0.90	0.8333	0.8333	1.0000	0.7127	0.6190	0.0456
0.95	0.8333	0.8333	1.0000	0.7535	0.6216	0.0410
1.00	0.8333	0.8333	1.0000	0.7902	0.6266	0.0368
1.10	0.8333	0.8333	1.0000	0.8525	0.6414	0.0297
1.20	0.8333	0.8333	1.0000	0.9017	0.6610	0.0239
1.30	0.8333	0.8333	1.0000	0.9395	0.6837	0.0192
1.40	0.8333	0.8333	1.0000	0.9684	0.7075	0.0155
1.50	0.8333	0.8333	1.0000	0.9901	0.7318	0.0125
1.60	0.8333	0.8333	1.0000	1.0000	0.7556	0.0101
1.70	0.8333	0.8333	1.0000	1.0000	0.7786	0.0081
1.80	0.8333	0.8333	1.0000	1.0000	0.8002	0.0066
1.90	0.8333	0.8333	1.0000	1.0000	0.8203	0.0053
2.00	0.8333	0.8333	1.0000	1.0000	1.0000	0.0044

Table A.7 Thermal bending calculation coefficient of three edges simply supported and one edge free under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

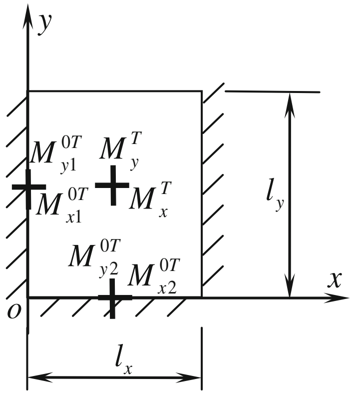
$$M_{y1}^{0T} = k_{y1} M^T, \quad M_{x2}^{0T} = k_{x2} M^T$$

$$w(x, y) = f\,\frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{y1}	k_{x2}	k_x	k_y	f
0.50	0.8333	0.8333	0.0950	−0.7857	−0.0300
0.55	0.8333	0.8333	0.1230	−0.7955	−0.0330
0.60	0.8333	0.8333	0.1513	−0.7979	−0.0368
0.65	0.8333	0.8333	0.0973	−0.7962	−0.0412
0.70	0.8333	0.8333	0.1935	−0.7903	−0.0461
0.75	0.8333	0.8333	0.0979	−0.7804	−0.0514
0.80	0.8333	0.8333	0.0983	−0.7665	−0.0568
0.85	0.8333	0.8333	0.0975	−0.7487	−0.0623
0.90	0.8333	0.8333	0.0963	−0.7248	−0.0676
0.95	0.8333	0.8333	0.2020	−0.7037	−0.0728
1.00	0.8333	0.8333	0.1893	−0.6769	−0.0777
1.10	0.8333	0.8333	0.1543	−0.6173	−0.0864
1.20	0.8333	0.8333	0.1103	−0.5513	−0.0935
1.30	0.8333	0.8333	0.0623	−0.4815	−0.0988
1.40	0.8333	0.8333	0.0132	−0.4099	−0.1025
1.50	0.8333	0.8333	−0.0340	−0.3378	−0.1048
1.60	0.8333	0.8333	−0.0774	−0.2662	−0.1058
1.70	0.8333	0.8333	−0.1161	−0.1963	−0.1057
1.80	0.8333	0.8333	−0.1500	−0.1282	−0.1046
1.90	0.8333	0.8333	−0.1787	−0.0625	−0.1029
2.00	0.8333	0.8333	−0.1878	0.0006	−0.1006

Table A.8 Thermal bending calculation coefficient of three edges clamped and one edge free



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T,\ M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T,\ M_{y1}^{0T} = k_{y1} M^T$$

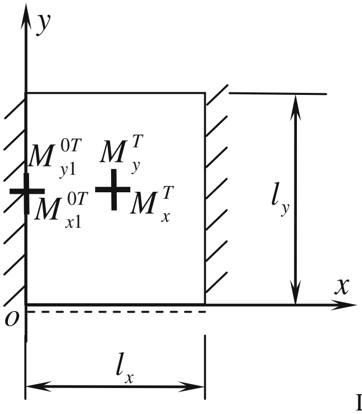
$$M_{x2}^{0T} = k_{x2} M^T,\ M_{y2}^{0T} = k_{y2} M^T$$

$$w(x, y) = f\frac{l_x^2 M^T}{D}\ \text{(mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_{x2}	k_{y2}	k_x	k_y	f
0.50	1.0000	0.8333	0.8333	1.0000	0.2301	0.9699	0.0133
0.55	1.0000	0.8333	0.8333	1.0000	0.2350	0.9579	0.0191
0.60	1.0000	0.8333	0.8333	1.0000	0.2408	0.9398	0.0256
0.65	1.0000	0.8333	0.8333	1.0000	0.2437	0.9198	0.0326
0.70	1.0000	0.8333	0.8333	1.0000	0.2435	0.8985	0.0398
0.75	1.0000	0.8333	0.8333	1.0000	0.2403	0.8765	0.0470
0.80	1.0000	0.8333	0.8333	1.0000	0.2343	0.8542	0.0541
0.85	1.0000	0.8333	0.8333	1.0000	0.2260	0.8319	0.0608
0.90	1.0000	0.8333	0.8333	1.0000	0.2155	0.8099	0.0670
0.95	1.0000	0.8333	0.8333	1.0000	0.2031	0.7884	0.0726
1.00	1.0000	0.8333	0.8333	1.0000	0.1893	0.7676	0.0777
1.10	1.0000	0.8333	0.8333	1.0000	0.1584	0.7288	0.0858
1.20	1.0000	0.8333	0.8333	1.0000	0.1248	0.6935	0.0913
1.30	1.0000	0.8333	0.8333	1.0000	0.0905	0.6622	0.0943
1.40	1.0000	0.8333	0.8333	1.0000	0.0564	0.6345	0.0951
1.50	1.0000	0.8333	0.8333	1.0000	0.0234	0.6103	0.0941
1.60	1.0000	0.8333	0.8333	1.0000	−0.0077	0.5893	0.0915
1.70	1.0000	0.8333	0.8333	1.0000	−0.0368	0.5712	0.0878
1.80	1.0000	0.8333	0.8333	1.0000	−0.0638	0.5556	0.0832
1.90	1.0000	0.8333	0.8333	1.0000	−0.0885	0.5421	0.0779
2.00	1.0000	0.8333	0.8333	1.0000	−0.1113	0.5308	0.0722

Table A.9 Thermal bending calculation coefficient of two opposite edges clamped and one edge simply supported and one edge free under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

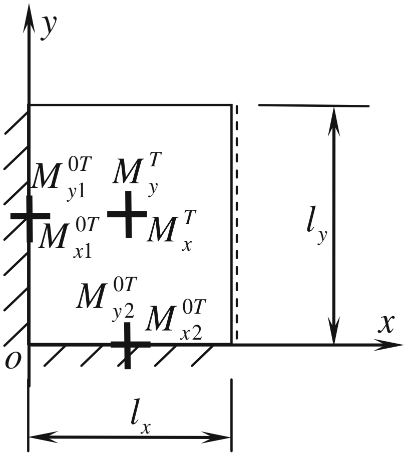
$$M_{x1}^{0T} = k_{x1} M^T, \quad M_{y1}^{0T} = k_{y1} M^T$$

$$w(x, y) = f \frac{l_x^2 M_T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_x	k_y	f
0.50	1.0000	0.8333	1.1670	−0.1714	0.2123
0.55	1.0000	0.8333	1.0194	−0.1120	0.1977
0.60	1.0000	0.8333	0.8720	−0.0543	0.1830
0.65	1.0000	0.8333	0.7274	0.0001	0.1686
0.70	1.0000	0.8333	0.5870	0.0513	0.1547
0.75	1.0000	0.8333	0.4551	0.0961	0.1415
0.80	1.0000	0.8333	0.3299	0.1368	0.1290
0.85	1.0000	0.8333	0.2131	0.1724	0.1174
0.90	1.0000	0.8333	0.1050	0.2031	0.1066
0.95	1.0000	0.8333	0.0055	0.2293	0.0966
1.00	1.0000	0.8333	−0.0853	0.2512	0.0875
1.10	1.0000	0.8333	−0.2433	0.2838	0.0716
1.20	1.0000	0.8333	−0.3732	0.3040	0.0586
1.30	1.0000	0.8333	−0.4781	0.3148	0.0480
1.40	1.0000	0.8333	−0.5629	0.3185	0.0393
1.50	1.0000	0.8333	−0.6310	0.3170	0.0323
1.60	1.0000	0.8333	−0.6853	0.3119	0.0266
1.70	1.0000	0.8333	−0.7288	0.3204	0.0219
1.80	1.0000	0.8333	−0.7635	0.2951	0.0181
1.90	1.0000	0.8333	−0.7911	0.2849	0.0150
2.00	1.0000	0.8333	−0.8129	0.2739	0.0125

Table A.10 Thermal bending calculation coefficient of the concrete rectangular thin plate with two adjacent edges clamped and one edge simply supported and one edge free under temperature disparity



$$\mu = \frac{1}{6},$$

$$M_x^T = k_x M^T,\quad M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T, M_{y1}^{0T} = k_{y1} M^T$$

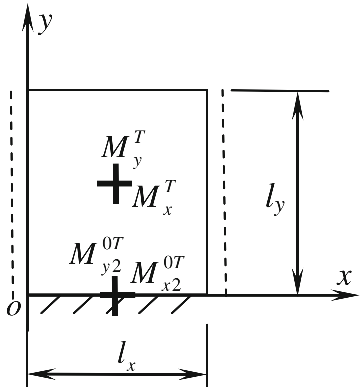
$$M_{x2}^{0T} = k_{x2} M^T,\quad M_{y2}^{0T} = k_{y2} M^T$$

$$w(x, y) = f\frac{l_x^2 M^T}{D}\ \text{(mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_{x2}	k_{y2}	k_x	k_y	f
0.50	1.0000	0.8333	0.8333	1.0000	0.6952	−0.3853	0.1754
0.55	1.0000	0.8333	0.8333	1.0000	0.6217	−0.3212	0.1672
0.60	1.0000	0.8333	0.8333	1.0000	0.5439	−0.2544	0.1585
0.65	1.0000	0.8333	0.8333	1.0000	0.4634	−0.1864	0.1494
0.70	1.0000	0.8333	0.8333	1.0000	0.3808	−0.1175	0.1403
0.75	1.0000	0.8333	0.8333	1.0000	0.2997	−0.0513	0.1310
0.80	1.0000	0.8333	0.8333	1.0000	0.2186	0.0140	0.1219
0.85	1.0000	0.8333	0.8333	1.0000	0.1390	0.0773	0.1129
0.90	1.0000	0.8333	0.8333	1.0000	0.0617	0.1379	0.1041
0.95	1.0000	0.8333	0.8333	1.0000	−0.0131	0.1956	0.0957
1.00	1.0000	0.8333	0.8333	1.0000	−0.0849	0.2503	0.0875
1.10	1.0000	0.8333	0.8333	1.0000	−0.2190	0.3500	0.0722
1.20	1.0000	0.8333	0.8333	1.0000	−0.3401	0.4375	0.0583
1.30	1.0000	0.8333	0.8333	1.0000	−0.4480	0.5132	0.0457
1.40	1.0000	0.8333	0.8333	1.0000	−0.5437	0.5786	0.0345
1.50	1.0000	0.8333	0.8333	1.0000	−0.6281	0.6345	0.0244
1.60	1.0000	0.8333	0.8333	1.0000	−0.7023	0.6823	0.0155
1.70	1.0000	0.8333	0.8333	1.0000	−0.7674	0.7231	0.0075
1.80	1.0000	0.8333	0.8333	1.0000	−0.8244	0.7579	0.0004
1.90	1.0000	0.8333	0.8333	1.0000	−0.8744	0.7876	−0.0059
2.00	1.0000	0.8333	0.8333	1.0000	−0.9182	0.8128	−0.0115

Table A.11 Thermal bending calculation coefficient of the concrete rectangular thin plate with two opposite edges simply supported and one edge clamped and one edge free under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

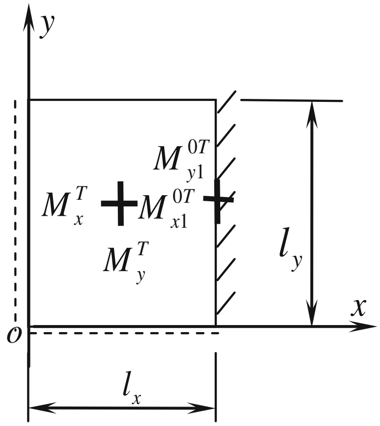
$$M_{x2}^{0T} = k_{x2} M^T, \quad M_{y2}^{0T} = k_{y2} M^T$$

$$w(x, y) = f\frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{y2}	k_{x2}	k_x	k_y	f
0.50	1.0000	0.8333	0.0656	−0.6708	0.1262
0.55	1.0000	0.8333	0.0508	−0.6212	0.1234
0.60	1.0000	0.8333	0.0307	−0.5672	0.1201
0.65	1.0000	0.8333	0.0056	−0.5097	0.1162
0.70	1.0000	0.8333	−0.0244	−0.4488	0.1119
0.75	1.0000	0.8333	−0.0563	−0.3881	0.1072
0.80	1.0000	0.8333	−0.0919	−0.3259	0.1022
0.85	1.0000	0.8333	−0.1297	−0.2637	0.0969
0.90	1.0000	0.8333	−0.1693	−0.2023	0.0916
0.95	1.0000	0.8333	−0.2098	−0.1420	0.0861
1.00	1.0000	0.8333	−0.2510	−0.0834	0.0806
1.10	1.0000	0.8333	−0.3336	0.0278	0.0697
1.20	1.0000	0.8333	−0.4143	0.1300	0.0591
1.30	1.0000	0.8333	−0.4913	0.2222	0.0490
1.40	1.0000	0.8333	−0.5635	0.3047	0.0395
1.50	1.0000	0.8333	−0.6302	0.3780	0.0307
1.60	1.0000	0.8333	−0.6913	0.4426	0.0226
1.70	1.0000	0.8333	−0.7469	0.4995	0.0151
1.80	1.0000	0.8333	−0.7971	0.5496	0.0083
1.90	1.0000	0.8333	−0.8424	0.5934	0.0021
2.00	1.0000	0.8333	−0.8829	0.6319	−0.0035

Table A.12 Thermal bending calculation coefficient of the concrete rectangular thin plate with two adjacent edges simply supported and one edge clamped and one edge free under temperature disparity



$$\mu = \frac{1}{6}$$

$$M_x^T = k_x M^T, \quad M_y^T = k_y M^T$$

$$M_{x1}^{0T} = k_{x1} M^T, \quad M_{y1}^{0T} = k_{y1} M^T$$

$$w(x, y) = f\,\frac{l_x^2 M^T}{D} \text{ (mid-span deflection)}$$

Lower temperature side is in tension

l_x/l_y	k_{x1}	k_{y1}	k_x	k_y	f
0.50	1.0000	0.8333	0.5378	−0.4566	0.1631
0.55	1.0000	0.8333	0.4490	−0.4199	0.1539
0.60	1.0000	0.8333	0.3591	−0.3670	0.1447
0.65	1.0000	0.8333	0.2700	−0.3230	0.1354
0.70	1.0000	0.8333	0.1822	−0.2799	0.1263
0.75	1.0000	0.8333	0.0995	−0.2408	0.1176
0.80	1.0000	0.8333	0.0199	−0.2035	0.1093
0.85	1.0000	0.8333	0.0552	−0.1691	0.1014
0.90	1.0000	0.8333	−0.1254	−0.1370	0.0939
0.95	1.0000	0.8333	−0.1907	−0.1091	0.0870
1.00	1.0000	0.8333	−0.2510	−0.0834	0.0806
1.10	1.0000	0.8333	−0.3575	−0.0398	0.0691
1.20	1.0000	0.8333	−0.4470	−0.0054	0.0594
1.30	1.0000	0.8333	−0.5211	0.0214	0.0512
1.40	1.0000	0.8333	−0.5824	0.0419	0.0443
1.50	1.0000	0.8333	−0.6328	0..0574	0.0385
1.60	1.0000	0.8333	−0.6741	0.0689	0.0336
1.70	1.0000	0.8333	−0.7081	0.0772	0.0295
1.80	1.0000	0.8333	−0.7359	0.0830	0.0260
1.90	1.0000	0.8333	−0.7587	0.0869	0.0230
2.00	1.0000	0.8333	−0.7774	0.0891	0.0205

Appendix B
Programs for the Rectangular Thin Plate with Four Edges Supported

Case 1: Four edges simply supported

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\frac{2\alpha_m y}{b}\sin\frac{m\pi x}{a} - \frac{M^T}{2D}(x-a)x$$

Let

$$a_1 = \sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\frac{2\alpha_m y}{b}\sin\frac{m\pi x}{a}$$

So

$$w = \left[-\frac{4}{\pi^3}a_1 - \frac{1}{2a^2}(x-a)x\right]\frac{a^2M^T}{D} = f\frac{a^2M^T}{D}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = 1/2/a/b$, $L = a/b$,

$a_1$is calculated as follows:

```
syms a b am c d L
num=1;sum_x1=0;m=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=1/2/L;
am=0.5*m*pi/L;
sum_x=1/(m^3*cosh(am))*cosh(m*pi*d-am)*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
```


X. Cheng, *Thermal Elastic Mechanics Problems of Concrete Rectangular Thin Plate*, Springer Tracts in Civil Engineering,
DOI 10.1007/978-981-10-4472-4

```
        sum_x1=sum_x;
        num=num+1;
        m=m+2;
        am=0.5*m*pi/L;
        sum_x2=1/(m^3*cosh(am))*cosh(m*pi*d-am)*sin(m*pi*c);
        sum_x=sum_x1+sum_x2;
    end
    sum_x
    num
```

(2) Bending moment

$$\begin{cases} M_x^T = \frac{4M^T}{\pi}(\mu - 1)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\ \cosh\alpha_m}\cosh\frac{m\pi y}{a}\sin\frac{m\pi x}{a} \\ M_y^T = \frac{4M^T}{\pi}(1 - \mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\ \cosh\alpha_m}\cosh\frac{m\pi y}{a}\sin\frac{m\pi x}{a} + (\mu - 1)M^T \\ M_{xy}^T = \frac{4M^T}{\pi}(1 - \mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\ \cosh\alpha_m}\sinh\frac{m\pi y}{a}\cos\frac{m\pi x}{a} \end{cases}$$

Let

$$b_1 = \sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\ \cosh\alpha_m}\cosh\frac{m\pi y}{a}\sin\frac{m\pi x}{a}$$

Hence

$$\begin{cases} M_x^T = \frac{4M^T}{\pi}(\mu - 1)b_1 \\ M_y^T = \frac{4M^T}{\pi}(1 - \mu)b_1 + (\mu - 1)M^T \\ M_{xy}^T = \frac{4M^T}{\pi}(1 - \mu)b_1 \end{cases}$$

That is

$$\begin{cases} M_x^T = \left[\frac{4}{\pi}(\mu - 1)b_1\right]M^T = k_{x1}M^T = k_x M^T \\ M_y^T = \left[\frac{4}{\pi}(1 - \mu)b_1 + (\mu - 1)\right]M^T = k_{y1}M^T = k_y M^T \quad (\text{where } \mu = 1/6, \text{ the same below}) \\ M_{xy}^T = \left[\frac{4}{\pi}(1 - \mu)b_1\right]M^T = k_{xy1}M^T = k_{xy}M^T \end{cases}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = 1/2/a/b$, $L = a/b$, there is,

b_1 is calculated as follows:

```
syms a b am c d L
num=1;sum_x1=0;m=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=1/2/L;
am=0.5*m*pi/L;
sum_x=1/(m*cosh(am))*cosh(m*pi*d-am)*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    am=0.5*m*pi/L;
    sum_x2=1/(m*cosh(am))*cosh(m*pi*d-am)*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
```

Case 2: Four edges clamped

(1) Deflection

$$w = -\frac{4a^2 M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$-\frac{16M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

That is

$$w = \left\{-\frac{4a^2 M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x\right\}$$
$$-\frac{16M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

In above equation, the first part can be obtained by Appendix B 2.1. For the second part, there is

$$w_2 = -\frac{16M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Let

$$f_2 = -\frac{16}{a^2\pi^4}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

$$a_2 = \sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Hence

$$w_2 = \left(-\frac{16}{\pi^4 a^2}a_2\right)\frac{a^2M^T}{D} = f_2\frac{a^2M^T}{D}$$

$$w = (f_1+f_2)\frac{a^2M^T}{D} = f\frac{a^2M^T}{D}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = 1/2/a/b$, $L = a/b$, there is, a_2 is calculated as follows:

$$a_2 = a^2\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{1}{mn}\left(m^2+L^2n^2\right)^{-1}\sin m\pi c\,\sin n\pi d = a^2c_1$$

$c_1$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:69

    for n=1:2:69

        sum_x=1/m/n/(m^2+n^2*L^2)*sin(m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;

    end

end
sum_x1
```

(2) Bending moment

$$\begin{cases} M_x^T = \frac{4M^T}{\pi}(\mu-1)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a} \\ \qquad -\frac{16M^T}{\pi^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_y^T = \frac{4M^T}{\pi}(1-\mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}+(\mu-1)M^T \\ \qquad -\frac{16M^T}{\pi^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ \qquad + M_{xy}^T = \frac{4M^T}{\pi}(1-\mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\cos\frac{m\pi x}{a} \\ \qquad +(1-\mu)\frac{16M^T}{\pi^2 ab}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

That is

$$\begin{cases} M_x^T = \left\{\frac{4M^T}{\pi}(\mu-1)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}\right\} \\ \qquad -\frac{16M^T}{\pi^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_y^T = \left\{\frac{4M^T}{\pi}(1-\mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}+(\mu-1)M^T\right\} \\ \qquad -\frac{16M^T}{\pi^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{xy}^T = \left\{\frac{4M^T}{\pi}(1-\mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\cos\frac{m\pi x}{a}\right\} \\ \qquad +(1-\mu)\frac{16M^T}{\pi^2 ab}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$\begin{cases} M_{x2}^T = -\frac{16M^T}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{y2}^T = -\frac{16M^T}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{xy2}^T = (1-\mu)\frac{16M^T}{\pi^2 ab}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

$$\begin{cases} M_{x2}^T = \left[-\frac{16}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\right]M^T \\ M_{y2}^T = \left[-\frac{16}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\right]M^T \\ M_{xy2}^T = \left[(1-\mu)\frac{16}{\pi^2 ab}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b}\right]M^T \end{cases}$$

Let

$$\begin{cases} \mathrm{b}_2 = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ \mathrm{b}_3 = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}, \\ \mathrm{b}_4 = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

Hence

$$\begin{cases} M_{x2}^T = -\frac{16}{\pi^2}b_2 M^T = k_{x2}M^T \\ M_{y2}^T = -\frac{16}{\pi^2}b_3 M^T = k_{y2}M^T \\ M_{xy2}^T = (1-\mu)\frac{16}{\pi^2 ab}b_4 M^T = k_{xy2}M^T \end{cases}$$

$$\begin{cases} M_x^T = (k_{x1}+k_{x2})M^T = k_x M^T \\ M_y^T = \left(k_{y1}+k_{y2}\right)M^T = k_y M^T \\ M_{xy}^T = \left(k_{xy1}+k_{xy2}\right)M^T = k_{xy}M^T \end{cases}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_2 = \frac{1}{b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{m^2 + \mu n^2 L^2}{mnL^2} \sin m\pi c \, \sin n\pi d = \frac{1}{b^2} d_1 \\ b_3 = \frac{1}{b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{\mu m^2 + n^2 L^2}{mnL^2} \sin m\pi c \, \sin n\pi d = \frac{1}{b^2} d_2 \\ b_4 = ab \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{L^2}{m^2 + n^2 L^2} \cos m\pi c \, \cos n\pi d = abd_3 \end{cases}$$

***d*₁is calculated as follows**:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:7999
    for n=1:2:7999
        sum_x=(m^2+u*n^2*L^2)/m/n/L^2*sin(m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

$d_2$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:10999
    for n=1:2:10999
        sum_x=(u*m^2+n^2*L^2)/m/n/L^2*sin(m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
```

```
sum_x1
```

d_3 is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:69
    for n=1:2:69
        sum_x=L^2/(m^2+n^2*L^2)*cos(m*pi*c)*cos(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

Case 3: One edge simply supported and three edges clamped

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

That is

$$w = \left\{-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x\right\}$$
$$-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$w_3 = -\frac{8a^2M^T}{\pi^4 D}\frac{1}{a^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Let

$$f_3 = -\frac{8}{\pi^4 a^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{1}{ij} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \left(\frac{2i^2}{a^2} + \frac{j^2}{b^2}\right) \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b}$$

$$a_3 = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{1}{ij} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \left(\frac{2i^2}{a^2} + \frac{j^2}{b^2}\right) \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b}$$

Hence

$$w_3 = \left[-\frac{8}{\pi^4 a^2} a_3\right] \frac{a^2 M^T}{D} = f_3 \frac{a^2 M^T}{D}$$

$$w = (f_1 + f_3) \frac{a^2 M^T}{D} = f \frac{a^2 M^T}{D}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = 1/2/a/b$, $L = a/b$, there is, a_2 is calculated as follows:

$$a_3 = a^2 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{1}{mn} \frac{2m^2 + n^2 L^2}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = a^2 c_2$$

$c_2$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:39

    for n=1:2:39

        sum_x=1/m/n/(m^2+n^2*L^2)^2*(2*m^2+n^2*L^2)*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;

    end

end
sum_x1
```

(2) Bending moment

$$
\begin{cases}
M_x^T = \dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a} \\
\qquad -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\dfrac{i^2}{a^2}+\mu\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{2i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
M_y^T = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}+(\mu-1)M^T \\
\qquad -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\mu\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{2i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
M_{xy}^T = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\cos\dfrac{m\pi x}{a} \\
\qquad +\dfrac{8(1-\mu)M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\dfrac{2i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b}
\end{cases}
$$

That is

$$
\begin{cases}
M_x^T = \left\{\dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}\right\} \\
\qquad -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\dfrac{i^2}{a^2}+\mu\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{2i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
M_y^T = \left\{\dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}+(\mu-1)M^T\right\} \\
\qquad -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\mu\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{2i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
M_{xy}^T = \left\{\dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\cos\dfrac{m\pi x}{a}\right\} \\
\qquad +\dfrac{8(1-\mu)M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\dfrac{2i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b}
\end{cases}
$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$\begin{cases} M_{x3}^T = -\dfrac{8M^T}{\pi^2} \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \sin\dfrac{i\pi x}{a} \sin\dfrac{j\pi y}{b} \\ M_{y3}^T = -\dfrac{8M^T}{\pi^2} \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \sin\dfrac{i\pi x}{a} \sin\dfrac{j\pi y}{b} \\ M_{xy3}^T = \dfrac{8(1-\mu)M^T}{\pi^2 ab} \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \cos\dfrac{i\pi x}{a} \cos\dfrac{j\pi y}{b} \end{cases}$$

$$\begin{cases} M_{x3}^T = \left[-\dfrac{8}{\pi^2} \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \sin\dfrac{i\pi x}{a} \sin\dfrac{j\pi y}{b}\right] M^T \\ M_{y3}^T = \left[-\dfrac{8}{\pi^2} \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \sin\dfrac{i\pi x}{a} \sin\dfrac{j\pi y}{b}\right] M^T \\ M_{xy3}^T = \left[\dfrac{8(1-\mu)}{\pi^2 ab} \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \cos\dfrac{i\pi x}{a} \cos\dfrac{j\pi y}{b}\right] M^T \end{cases}$$

Let

$$\begin{cases} b_5 = \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \sin\dfrac{i\pi x}{a} \sin\dfrac{j\pi y}{b} \\ b_6 = \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \sin\dfrac{i\pi x}{a} \sin\dfrac{j\pi y}{b} \\ b_7 = \displaystyle\sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2} \cos\dfrac{i\pi x}{a} \cos\dfrac{j\pi y}{b} \end{cases}$$

Hence

$$\begin{cases} M_{x3}^T = -\dfrac{8}{\pi^2} b_5 M^T = k_{x3} M^T \\ M_{y3}^T = -\dfrac{8}{\pi^2} b_6 M^T = k_{y3} M^T \\ M_{xy3}^T = \dfrac{8(1-\mu)}{\pi^2 ab} b_7 M^T = k_{xy3} M^T \end{cases}$$

$$\begin{cases} M_x^T = (k_{x1} + k_{x3})M^T = k_x M^T \\ M_y^T = \left(k_{y1} + k_{y3}\right)M^T = k_y M^T \\ M_{xy}^T = \left(k_{xy1} + k_{xy3}\right)M^T = k_{xy} M^T \end{cases}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_5 = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{(m^2 + \mu n^2 L^2) \times (2m^2 + n^2 L^2)}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = d_4 \\ b_6 = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{(\mu m^2 + n^2 L^2) \times (2m^2 + n^2 L^2)}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = d_5 \\ b_7 = a^2 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{2m^2 + n^2 L^2}{(m^2 + n^2 L^2)^2} \cos m\pi c \, \cos n\pi d = a^2 d_6 \end{cases}$$

d_4 is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:7999
    for n=1:2:7999
        sum_x=(m^2+u*n^2*L^2)/(m^2+n^2*L^2)^2*(2*m^2+n^2*L^2)*sin
        (m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

d_5 is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:1999
    for n=1:2:1999
```

```
sum_x=(u*m^2+n^2*L^2)/m/n(m^2+n^2*L^2)^2*(2*m^2+n^2*L^2)*sin
(m*pi*c)*sin(n*pi*d);
    sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

$d_6$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:39
    for n=1:2:39
sum_x=1/(m^2+n^2*L^2)^2*(2*m^2+n^2*L^2)*cos(m*pi*c)*cos(n*pi*d);
    sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

Case 4: Three edges simply supported and one edge clamped

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$-\frac{8M^T}{\pi^4 Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

That is

$$w = \left\{-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x\right\}$$
$$-\frac{8M^T}{\pi^4 Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$w_4 = -\frac{8M^T}{\pi^4 Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Let

$$f_4 = -\frac{8}{\pi^4 a^2 b^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{n}{m}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{-2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$a_4 = \sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{n}{m}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{-2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

Hence

$$w_4 = -\frac{8}{\pi^4 a^2 b^2}a_4\frac{a^2 M^T}{D} = f_4\frac{a^2 M^T}{D}$$

$$w = (f_1 + f_4)\frac{a^2 M^T}{D} = f\frac{a^2 M^T}{D}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = 1/2/a/b$, $L = a/b$, there is, a_4 is calculated as follows:

$$a_4 = a^4\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{n}{m}\frac{1}{\left(m^2+n^2L^2\right)^2}\sin m\pi c\,\sin n\pi d = a^4 c_3$$

$c_3$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:39

    for n=1:2:39

        sum_x=n/m/(m^2/L^2+n^2)^2*sin(m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;

    end
```

```
end
sum_x1
```

(2) Bending moment

$$\begin{cases} M_x^T = \dfrac{4M^T}{\pi}(\mu - 1)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} - \alpha_m\right)\sin\dfrac{m\pi x}{a} \\ \qquad - \dfrac{8M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty} \dfrac{j}{i}\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{i^2}{a^2} + \dfrac{\mu j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_y^T = \dfrac{4M^T}{\pi}(1 - \mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\,\pi\,y}{a} + \alpha_m\right)\sin\dfrac{m\,\pi\,x}{a} + (\mu - 1)M^T \\ \qquad - \dfrac{8M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty} \dfrac{j}{i}\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{\mu i^2}{a^2} + \dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{i\pi y}{b} \\ M_{xy}^T = \dfrac{4M^T}{\pi}(1 - \mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} + \alpha_m\right)\cos\dfrac{m\pi x}{a} \\ \qquad - (\mu - 1)\dfrac{8M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty} j^2\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

That is

$$\begin{cases} M_x^T = \left\{\dfrac{4M^T}{\pi}(\mu - 1)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} - \alpha_m\right)\sin\dfrac{m\pi x}{a}\right\} \\ \qquad - \dfrac{8M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty} \dfrac{j}{i}\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{i^2}{a^2} + \dfrac{\mu j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_y^T = \left\{\dfrac{4M^T}{\pi}(1 - \mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} + \alpha_m\right)\sin\dfrac{m\pi x}{a} + (\mu - 1)M^T\right\} \\ \qquad - \dfrac{8M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty} \dfrac{j}{i}\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{\mu i^2}{a^2} + \dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{i\pi y}{b} \\ M_{xy}^T = \left\{\dfrac{4M^T}{\pi}(1 - \mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} + \alpha_m\right)\cos\dfrac{m\pi x}{a}\right\} \\ \qquad - (\mu - 1)\dfrac{8M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty} j^2\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$\begin{cases} M_{x4}^T = -\dfrac{8M^T}{\pi^2 b^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_{y4}^T = -\dfrac{8M^T}{\pi^2 b^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{i\pi y}{b} \\ M_{xy4}^T = -(\mu-1)\dfrac{8M^T}{\pi^2 ab}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}j^2\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

$$\begin{cases} M_{x4}^T = \left[-\dfrac{8}{\pi^2 b^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b}\right]M^T \\ M_{y4}^T = \left[-\dfrac{8}{\pi^2 b^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{i\pi y}{b}\right]M^T \\ M_{xy4}^T = \left[-(\mu-1)\dfrac{8M^T}{\pi^2 ab}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}j^2\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b}\right]M^T \end{cases}$$

Let

$$\begin{cases} b_8 = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ b_9 = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{i\pi y}{b} \\ b_{10} = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}j^2\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

Hence

$$\begin{cases} M_{x4}^T = -\dfrac{8}{\pi^2 b^2}b_8 M^T = k_{x4}M^T \\ M_{y4}^T = -\dfrac{8}{\pi^2 b^2}b_9 M^T = k_{y4}M^T \\ M_{xy4}^T = -(\mu-1)\dfrac{8M^T}{\pi^2 ab}b_{10}M^T = k_{xy4}M^T \end{cases}$$

$$\begin{cases} M_x^T = (k_{x1} + k_{x4})M^T = k_x M^T \\ M_y^T = \left(k_{y1} + k_{y4}\right)M^T = k_y M^T \\ M_{xy}^T = \left(k_{xy1} + k_{xy4}\right)M^T = k_{xy} M^T \end{cases}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_8 = a^2 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{n}{m} \frac{m^2 + \mu n^2 L^2}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_7 \\ b_9 = a^2 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{n}{m} \frac{\mu m^2 + n^2 L^2}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_8 \\ b_{10} = a^4 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} n^2 \frac{1}{(m^2 + n^2 L^2)^2} \cos m\pi c \, \cos n\pi d = a^4 d_9 \end{cases}$$

d_7 is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:1999

    for n=1:2:1999

        sum_x=n/m(m^2+u*n^2*L^2)/(m^2+n^2*L^2)^2*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;

    end

end
sum_x1
```

d_8 is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:7001
```

```
    for n=1:2:7001
        sum_x=n/m(u*m^2+n^2*L^2)/(m^2+n^2*L^2)^2*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

$d_9$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:7999
    for n=1:2:7999
        sum_x=n^2/(m^2+n^2*L^2)^2*cos(m*pi*c)*cos(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

Case 5: Two adjacent edges simply supported and two edges clamped

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

That is

$$w = \left\{-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x\right\}$$
$$-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$w_5 = -\frac{8a^2M^T}{\pi^4 D}\frac{1}{a^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Let

$$f_5 = -\frac{8}{\pi^4 a^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

$$a_5 = \sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Hence

$$w_5 = \left(-\frac{8}{\pi^4 a^2}a_5\right)\frac{a^2M^T}{D} = f_5\frac{a^2M^T}{D}$$

$$w = (f_1 + f_5)\frac{a^2M^T}{D} = f\frac{a^2M^T}{D}$$

Taking $x = a/2$, y = $b/2$, $c = x/a$, $d = y/b$, $L = a/b$, so

$$a_5 = a^2\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{mn}\frac{1}{m^2+n^2L^2}\sin m\pi c\,\sin n\pi d = a^2c_4$$

$c_4$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:69

    for n=1:2:69

        sum_x=1/m/n/(m^2+n^2*L^2)*sin(m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;

    end

end
sum_x1
```

(2) Bending moment

$$\begin{cases} M_x^T = \dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a} \\ \qquad -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{\mu j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_y^T = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}-(\mu-1)M^T \\ \qquad +\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{\mu i^2}{a^2}+\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{xy}^T = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\cos\frac{m\pi x}{a} \\ \qquad -(\mu-1)\dfrac{8M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

That is

$$\begin{cases} M_x^T = \left\{\dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}\right\} \\ \qquad -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{\mu j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_y^T = \left\{\dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}-(\mu-1)M^T\right\} \\ \qquad +\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{\mu i^2}{a^2}+\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{xy}^T = \left\{\dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\cos\frac{m\pi x}{a}\right\} \\ \qquad -(\mu-1)\dfrac{8M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$\begin{cases} M_{x5}^T = -\dfrac{8M^T}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_{y5}^T = \dfrac{8M^T}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_{xy5}^T = -(\mu-1)\dfrac{8M^T}{\pi^2 ab}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

$$\begin{cases} M_{x5}^T = \left[-\dfrac{8}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b}\right]M^T \\ M_{y5}^T = \left[\dfrac{8M_T}{\pi^2}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b}\right]M^T \\ M_{xy5}^T = \left[-(\mu-1)\dfrac{8M^T}{\pi^2 ab}\sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b}\right]M^T \end{cases}$$

Let

$$\begin{cases} b_{11} = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ b_{12} = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ b_{13} = \sum\limits_{i=1,3,\ldots}^{\infty}\sum\limits_{j=1,3,\ldots}^{\infty}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

Hence

$$\begin{cases} M_{x5}^T = -\dfrac{8}{\pi^2}b_{11}M^T = k_{x5}M^T \\ M_{y5}^T = \dfrac{8}{\pi^2}b_{12}M^T = k_{y5}M^T \\ M_{xy5}^T = -(\mu-1)\dfrac{8}{\pi^2 ab}b_{13}M^T = k_{xy5}M^T \end{cases}$$

$$\begin{cases} M_x^T = (k_{x1} + k_{x5})M^T = k_x M^T \\ M_y^T = \left(k_{y1} + k_{y5}\right)M^T = k_y M^T \\ M_{xy}^T = \left(k_{xy1} + k_{xy5}\right)M^T = k_{xy} M^T \end{cases}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_{11} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{1}{mn} \dfrac{m^2 + \mu n^2 L^2}{m^2 + n^2 L^2} \sin m\pi c \, \sin n\pi d = d_{10} \\ b_{12} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{1}{mn} \dfrac{\mu m^2 + n^2 L^2}{m^2 + n^2 L^2} \sin m\pi c \, \sin n\pi d = d_{11} \\ b_{13} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{1}{m^2 + n^2 L^2} \cos m\pi c \, \cos n\pi d = a^2 d_{12} \end{cases}$$

$d_{10}$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:5999

    for n=1:2:5999

        sum_x=(m^2+u*n^2*L^2)/m/n/(m^2+n^2*L^2)*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;

    end

end
sum_x1
```

$d_{11}$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:5001
```

```
    for n=1:2:5001
        sum_x=(u*m^2+n^2*L^2)/m/n/(m^2+n^2*L^2)*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

d_{12}**is calculated as follows**:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:69
    for n=1:2:69
        sum_x=1/(m^2+n^2*L^2)*cos(m*pi*c)*cos(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

Case 6: Two opposite edges simply supported and two edges clamped

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$-\frac{16M^T}{\pi^4b^2D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

That is

$$w = \left\{-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x\right\}$$
$$-\frac{16M^T}{\pi^4b^2D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$w_6 = -\frac{16M^T}{\pi^4 b^2 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Let

$$f_6 = -\frac{16}{\pi^4 a^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

$$a_6 = \sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

Hence

$$w_6 = -\frac{16}{\pi^4 a^2}a_6\frac{a^2 M^T}{D} = f_6\frac{a^2 M^T}{D}$$

$$w = (f_1 + f_6)\frac{a^2 M^T}{D} = f\frac{a^2 M^T}{D}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = 1/2/a/b$, $L = a/b$, so

$$a_6 = a^4\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{n}{m}\frac{1}{(m^2+n^2L^2)^2}\sin m\pi c\,\sin n\pi d = a^4 c_5$$

$c_5$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:39

    for n=1:2:39

        sum_x=n/m/(m^2+n^2 L^2)^2*sin(m*pi*c)*sin(n*pi*d);
        sum_x1=sum_x1+sum_x;

    end

end
sum_x1
```

(2) Bending moment

$$\begin{cases} M_x^T = \dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a} \\ \qquad -\dfrac{16M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_y^T = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}-(\mu-1)M^T \\ \qquad -\dfrac{16M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_{xy}^T = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\cos\dfrac{m\pi x}{a} \\ \qquad -(\mu-1)\dfrac{16M^T}{\pi\, ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}j^2\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

That is

$$\begin{cases} M_x^T = \left\{\dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\ \cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}\right\} \\ \qquad -\dfrac{16M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{i^2}{a^2}+\dfrac{\mu j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_y^T = \left\{\dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}-(\mu-1)M^T\right\} \\ \qquad -\dfrac{16M^T}{\pi^2 b^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{j}{i}\left(\dfrac{\mu i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\ M_{xy}^T = \left\{\dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\cos\dfrac{m\pi x}{a}\right\} \\ \qquad -(\mu-1)\dfrac{16M^T}{\pi^2 ab}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}j^2\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-1}\cos\dfrac{i\pi x}{a}\cos\dfrac{j\pi y}{b} \end{cases}$$

In above equation, the first part can be obtained by Case 1. For the second part, there is

$$\begin{cases} M_{x6}^T = -\frac{16M^T}{\pi^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i}\left(\frac{i^2}{a^2} + \frac{\mu j^2}{b^2}\right)\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{y6}^T = -\frac{16M^T}{\pi^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i}\left(\frac{\mu i^2}{a^2} + \frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ M_{xy6}^T = -(\mu - 1)\frac{16M^T}{\pi^2 ab} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} j^2\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-1} \cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

$$\begin{cases} M_{x6}^T = \left[-\frac{16}{\pi^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i}\left(\frac{i^2}{a^2} + \frac{\mu j^2}{b^2}\right)\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\right] M^T \\ M_{y6}^T = \left[-\frac{16}{\pi^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i}\left(\frac{\mu i^2}{a^2} + \frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\right] M^T \\ M_{xy6}^T = \left[-(\mu - 1)\frac{16}{\pi^2 ab} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} j^2\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-1} \cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b}\right] M^T \end{cases}$$

Let

$$\begin{cases} b_{14} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i}\left(\frac{i^2}{a^2} + \frac{\mu j^2}{b^2}\right)\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ b_{15} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i}\left(\frac{\mu i^2}{a^2} + \frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-2} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ b_{16} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} j^2\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)^{-1} \cos\frac{i\pi x}{a}\cos\frac{j\pi y}{b} \end{cases}$$

Hence

$$\begin{cases} M_{x6}^T = -\frac{16}{\pi^2 b^2} b_{14} M^T = k_{x6} M^T \\ M_{y6}^T = -\frac{16}{\pi^2 b^2} b_{15} M^T = k_{y6} M^T \\ M_{xy6}^T = -(\mu - 1)\frac{16}{\pi^2 ab} b_{16} M^T = k_{xy6} M^T \end{cases}$$

$$\begin{cases} M_x^T = (k_{x1} + k_{x5}) M^T = k_x M^T \\ M_y^T = \left(k_{y1} + k_{y5}\right) M^T = k_y M^T \\ M_{xy}^T = \left(k_{xy1} + k_{xy5}\right) M^T = k_{xy} M^T \end{cases}$$

Taking $x = a/2$, $y = b/2$, $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_{14} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \frac{n}{m} \frac{m^2 + \mu n^2 L^2}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{13} \\ b_{15} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \frac{n}{m} \frac{\mu m^2 + n^2 L^2}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{14} \\ b_{16} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} n^2 \frac{1}{m^2 + n^2 L^2} \cos m\pi c \, \cos n\pi d = a^2 d_{15} \end{cases}$$

$d_{13}$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:5999
    for n=1:2:5999
        sum_x=n/m(m^2+u*n^2*L^2)/(m^2+n^2*L^2)^2*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

$d_{14}$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:9999
    for n=1:2:9999
        sum_x=n/m(u*m^2+n^2*L^2)/(m^2+n^2*L^2)^2*sin(m*pi*c)*sin
        (n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

$d_{15}$is calculated as follows:

```
syms a b am c d L
sum_x1=0;
u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
for m=1:2:69
    for n=1:2:69
        sum_x=n^2/(m^2+n^2*L^2)*cos(m*pi*c)*cos(n*pi*d);
        sum_x1=sum_x1+sum_x;
    end
end
sum_x1
```

Appendix C
Programs for the Rectangular Thin Plate with One edges Free

Case 1: Three edges simply supported and one edge free

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\left[\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}\right]-\frac{M^T}{2D}(x-a)x$$
$$+\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}$$
$$\times\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\frac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\}$$

(where, $\mu = 1/6$, the same below)

In above equation, the first part can be obtained by Case 1. For the second part, Taking $x = a/2$, $y = b/2$, there is

$$\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\frac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\}$$
$$=0$$

Therefore, for the rectangular thin plate with three edges simply supported and one edge free, the deflection solution in center point of the plate is the same with that with four edges simply supported.

X. Cheng, *Thermal Elastic Mechanics Problems of Concrete Rectangular Thin Plate*, Springer Tracts in Civil Engineering,
DOI 10.1007/978-981-10-4472-4

(2) Bending moment

$$\begin{cases} M_x = \frac{4M^T}{\pi}(\mu-1)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a} \\ \quad +\frac{2(3-2\mu)M^T}{\pi}\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left[\begin{pmatrix}\beta_m\coth\beta_m \\ +2\frac{1+\mu}{1-\mu}\end{pmatrix}\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\} \\ M_y = \frac{4M^T}{\pi}(1-\mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}+(\mu-1)M^T \\ \quad +\frac{2(3-2\mu)M^T}{\pi}\times\sum_{m=1,3,\ldots}^{\infty}\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \end{cases}$$

Taking $x = a/2$, $y = b/2$, so

$$\begin{pmatrix} \frac{2(3-2\mu)M^T}{\pi}\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left[\begin{pmatrix}\beta_m\coth\beta_m \\ +2\frac{1+\mu}{1-\mu}\end{pmatrix}\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\}=0 \\ \frac{2(3-2\mu)M^T}{\pi}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\frac{\frac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\frac{2\alpha m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}=0 \end{pmatrix}$$

Therefore, for the rectangular thin plate with three edges simply supported and one edge free, the bending moment solution in center point of the plate is the same with that with four edges simply supported.

Case 2 Three edges clamped and one edge free

(1) Deflection

$$\begin{aligned} w = &-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x \\ &+\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\frac{\frac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\ &-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ &-\frac{8M^Ta^3}{\pi^4 Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\frac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\begin{array}{c}\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}- \\ \frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \end{aligned}$$

Taking $x = a/2$, $y = b/2$, so

$$\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\dfrac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}$$
$$=0$$

Namely

$$w=-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$-\frac{8M^Ta^3}{\pi^4 Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\begin{array}{r}\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}- \\ \dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\}$$

That is

$$w=\left\{-\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x\right\}$$
$$-\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$-\frac{8M^Ta^3}{\pi^4 Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\begin{array}{r}\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}- \\ \dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\}$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, there is

$$w_7 = -\frac{8M^T}{\pi^4 D}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$-\frac{8M^T a^3}{\pi^4 Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}$$

Let

$$f_7 = -\frac{8}{\pi^4 a^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$-\frac{8a}{\pi^4 b(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}$$

$$a_7 = \sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$

$$a_8 = \sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum\limits_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}$$

Hence

$$w_7 = \left(-\frac{8b^2}{\pi^4 a^2} a_7 - \frac{8a}{\pi^4 b(1-\mu)} a_8\right) \frac{a^2 M^T}{D} = f_7 \frac{a^2 M^T}{D}$$

$$w = (f_1 + f_7) \frac{a^2 M^T}{D} = f \frac{a^2 M^T}{D}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$a_7 = b^2 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{L}{mn} \frac{2m^2 + n^2 \times L^2}{m^2 + n^2 \times L^2} \sin m\pi c \, \sin n\pi d = b^2 c_6$$

$$a_8 = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{2m^2 + L^2 \times k^2}{m^4} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)^2}}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\left(\dfrac{2}{L-\mu} + \beta_m \coth \beta_m\right) \sinh \dfrac{m\pi d}{L} - \dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} \right] \sin m\pi c \end{array} \right\} = c_7$$

$c_6$is calculated as follows:

```
syms a b am c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
am=0.5*m*pi/L;
sum_x=L*(2*m^2+n^2*L^2)sin(m*pi*c)sin(n*pi*d)/m/n/(m^2+n^2*L^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    am=0.5*m*pi/L;
    sum_x2=L*(2*m^2+n^2*L^2)sin(m*pi*c)sin(n*pi*d)/m/n/(m^2+n^2*L^2);
    sum_x=sum_x1+sum_x2;
```

```
end
sum_x
num
```

c_7**is calculated as follows**:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1; u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;
sum_x=(2*m^2+L^2*k^2)*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^4*(m^2
+k^2*L^2)^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth
(bm))*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    bm=m*pi/L;
    sum_x2=(2*m^2+L^2*k^2)*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^4*(m^2
    +k^2*L^2)^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth
    (bm))*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
k
```

(2) Bending moment

$$
\begin{cases}
M_x = \dfrac{4M^T}{\pi}(\mu - 1)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} - \alpha_m\right)\sin\dfrac{m\pi x}{a} \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b} - \alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2} + \beta_m}\left[\begin{array}{c}\left(\beta_m\coth\beta_m + 2\dfrac{1+\mu}{1-\mu}\right) \\ \times\sinh\dfrac{m\pi y}{a} - \dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\dfrac{m\pi x}{a} \\
\quad - \dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\dfrac{i^2}{a^2} + \mu\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
\quad - \dfrac{8M^T a}{\pi^2 b}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{c}\dfrac{\sinh\beta_m \sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2} + \frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2} + (2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times\left[\begin{array}{c}\left(\beta_m\coth\beta_m + 2\frac{1+\mu}{1-\mu}\right)\times \\ \sinh\frac{m\pi y}{a} - \frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
M_y = \dfrac{4M^T}{\pi}(1 - \mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a} - \alpha_m\right)\sin\dfrac{m\pi x}{a} + (\mu - 1)M^T \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty} \dfrac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b} - \alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2} + \beta_m}\left(\begin{array}{c}\frac{m\pi y}{a}\cosh\frac{m\pi y}{a} \\ -\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\end{array}\right)\sin\dfrac{m\pi x}{a} \\
\quad - \dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{1}{ij}\left(\mu\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2}\right)^{-2}\left(\dfrac{2i^2}{a^2} + \dfrac{j^2}{b^2}\right)\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
\quad - \dfrac{8M^T a}{\pi^2 b}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{c}\dfrac{\sinh\beta_m \sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2} + \frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2} + (2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a} - \beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\}
\end{cases}
$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero. so

$$\begin{cases} M_{x7} = -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ \qquad -\dfrac{8M^Ta}{\pi^2 b}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\begin{array}{l}\left(\beta_m\coth\beta_m+2\dfrac{1+\mu}{1-\mu}\right)\times \\ \sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\ M_{y7} = -\dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ \qquad -\dfrac{8M^Ta}{\pi^2 b}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\} \end{cases}$$

Let

$$\begin{cases} b_{17} = \displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ b_{18} = \displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{2i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\ b_{19} = \displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\begin{array}{l}\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a} \\ -\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\ b_{20} = \displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left(\frac{2m^2}{a^2}+\frac{k^2}{b^2}\right)\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\} \end{cases}$$

Hence

$$\begin{cases} M_{x7} = -\dfrac{8M^T}{\pi^2} \times b_{17} - \dfrac{8M^T a}{\pi^2 b} \times b_{19} = k_{x7} M^T \\ M_{y7} = -\dfrac{8M^T}{\pi^2} \times b_{18} - \dfrac{8M^T a}{\pi^2 b} \times b_{20} = k_{y7} M^T \end{cases}$$

$$\begin{cases} M_x^T = (k_{x1} + k_{x7}) M^T = k_x M^T \\ M_y^T = \left(k_{y1} + k_{y7}\right) M^T = k_y M^T \end{cases}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_{17} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{2m^2 + n^2 \times L^2}{mn} \dfrac{m^2 + \frac{1}{6} \times L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = d_{16} \\ b_{18} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{2m^2 + n^2 \times L^2}{mn} \dfrac{\frac{1}{6} \times m^2 + L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = d_{17} \\ b_{19} = \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum\limits_{k=1,3,\ldots}^{\infty} \frac{2m^2 + L^2 \times k^2}{m^4} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\frac{2}{L-\mu} + \beta_m \coth \beta_m \right) \sinh \frac{m\pi d}{L} - \\ \frac{m\pi d}{L} \cosh \frac{m\pi d}{L} \end{array} \right] \sin m\pi c \end{array} \right\} = d_{18} \\ b_{20} = \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum\limits_{k=1,3,\ldots}^{\infty} \frac{2m^2 + L^2 \times k^2}{m^2} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi d}{L} \cosh \frac{m\pi d}{L} - \beta_m \coth \beta_m \sinh \frac{m\pi d}{L} \right) \sin m\pi c \end{array} \right\} = d_{19} \end{cases}$$

$d_{16}$is calculated as follows:

```
syms a b am c d L
num=1;sum_x1=0;m=1;n=1;u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=(m^2+u*n^2*L^2)*(2*m^2+n^2*L^2)sin(m*pi*c)sin(n*pi*d)/m/n/
(m^2+n^2*L^2)^2;
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
```

```
    m=m+2;
    am=0.5*m*pi/L;
sum_x2=(m^2+u*n^2*L^2)*(2*m^2+n^2*L^2)sin(m*pi*c)sin(n*pi*d)/m/n/
(m^2+n^2*L^2)^2;
    sum_x=sum_x1+sum_x2;
end
sum_x
num
```

d_{17}**is calculated as follows**:

```
syms a b am c d L
num=1;sum_x1=0;m=1;n=1;u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=(u*m^2+n^2*L^2)*(2*m^2+n^2*L^2)sin(m*pi*c)sin(n*pi*d)/m/n/
(m^2+n^2*L^2)^2;
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    am=0.5*m*pi/L;
sum_x2=(u*m^2+n^2*L^2)*(2*m^2+n^2*L^2)sin(m*pi*c)sin(n*pi*d)/m/n/
(m^2+n^2*L^2)^2;
    sum_x=sum_x1+sum_x2;
end
sum_x
num
```

d_{18}**is calculated as follows**:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1; u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;
sum_x=(2*m^2+L^2*k^2)*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^4*(m^2
+k^2*L^2)^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth
(bm))*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
```

```
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    bm=m*pi/L;
    sum_x2=(2*m^2+L^2*k^2)*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^4*(m^2
    +k^2*L^2)^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth
    (bm))*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
k
```

$d_{19}$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1; u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;
sum_x=(2*m^2+L^2*k^2)*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2
+k^2*L^2)^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh
(m*pi*d/L))-bm*coth(bm)*sinh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    bm=m*pi/L;
    sum_x2=(2*m^2+L^2*k^2)*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2
    +k^2*L^2)^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh
    (m*pi*d/L))-bm*coth(bm)*sinh(m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
k
```

Case 3: Two opposite edges clamped and one edge simply supported and one edge free

(1) Deflection

$$
\begin{aligned}
w = & -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x \\
& + \frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\dfrac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
& - \frac{16M^T}{\pi^4Da^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{i}{j}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
& + \frac{16M^Ta}{\pi^4Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}
\end{aligned}
$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$
\begin{aligned}
w_8 = & -\frac{16M^T}{\pi^4Da^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{i}{j}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
& + \frac{16M^Ta}{\pi^4Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}
\end{aligned}
$$

Let

$$f_8 = -\frac{16}{\pi^4 a^4}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{i}{j}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$+\frac{16}{\pi^4 ab(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\dfrac{m\pi x}{a}\end{array}\right\}$$

$$\left\{\begin{array}{l}a_9=\dfrac{1}{a^4}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{i}{j}\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b}\\ a_{10}=\dfrac{1}{ab}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}\end{array}\right.$$

Hence

$$w_8 = -\frac{16a^2M^T}{\pi^4 D}\times a_9+\frac{16a^2M^T}{\pi^4 D(1-\mu)}\times a_{10}=f\,\frac{a^2M^T}{D}$$

$$w=(f_1+f_8)\frac{a^2M^T}{D}=f\,\frac{a^2M^T}{D}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$a_9 = a^4 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{m}{n} \left(\frac{1}{m^2 + n^2 \times L^2} \right)^2 \sin m\pi c \, \sin n\pi d = a^4 c_8$$

$$a_{10} = ab \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \dfrac{L}{m^2} \dfrac{k^2 \times L^2 + (2-\mu)m^2}{(m^2 + k^2 \times L^2)^2}}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi d}{L} - \\ \qquad \dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} \end{array} \right] \sin m\pi c \end{array} \right\} = abc_9$$

c_8 is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*m/(n*(m^2+n^2*L^2)^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*m/(n*(m^2+n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
n
```

$c_9$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)^2)/((3
+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh(m*pi*d/L)-
m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)^2)/
    ((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh
    (m*pi*d/L)-m*pi*d/L*cosh (m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
k
```

(2) Bending moment

$$
\begin{cases}
M_x = \dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a} \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{\dfrac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\left[\begin{array}{c}\left(\beta_m\coth\beta_m+2\dfrac{1+\mu}{1-\mu}\right)\\ \times\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\dfrac{m\pi x}{a} \\
\quad - \dfrac{16M^T}{\pi^2 a^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{i}{j}\left(\dfrac{i^2}{a^2}+\mu\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
\quad - \dfrac{16M^T}{\pi^2 ab}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\begin{array}{c}\left(\beta_m\coth\beta_m+2\dfrac{1+\mu}{1-\mu}\right)\times\\ \sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
M_y = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{1}{m\cosh\alpha_m}\cosh\left(\dfrac{m\pi y}{a}-\alpha_m\right)\sin\dfrac{m\pi x}{a}+(\mu-1)M^T \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\dfrac{\dfrac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\left(\begin{array}{l}\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\\ -\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\end{array}\right)\sin\dfrac{m\pi x}{a} \\
\quad + \dfrac{16M^T}{\pi^2 a^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\dfrac{i}{j}\left(\mu\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)\left(\dfrac{i^2}{a^2}+\dfrac{j^2}{b^2}\right)^{-2}\sin\dfrac{i\pi x}{a}\sin\dfrac{j\pi y}{b} \\
\quad - \dfrac{16M^T}{\pi^2 ab}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\}
\end{cases}
$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$
\begin{cases}
M_{x8} = -\dfrac{16M^T}{\pi^2 a^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\
\qquad - \dfrac{16M^T}{\pi^2 ab} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \frac{1+\mu}{1-\mu} \right) \times \\ \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\
M_{y8} = \dfrac{16M^T}{\pi^2 a^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\
\qquad - \dfrac{16M^T}{\pi^2 ab} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\}
\end{cases}
$$

Let

$$
\begin{cases}
b_{21} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\
b_{22} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\
b_{23} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\dfrac{m^2}{a^2} + \dfrac{k^2}{b^2} \right)^{-2} \left[\dfrac{k^2}{b^2} + (2-\mu) \dfrac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \frac{1+\mu}{1-\mu} \right) \times \\ \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\
b_{24} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\}
\end{cases}
$$

$$\begin{cases} M_{x8} = -\frac{16M^T}{\pi^2} \times b_{21} - \frac{16M^T}{\pi^2} \times b_{23} = k_{x8}M^T \\ M_{y8} = \frac{16M^T}{\pi^2} \times b_{22} - \frac{16M^T}{\pi^2} \times b_{24} = k_{y8}M^T \end{cases}$$

Taking $c=x/a$, $d=y/b$, $L=a/b$, there is

$$\begin{cases} b_{21} = a^2 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{m}{n} \frac{m^2 + \frac{1}{6} \times L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{20} \\ b_{22} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{m}{n} \frac{\frac{1}{6} \times m^2 + L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{21} \\ b_{23} = ab \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} L \frac{k^2 \times L^2 + (2-\frac{1}{6})m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2\frac{1+\mu}{1-\mu}\right) \times \\ \sinh \frac{m\pi d}{L} - \frac{m\pi d}{L} \cosh \frac{m\pi d}{L} \end{array} \right] \sin \frac{m\pi c}{L} \end{array} \right\} = abd_{22} \\ b_{24} = ab \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} L \frac{k^2 \times L^2 + (2-\frac{1}{6})m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi d}{L} \cosh \frac{m\pi d}{L} - \beta_m \coth \beta_m \sinh \frac{m\pi d}{L}\right) \sin m\pi c \end{array} \right\} = abd_{23} \end{cases}$$

$d_{20}$is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*m*(m^2+1/6*n^2*L^2)/(n*(m^2+n^2*L^2)
^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*m*(m^2+1/6*n^2*L^2)/(n*(m^2
    +n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;
```

```
end
sum_x
num
m
n
```

d_{21} **is calculated as follows**:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*m*(1/6*m^2+n^2*L^2)/(n*(m^2+n^2*L^2)
^2);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*m*(1/6*m^2+n^2*L^2)/(n*(m^2
    +n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
n
```

d_{22} **is calculated as follows**:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*sin(m*pi*c)*((2*(1+1/6)/(1-1/6)+bm*coth(bm))
*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L));
while abs(sum_x-sum_x1)>=1.0e-05
```

```
    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
    (1-1/6)*(sinh(2*bm)/2)+bm)*sin(m*pi*c)*((2*(1+1/6)/(1-1/6)+bm*coth
    (bm))*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L));
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
k
```

$d_{23}$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the waLue of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L)-bm*coth(bm)*sinh
(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
    (1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L)-bm*coth(bm)
    *sinh(m*pi*d/L))*sin(m*pi*c);
end
sum_x
num
m
k
```

Case 4: Two adjacent edges clamped and one edge simply supported and one edge free

(1) Deflection

$$
\begin{aligned}
w = & -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x \\
& +\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\dfrac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
& +\frac{8M^T}{\pi^4Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\times\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
& -\frac{8M^Ta^3}{\pi^4Db^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{k^2}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}
\end{aligned}
$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$
\begin{aligned}
w_9 = & \frac{8M^T}{\pi^4Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\times\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
& -\frac{8M^Ta^3}{\pi^4Db^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{k^2}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}
\end{aligned}
$$

Let

$$f_9 = \frac{8}{\pi^4 a^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \times \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$- \frac{8a}{\pi^4 b^3 (1-\mu)} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2}{m^4} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi y}{a} - \\ \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\}$$

$$a_{11} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$a_{12} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi y}{a} - \\ \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\}$$

Hence

$$w_9 = \frac{8M^T}{\pi^4 D b^2} a_{11} - \frac{8M^T a^3}{\pi^4 D b^3 (1-\mu)} a_{12} = f_9 \frac{a^2 M^T}{D}$$

$$w = (f_1 + f_9) \frac{a^2 M^T}{D} = f \frac{a^2 M^T}{D}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$a_{11} = a^4 \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{n}{m} \left(\frac{1}{m^2 + n^2 \times L^2} \right)^2 \sin m\pi c \, \sin n\pi d = a^4 c_{10}$$

$$a_{12} = ab \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2 * L}{m^2} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\frac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \frac{m\pi d}{L} - \\ \frac{m\pi d}{L} \cosh \frac{m\pi d}{L} \end{array} \right] \sin m\pi c \end{array} \right\} = abc_{11}$$

c_{10} is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*n/(m*(m^2+n^2*L^2)^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2= sin(m*pi*c)*sin(n*pi*d)*n/(m*(m^2+n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
n
```

$c_{11}$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=k^2L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)^2)/((3
+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh(m*pi*d/L)-
m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=k^2L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)
    ^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh
    (m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
k
```

(2) Bending moment

$$
\begin{cases}
M_x = \dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a} \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{\dfrac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\left[\begin{array}{l}\left(\beta_m\coth\beta_m+2\dfrac{1+\mu}{1-\mu}\right)\\ \times\ \sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a} \\
\quad + \dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{i^2}{a^2}+\frac{\mu j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
\quad - \dfrac{8M^T a}{\pi^2 b}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-1}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\begin{array}{l}\left(\beta_m\coth\beta_m+2\frac{1+\mu}{1-\mu}\right)\\ \times\ \sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
M_y = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}+(\mu-1)M^T \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{\dfrac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\left(\begin{array}{l}\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\\ -\ \beta_m\coth\beta_m\sinh\dfrac{m\pi y}{a}\end{array}\right)\sin\frac{m\pi x}{a} \\
\quad + \dfrac{8M^T}{\pi^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{1}{ij}\left(\frac{\mu i^2}{a^2}+\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-1}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
\quad - \dfrac{8M^T a}{\pi^2 b}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-1}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\}
\end{cases}
$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x=a/2$, $y=b/2$, the second part is zero.

For the last two part, there is

$$\begin{cases} M_{x9} = \dfrac{8M^T}{\pi^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\dfrac{i^2}{a^2} + \dfrac{\mu j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-1} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ \qquad - \dfrac{8M^T a}{\pi^2 b} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-1} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \frac{1+\mu}{1-\mu} \right) \\ \times \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\ M_{y9} = \dfrac{8M^T}{\pi^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\dfrac{\mu i^2}{a^2} + \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-1} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ \qquad - \dfrac{8M^T a}{\pi^2 b} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-1} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\} \end{cases}$$

Let

$$\begin{cases} b_{25} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-1} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ b_{26} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \dfrac{1}{ij} \left(\dfrac{\mu i^2}{a^2} + \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-1} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ b_{27} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-1} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \dfrac{1+\mu}{1-\mu} \right) \\ \times \sinh \dfrac{m\pi y}{a} - \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\ b_{28} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-1} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\} \end{cases}$$

$$\begin{cases} M_{x9} = \dfrac{8M^T}{\pi^2} \times b_{25} - \dfrac{8M^T a}{\pi^2 b} \times b_{27} = k_{x9} M^T \\ M_{y9} = \dfrac{8M^T}{\pi^2} \times b_{26} - \dfrac{8M^T a}{\pi^2 b} \times b_{28} = k_{y9} M^T \end{cases}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_{25} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{1}{mn} \dfrac{m^2 + \frac{1}{6} \times L^2 \times n^2}{m^2 + L^2 \times n^2} \sin m\pi c \, \sin n\pi d = d_{24} \\ b_{26} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{1}{mn} \dfrac{\frac{1}{6} \times m^2 + L^2 \times n^2}{m^2 + L^2 \times n^2} \sin m\pi c \, \sin n\pi d = d_{25} \\ b_{27} = \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi d}{L} \\ - \dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} \end{array} \right] \sin m\pi c \end{array} \right\} = d_{26} \\ b_{28} = \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi d}{L} \cosh \frac{m\pi d}{L} - \beta_m \coth \beta_m \sinh \frac{m\pi d}{L} \right) \sin m\pi c \end{array} \right\} = d_{27} \end{cases}$$

$d_{24}$is calculated as follows:

```
syms a b am c d L
num=1;sum_x1=0;m=1;n=1;u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=(m^2+u*n^2*L^2)*sin(m*pi*c)sin(n*pi*d)/m/n/(m^2+n^2*L^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    am=0.5*m*pi/L;

sum_x2=(m^2+u*n^2*L^2)*sin(m*pi*c)sin(n*pi*d)/m/n/(m^2+n^2*L^2);

    sum_x=sum_x1+sum_x2;
```

```
end
sum_x
num
```

d_{25}**is calculated as follows**:

```
syms a b am c d L
num=1;sum_x1=0;m=1;n=1;u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=(u*m^2+n^2*L^2)*sin(m*pi*c)sin(n*pi*d)/m/n/(m^2+n^2*L^2);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    am=0.5*m*pi/L;
sum_x2=(u*m^2+n^2*L^2)*sin(m*pi*c)sin(n*pi*d)/m/n/(m^2+n^2*L^2);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
```

d_{26}**is calculated as follows**:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;
sum_x=sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2))/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(11/6)+bm*coth(bm))*sinh(m*pi*d/L)
m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    bm=m*pi/L;
```

```
sum_x2=sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2))/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(11/6)+bm*coth(bm))*sinh(m*pi*d/L)
m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);

    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
k
```

d_{27} is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the walue of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/m^2/(m^2+k^2*L^2)^2/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L)-bm*coth(bm)*sinh
(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=      sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/m^2/(m^2+k^2*L^2)^2/((3
    +1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L)-bm*coth
    (bm)*sinh(m*pi*d/L))*sin(m*pi*c);end
    sum_x

num
m
k
```

Case 5: Two opposite edges simply supported and one edge clamped and one edge free

(1) Deflection

$$w = -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x$$
$$+\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\frac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\}$$
$$+\frac{8M^T}{\pi^4Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\times\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$-\frac{8M^Ta^3}{\pi^4Db^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{k^2}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\}$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$w_{10} = \frac{8M^T}{\pi^4Db^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\times\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$$
$$-\frac{8M^Ta^3}{\pi^4Db^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\frac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{k^2}{m^4}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m} \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\right\}$$

Let

$$f_{10} = \frac{8}{\pi^4 a^2 b^2} \sum_{i=1,3,}^{\infty} \sum_{j=1,3,\dots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \times \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$- \frac{8a}{\pi^4 b^3 (1-\mu)} \sum_{m=1,3,\dots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\dots}^{\infty} \frac{k^2}{m^4} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi y}{a} - \\ \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\}$$

$$a_{13} = \sum_{i=1,3,\dots}^{\infty} \sum_{j=1,3,\dots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$a_{14} = \sum_{m=1,3,\dots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\dots}^{\infty} \frac{k^2}{m^4} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi y}{a} - \\ \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\}$$

Hence

$$w_{10} = \frac{8M^T}{\pi^4 D b^2} a_{13} - \frac{8M^T a^3}{\pi^4 D b^3 (1-\mu)} a_{14} = f_{10} \frac{a^2 M^T}{D}$$

$$w = (f_1 + f_{10}) \frac{a^2 M^T}{D} = f \frac{a^2 M^T}{D}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$a_{13} = a^4 \sum_{i=1,3,\dots}^{\infty} \sum_{j=1,3,\dots}^{\infty} \frac{n}{m} \left(\frac{1}{m^2 + n^2 \times L^2} \right)^2 \sin m\pi c \, \sin n\pi d = a^4 c_{12}$$

$$a_{14} = ab \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2 \times L}{m^4} \frac{k^2 \times L^2 + (2-\mu)m^2}{(m^2 + k^2 \times L^2)^2}}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi d}{L} - \dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} \right] \sin m\pi c \end{array} \right\} = abc_{13}$$

$c_{12}$is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*n/(m*(m^2+n^2*L^2)^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*n/(m*(m^2+n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
n
```

$c_{13}$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=k^2*L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^4*(m^2+k^2*L^2)^2)/
((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh
(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    k=k+2;
```

```
        m=m+2;
        bm=m*pi/L;
        sum_x2= k^2*L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^4*(m^2+k^2*L^2)
        ^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh
        (m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
        sum_x=sum_x1+sum_x2;
    end
    sum_x
    num
    m
    k
```

(2) Bending moment

$$\left\{\begin{aligned}
M_x =& \frac{4M^T}{\pi}(\mu-1)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}\\
&+\frac{2(3-2\mu)M^T}{\pi}\sum_{m=1,3,\ldots}^{\infty}\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left[\begin{array}{c}\left(\beta_m\coth\beta_m+2\frac{1+\mu}{1-\mu}\right)\\ \times\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\\
&+\frac{8M^T}{\pi^2 b^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\left(\frac{i^2}{a^2}+\frac{\mu j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\\
&-\frac{8M^T a}{\pi^2 b^3}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{c}\frac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{k^2}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\begin{array}{c}\left(\beta_m\coth\beta_m+2\frac{1+\mu}{1-\mu}\right)\\ \times\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\}\\
M_y =& \frac{4M^T}{\pi}(1-\mu)\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}+(\mu-1)M^T\\
&+\frac{2(3-2\mu)M^T}{\pi}\sum_{m=1,3,\ldots}^{\infty}\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left(\begin{array}{c}\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\\ -\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\end{array}\right)\sin\frac{m\pi x}{a}\\
&+\frac{8M^T}{\pi^2 b^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{j}{i}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\times\left(\frac{\mu i^2}{a^2}+\frac{j^2}{b^2}\right)\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}\\
&-\frac{8M^T a}{\pi^2 b^3}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{c}\frac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{k^2}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\}
\end{aligned}\right.$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$\begin{cases} M_{x10} = \frac{8M^T}{\pi^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \left(\frac{i^2}{a^2} + \frac{\mu j^2}{b^2} \right) \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \\ \quad - \frac{8M^T a}{\pi^2 b^3} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \frac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{c} \left(\beta_m \coth \beta_m + 2 \frac{1+\mu}{1-\mu} \right) \\ \times \sinh \frac{m\pi y}{a} - \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\ M_{y10} = \frac{8M^T}{\pi^2 b^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \times \left(\frac{\mu i^2}{a^2} + \frac{j^2}{b^2} \right) \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \\ \quad - \frac{8M^T a}{\pi^2 b^3} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \frac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\} \end{cases}$$

Let

$$\begin{cases} b_{29} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \mu \frac{j^2}{b^2} \right) \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \\ b_{30} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{j}{i} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \times \left(\frac{\mu i^2}{a^2} + \frac{j^2}{b^2} \right) \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b} \\ b_{31} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \frac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(2 \frac{1+\mu}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \frac{m\pi y}{a} - \\ \frac{m\pi y}{a} \cosh \frac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\ b_{32} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \frac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\} \end{cases}$$

Hence

$$\begin{cases} M_{x10} = \dfrac{8M^T}{\pi^2 b^2} b_{29} - \dfrac{8M^T a}{\pi^2 b^3} b_{31} = k_{x10} M^T \\ M_{y10} = \dfrac{8M^T}{\pi^2 b^2} b_{30} - \dfrac{8M^T a}{\pi^2 b^3} b_{32} = k_{y10} M^T \end{cases}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_{29} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{n}{m} \dfrac{m^2 + \frac{1}{6} \times L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{28} \\ b_{30} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{n}{m} \dfrac{\mu m^2 + n^2 L^2}{(m^2 + n^2 L^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{29} \\ b_{31} = ab \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2 * L}{m^2} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(2\dfrac{1+\mu}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi d}{L} - \\ \dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} \end{array} \right] \sin m\pi c \end{array} \right\} = abd_{30} \\ b_{32} = ab \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{k^2 * L}{m^2} \frac{k^2 \times L^2 + (2-\mu) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} - \beta_m \coth \beta_m \sinh \dfrac{m\pi d}{L} \right) \sin m\pi c \end{array} \right\} = abd_{31} \end{cases}$$

$d_{28}$is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
```

```
sum_x=sin(m*pi*c)*sin(n*pi*d)*n*(m^2+1/6*n^2*L^2)/(m*(m^2+n^2*L^2)
^2);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*n*(m^2+1/6*n^2*L^2)/(m*(m^2
    +n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
n
```

$d_{29}a_{y3}$**is calculated as follows**:

```
syms a b am c d L
num=1;sum_x1=0;m=1;n=1;u=1/6;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=(u*m^2+n^2*L^2)*sin(m*pi*c)sin(n*pi*d)/m*n/(m^2+n^2*L^2)^2;
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    am=0.5*m*pi/L;
    sum_x2=(u*m^2+n^2*L^2)*sin(m*pi*c)sin(n*pi*d)/m*n/(m^2+n^2*L^2)^2;
    sum_x=sum_x1+sum_x2;
end
sum_x
num
```

d_{30}**is calculated as follows**:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
```

```
bm=m*pi/L;%bm=m*pi/L;
sum_x=k^2*L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)^2)/
((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2(1+1/6)/(1-1/6)+bm*coth(bm))*sinh
(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=  k^2*L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)
    ^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2(1+1/6)/(1-1/6)+bm*coth(bm))
    *sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;

end
sum_x
num
m
k
```

d_{31} is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=k^2*L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)^2)/
((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L))-bm*coth
(bm)*sinh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    k=k+2;
```

```
        m=m+2;
        bm=m*pi/L;
        sum_x2= k^2*L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2*(m^2+k^2*L^2)
        ^2)/((3+1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L))-
        bm*coth(bm)*sinh(m*pi*d/L))*sin(m*pi*c);
        sum_x=sum_x1+sum_x2;
    end
    sum_x
    num
    m
    k
```

Case 6: Two adjacent edges simply supported and one edge clamped and one edge free

(1) Deflection

$$\begin{aligned}
w = & -\frac{4a^2M^T}{D\pi^3}\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m^3\cosh\alpha_m}\cosh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)\sin\frac{m\pi x}{a}-\frac{M^T}{2D}(x-a)x \\
& +\frac{2a^2(3-2\mu)M^T}{D\pi^3(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\dfrac{\sinh\beta_m}{m^3\cosh\alpha_m}\sinh\left(\dfrac{2\alpha_m y}{b}-\alpha_m\right)}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\frac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
& +\frac{8M^T}{\pi^4Da^2}\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{i}{j}\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
& -\frac{8M^Ta}{\pi^4Db(1-\mu)}\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}-\dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a}\end{array}\right\}
\end{aligned}$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$w_{11} = \frac{8M^T}{\pi^4 D a^2} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{i}{j} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$- \frac{8M^T a}{\pi^4 D b(1-\mu)} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{r} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi y}{a} - \\ \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \dfrac{m\pi x}{a} \end{array} \right\}$$

Let

$$f_{11} = \frac{8}{\pi^4 a^4} \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{i}{j} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$- \frac{8}{\pi^4 ab(1-\mu)} \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum\limits_{k=1,3,\ldots}^{\infty} \frac{1}{m^2} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\dfrac{2}{1-\mu} + \beta_m \coth \beta_m \right) \sinh \dfrac{m\pi y}{a} \\ - \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\}$$

$$a_{15} = \sum_{i=1,3,\ldots}^{\infty} \sum_{j=1,3,\ldots}^{\infty} \frac{i}{j} \left(\frac{i^2}{a^2} + \frac{j^2}{b^2} \right)^{-2} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{b}$$

$$a_{16} = \sum_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh\beta_m \sum_{k=1,3,\ldots}^{\infty} \frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi y}{a}- \dfrac{m\pi y}{a}\cosh\dfrac{m\pi y}{a}\right]\sin\frac{m\pi x}{a} \end{array} \right\}$$

Hence

$$w_{11} = \frac{8a^2M^T}{\pi^4 D}\times a_{15} - \frac{8a^2M^T}{\pi^4 D(1-\mu)}\times a_{16} = f_{11}\frac{a^2M^T}{D}$$

$$w = (f_1 + f_{11})\frac{a^2M^T}{D} = f\frac{a^2M^T}{D}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$a_{15} = a^4 \sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{n}{m}\left(\frac{1}{m^2+n^2\times L^2}\right)^2 \sin m\pi c\,\sin n\pi d = a^4 c_{14}$$

$$a_{16} = ab\sum_{m=1,3,\ldots}^{\infty}\left\{ \begin{array}{l} \dfrac{\sinh\beta_m \sum_{k=1,3,\ldots}^{\infty}\frac{L}{m^2}\frac{k^2\times L^2+(2-\mu)m^2}{(m^2+k^2\times L^2)^2}}{\dfrac{3+\mu}{1-\mu}\dfrac{\sinh 2\beta_m}{2}+\beta_m} \\ \times\left[\left(\dfrac{2}{1-\mu}+\beta_m\coth\beta_m\right)\sinh\dfrac{m\pi d}{L} - \dfrac{m\pi d}{L}\cosh\dfrac{m\pi d}{L}\right]\sin m\pi c \end{array}\right\} = abc_{15}$$

$c_{14}$is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
```

```
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*n/(m*(m^2+n^2*L^2)^2);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*n/(m*(m^2+n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
n
```

$c_{15}$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=L/m^2*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh(m*pi*d/L)-
m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=L/m^2*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3
    +1/6)/(1-1/6)*(sinh(2*bm)/2)+bm)*((2/(1-1/6)+bm*coth(bm))*sinh
    (m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L))*sin(m*pi*c);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
k
```

(2) Bending moment

$$
\begin{cases}
M_x = \dfrac{4M^T}{\pi}(\mu-1)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a} \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left[\begin{array}{l}\left(\beta_m\coth\beta_m+2\frac{1+\mu}{1-\mu}\right)\\ \times\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a} \\
\quad + \dfrac{8M^T}{\pi^2a^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{i}{j}\left(\frac{i^2}{a^2}+\mu\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
\quad - \dfrac{8M^T}{\pi^2ab}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left[\begin{array}{c}\left(\beta_m\coth\beta_m+2\frac{1+\mu}{1-\mu}\right)\\ \times\sinh\frac{m\pi y}{a}-\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\end{array}\right]\sin\frac{m\pi x}{a}\end{array}\right\} \\
M_y = \dfrac{4M^T}{\pi}(1-\mu)\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{1}{m\cosh\alpha_m}\cosh\left(\frac{m\pi y}{a}-\alpha_m\right)\sin\frac{m\pi x}{a}+(\mu-1)M^T \\
\quad + \dfrac{2(3-2\mu)M^T}{\pi}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\frac{\frac{\sinh\beta_m}{m\cosh\alpha_m}\sinh\left(\frac{2\alpha_m y}{b}-\alpha_m\right)}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\left(\begin{array}{l}\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}\\ -\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\end{array}\right)\sin\frac{m\pi x}{a} \\
\quad + \dfrac{8M^T}{\pi^2a^2}\displaystyle\sum_{i=1,3,\ldots}^{\infty}\sum_{j=1,3,\ldots}^{\infty}\frac{i}{j}\left(\mu\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)\left(\frac{i^2}{a^2}+\frac{j^2}{b^2}\right)^{-2}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b} \\
\quad - \dfrac{8M^T}{\pi^2ab}\displaystyle\sum_{m=1,3,\ldots}^{\infty}\left\{\begin{array}{l}\dfrac{\sinh\beta_m\sum_{k=1,3,\ldots}^{\infty}\left(\frac{m^2}{a^2}+\frac{k^2}{b^2}\right)^{-2}\left[\frac{k^2}{b^2}+(2-\mu)\frac{m^2}{a^2}\right]}{\frac{3+\mu}{1-\mu}\frac{\sinh 2\beta_m}{2}+\beta_m}\\ \times\left(\frac{m\pi y}{a}\cosh\frac{m\pi y}{a}-\beta_m\coth\beta_m\sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a}\end{array}\right\}
\end{cases}
$$

In above equation, the first part can be obtained by Appendix B Case 1. For the second part, Taking $x = a/2$, $y = b/2$, the second part is zero.

For the last two part, there is

$$\begin{cases} M_{x11} = \dfrac{8M^T}{\pi^2 a^2} \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ \qquad - \dfrac{8M^T}{\pi^2 ab} \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \dfrac{1+\mu}{1-\mu} \right) \\ \times \sinh \dfrac{m\pi y}{a} - \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\ M_{y11} = \dfrac{8M^T}{\pi^2 a^2} \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ \qquad - \dfrac{8M^T}{\pi^2 ab} \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\} \end{cases}$$

Let

$$\begin{cases} b_{33} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\dfrac{i^2}{a^2} + \mu \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ b_{34} = \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{i}{j} \left(\mu \dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right) \left(\dfrac{i^2}{a^2} + \dfrac{j^2}{b^2} \right)^{-2} \sin \dfrac{i\pi x}{a} \sin \dfrac{j\pi y}{b} \\ b_{35} = \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \dfrac{1+\mu}{1-\mu} \right) \\ \times \sinh \dfrac{m\pi y}{a} - \dfrac{m\pi y}{a} \cosh \dfrac{m\pi y}{a} \end{array} \right] \sin \frac{m\pi x}{a} \end{array} \right\} \\ b_{36} = \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} \left(\frac{m^2}{a^2} + \frac{k^2}{b^2} \right)^{-2} \left[\frac{k^2}{b^2} + (2-\mu) \frac{m^2}{a^2} \right]}{\dfrac{3+\mu}{1-\mu} \dfrac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi y}{a} \cosh \frac{m\pi y}{a} - \beta_m \coth \beta_m \sinh \frac{m\pi y}{a} \right) \sin \frac{m\pi x}{a} \end{array} \right\} \end{cases}$$

Hence

$$\begin{cases} M_{x11} = \dfrac{8M^T}{\pi^2} \times b_{33} - \dfrac{8M^T}{\pi^2} \times b_{35} = k_{x11}M^T \\ M_{y11} = \dfrac{8M^T}{\pi^2} \times b_{34} - \dfrac{8M^T}{\pi^2} \times b_{36} = k_{y11}M^T \end{cases}$$

Taking $c = x/a$, $d = y/b$, $L = a/b$, there is

$$\begin{cases} b_{33} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{m}{n} \dfrac{m^2 + \frac{1}{6} \times L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{32} \\ b_{34} = a^2 \sum\limits_{i=1,3,\ldots}^{\infty} \sum\limits_{j=1,3,\ldots}^{\infty} \dfrac{m}{n} \dfrac{\frac{1}{6} \times m^2 + L^2 \times n^2}{(m^2 + L^2 \times n^2)^2} \sin m\pi c \, \sin n\pi d = a^2 d_{33} \\ b_{35} = ab \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} L \frac{k^2 \times L^2 + \left(2 - \frac{1}{6}\right) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left[\begin{array}{l} \left(\beta_m \coth \beta_m + 2 \dfrac{1+\mu}{1-\mu} \right) \\ \times \, \sinh \dfrac{m\pi d}{L} - \dfrac{m\pi d}{L} \cosh \dfrac{m\pi d}{L} \end{array} \right] \sin \frac{m\pi c}{L} \end{array} \right\} = abd_{34} \\ b_{36} = ab \sum\limits_{m=1,3,\ldots}^{\infty} \left\{ \begin{array}{l} \dfrac{\sinh \beta_m \sum_{k=1,3,\ldots}^{\infty} L \frac{k^2 \times L^2 + \left(2 - \frac{1}{6}\right) m^2}{(m^2 + k^2 \times L^2)^2}}{\frac{3+\mu}{1-\mu} \frac{\sinh 2\beta_m}{2} + \beta_m} \\ \times \left(\frac{m\pi d}{L} \cosh \frac{m\pi d}{L} - \beta_m \coth \beta_m \sinh \frac{m\pi d}{L} \right) \sin m\pi c \end{array} \right\} = abd_{35} \end{cases}$$

$d_{32}$is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*m*(m^2+1/6*n^2*L^2)/(n*(m^2+n^2*L^2)
^2);
while abs(sum_x-sum_x1)>=1.0e-05

    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
```

```
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*m*(m^2+1/6*n^2*L^2)/(n*(m^2
    +n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
n
```

d_{33} is calculated as follows:

```
syms a b c d L
num=1;sum_x1=0;m=1;n=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
sum_x=sin(m*pi*c)*sin(n*pi*d)*m*(1/6*m^2+n^2*L^2)/(n*(m^2+n^2*L^2)
^2);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    m=m+2;
    n=n+2;
    sum_x2=sin(m*pi*c)*sin(n*pi*d)*m*(1/6*m^2+n^2*L^2)/(n*(m^2
    +n^2*L^2)^2);
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
n
```

d_{34} is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the value of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*sin(m*pi*c)*((2*(1+1/6)/(1-1/6)+bm*coth(bm))
*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L));
while abs(sum_x-sum_x1)>=1.0e-05
```

```
    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
    (1-1/6)*(sinh(2*bm)/2)+bm)*sin(m*pi*c)*((2*(1+1/6)/(1-1/6)+bm*coth
    (bm))*sinh(m*pi*d/L)-m*pi*d/L*cosh(m*pi*d/L));
    sum_x=sum_x1+sum_x2;
end
sum_x
num
m
k
```

$d_{35}$is calculated as follows:

```
syms a b bm c d L
num=1;sum_x1=0;m=1;k=1;
L=input('enter the value of the ratio of a to b>');
c=input('enter the value of the ratio of x_axis coordinate to a>');
d=input('enter the walue of the ratio of y_axis coordinate to b>');
bm=m*pi/L;%bm=m*pi/L;
sum_x=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
(1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L)-bm*coth(bm)*sinh
(m*pi*d/L))*sin(m*pi*c);
while abs(sum_x-sum_x1)>=1.0e-05
    sum_x1=sum_x;
    num=num+1;
    k=k+2;
    m=m+2;
    bm=m*pi/L;
    sum_x2=L*sinh(bm)*(k^2*L^2+(2-1/6)*m^2)/(m^2+k^2*L^2)^2/((3+1/6)/
    (1-1/6)*(sinh(2*bm)/2)+bm)*(m*pi*d/L*cosh(m*pi*d/L)-bm*coth(bm)
    *sinh(m*pi*d/L))*sin(m*pi*c);
end
sum_x
num
m
k
```

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X. Cheng, *Thermal Elastic Mechanics Problems of Concrete Rectangular Thin Plate*, Springer Tracts in Civil Engineering,
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