Epilogue

We focused the aim of this book on introduction to surface-knots, and we have not discussed here many advanced topics. This epilogue is devoted to short comments on biquandles, invariants of surface-knots via marked graph diagrams,[1] problems on P^2-knots and a concluding remark.

The notion of a quandle is generalized to the notion of a biquandle.[2] The fundamental biquandle for an oriented surface-knot was introduced by T. Carrell [15] in 2009. Although it is defined by using a diagram in 3-space, S. Ashihara [8] introduced, under the supervision of the author, a method of computing, or redefining, the fundamental biquandle of an oriented surface-knot by using a marked graph diagram. J. Kim, Y. Joung, and S.Y. Lee [102] used the method for computing, or redefining, the Alexander biquandles of oriented surface-knots via marked graph diagrams. Computation of invariants related to quandles and biquandles via marked graph diagrams is convenient in many situations. For example, S. Kamada, J. Kim, and S.Y. Lee [84] used marked graph diagrams to compute quandle cocycle invariants of surface-knots.[3]

Besides invariants related to quandles and biquandles, some invariants may be defined, redefined or computed via marked graph diagrams. For example, the knot group is easily computed via a marked graph diagram even if the surface-knot is non-orientable. One of the most attractive approaches to construction of surface-knot invariants via marked graph diagrams is the one due to S.Y. Lee in the 2008 and 2009 papers [107, 108].[4]

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[1] Here we call a ch-diagram a marked graph diagram.

[2] Refer to L.H. Kauffman and D.E. Radford [89] and R. Fenn, M. Jordan-Santana and L.H. Kauffman [32].

[3] J. Kim is now visiting Osaka and working on biquandle cocycle invariants of surface-knots.

[4] For example, refer to Y. Joung, S. Kamada and S.Y. Lee [63] or Y. Joung, S. Kamada, A. Kawauchi and S.Y. Lee [62].

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Further research on non-orientable surface-knots is hoped for. Here are two challenging problems: Kinoshita's problem on P^2-knots (Sect. 1.4) and a problem asking whether $P_+\#\tau^m(K)$ and $P_+\#\tau^n(K)$ are equivalent when $m \equiv n \mod 2$ for every knot K. Here $\tau^m(K)$ is the m-twist spun K and P_+ is the standard projective plane. This problem when m is an odd integer was proposed by P. Melvin for R. Kirby's collection of problems [104]. It is still open even if K is the trefoil knot. A technique used in O. Viro [177] and Y. Bae, J.S. Carter, S. Choi, and S. Kim [9] might be helpful.[5]

As seen in this book, surface-knots may be described using various methods: movies (in the motion picture method), marked graph diagrams, surface diagrams, braid monodormies and braid charts (in 2-dimensional braid presentations). Enumeration and computation of invariants via movies, marked graph diagrams, and braid charts would be important in further research on surface-knots.

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[5]The argument in Y. Marumoto and Y. Nakanishi [118] is also interesting.

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