Appendix A
SU(3) Clebsch-Gordan Series 8⊗35

In this appendix, we provide isoscalar factors of SU(3) Clebsch-Gordan series **8**⊗**35** which are extensions of the previous works of de Swart, McNamee and Chilton and play practical roles in current ongoing strange flavor hadron physics researches. To this end, we pedagogically study the SU(3) Lie algebra, its spin symmetries and eigenvalues for irreducible representations. We also evaluate values of Wigner *D* functions related with the isoscalar factors, which are immediately applicable to strange flavor hadron phenomenology. Exploiting SU(3) group properties associated with spin symmetries, we investigate decuplet-to-octet transition magnetic moments and baryon octet and decuplet magnetic moments in a flavor symmetric limit, to construct Coleman-Glashow type sum rules [293].

Now, after constructing the magnetic moments of the octet and the decuplet baryons, we formulate sum rules among the magnetic moments, which produce strange form factor predictions successively [58, 118–120, 129, 131]. In chiral theory, we need practically SU(3) flavor group analysis to construct theoretical hadron physics formula. We note that the SU(3) group structure [294–298] is generic property shared by chiral models which exploit a hedgehog ansatz solution corresponding to little group $SU(2) \times Z_2$ [261].

The SU(3) isoscalar factors are given in Refs. [299, 300] which are beneficial to strange flavor related physics. However, in order to perform strange hadron physics researches involving predictions of ongoing experimental data, we have necessities to update information of the SU(3) isoscalar factors.

Now, in this appendix we list up explicit values of the SU(3) isoscalar factors for Clebsch-Gordan series **8**⊗**35**, which are absent in the previous works [299, 300], and parts of which are necessary and useful in ongoing researches. As heuristic applications of the isoscalar factors for the series **8**⊗**35**, we also evaluate and summarize values of the Wigner *D* functions, parts of which can be directly applied to the strange flavor hadron physics of interest. We apply these SU(3) group

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properties related with spin symmetries to the baryon octet and decuplet magnetic moments and to the decuplet-to-octet transition magnetic moments, to obtain their Coleman-Glashow [222] type sum rules [293].

We start with SU(3) group Lie algebra associated with eight generators λ_a ($a = 1, 2, \ldots, 8$). These generators can be expressed by using Gell-Mann matrices satisfying

$$\begin{aligned} \mathrm{tr}(\lambda_a \lambda_b) &= 2\delta_{ab}, \\ [\lambda_a, \lambda_b] &= 2if_{abc}\lambda_c, \\ f_{abc} &= \frac{1}{4i}\mathrm{tr}([\lambda_a, \lambda_b]\lambda_c). \end{aligned} \tag{A.1}$$

In hadron physics, we have

$$\begin{aligned} \hat{I}_i &= \frac{1}{2}\lambda_i, \quad i = 1, 2, 3, \\ \hat{Y} &= \frac{1}{\sqrt{3}}\lambda_8, \end{aligned} \tag{A.2}$$

which are the isospin generators and the hypercharge one, respectively. In particular, combining the diagonal generators λ_3 and λ_8 one can construct the electromagnetic charge operator $\hat{Q}_{EM}$ given by the following Gell-Mann-Nishijima relation

$$\hat{Q}_{EM} = e\left(\hat{I}_3 + \frac{1}{2}\hat{Y}\right) = \frac{e}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right), \tag{A.3}$$

where e ($e > 0$) is magnitude of an electron charge. The other four generators λ_M ($M = 4, 5, 6, 7$) connect the isospins and the hypercharge to yield enlarged group SU(3) coming from SU(2) × U(1). The finite SU(3) transformation is given as,

$$U = e^{-i\rho\lambda_8/\sqrt{3}} e^{-i\alpha\lambda_3/2} e^{-i\beta\lambda_2/2} e^{-i\gamma\lambda_3/2} e^{-i(\delta\lambda_4 + \delta'\lambda_5 + \epsilon\lambda_6 + \epsilon'\lambda_7)}, \tag{A.4}$$

which can be also rewritten in the form [301, 302]

$$U = e^{-i\alpha\lambda_3/2} e^{-i\beta\lambda_2/2} e^{-i\gamma\lambda_3/2} e^{-i\rho\lambda_8/\sqrt{3}} e^{-i\delta\lambda_4} e^{-i\alpha'\lambda_3/2} e^{-i\beta'\lambda_2/2} e^{-i\gamma'\lambda_3/2}. \tag{A.5}$$

Here, the angle variables δ', ϵ and ϵ' in Eq. (A.4) are reshuffled to yield the new angle variables α', β' and γ' in Eq. (A.5), and we have used the identity

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!}[A, [A, B]] + \cdots . \tag{A.6}$$

The SU(3) group has two Casimir operators C_2 and C_3 which are given in terms of λ_a as follows

$$C_2 = \frac{1}{4}\sum_{a=1}^{8}\lambda_a^2,$$

$$C_3 = \frac{1}{4}\lambda_1(\{\lambda_4,\lambda_6\}+\{\lambda_5,\lambda_7\}) + \frac{1}{4}\lambda_2(-\{\lambda_4,\lambda_7\}+\{\lambda_5,\lambda_6\})$$
$$+\frac{1}{4}\lambda_3(\lambda_4^2+\lambda_5^2-\lambda_6^2-\lambda_7^2) + \frac{1}{\sqrt{3}}\lambda_8\left(\frac{1}{2}(\lambda_1^2+\lambda_2^2+\lambda_3^2)\right.$$
$$\left.-\frac{1}{6}\lambda_8^2 - 1 - \frac{1}{4}(\lambda_4^2+\lambda_5^2+\lambda_6^2+\lambda_7^2)\right). \tag{A.7}$$

Next, we use (λ, μ) and (Y, I, I_3) to denote irreducible representation and the state within the irreducible representation. For instance, $\{\lambda_1, \lambda_2, \lambda_3\}$ are the basis states chosen such that

$$I^2 = \frac{1}{4}(\lambda_1^2+\lambda_2^2+\lambda_3^2), \quad [I_i, I_j] = \epsilon_{ijk} I_k, \tag{A.8}$$

and then SU(3) has the isospin rotation group SU(2) as a subgroup, as expected. In SU(3) algebra, λ_3 and λ_8 are diagonal and satisfy

$$\langle YII_3|e^{-i\alpha\lambda_3/2}e^{-i\beta\lambda_2/2}e^{-i\gamma\lambda_3/2}|Y'I'I_3'\rangle = D^I_{I_3 I_3'}(\alpha,\beta,\gamma)\delta_{YY'}\delta_{II'}, \tag{A.9}$$

$$\langle YII_3|e^{-i\rho\lambda_8/\sqrt{3}}|Y'I'I_3'\rangle = e^{-i\rho Y'}\delta_{YY'}\delta_{II'}\delta_{I_3 I_3'}. \tag{A.10}$$

In order to discuss the I-, U- and V-spin symmetries of the SU(3) group, we introduce the root diagram approach to the construction of the Lie algebra of the SU(3) group which has eight generators. Since the rank of the SU(3) group is two, one can have the Cartan subalgebra [296, 297], the set of two commuting generators H_i $(i = 1, 2)$

$$[H_1,\ H_2] = 0, \tag{A.11}$$

and the other generators E_α $(\alpha = \pm1, \pm2, \pm3)$ satisfying the commutator relations

$$[H_i,\ E_\alpha] = e_i^\alpha E_\alpha,$$
$$\left[E_\alpha,\ E_\beta\right] = c_{\alpha\beta}E_\gamma,$$
$$[E_\alpha,\ E_{-\alpha}] = e_i^\alpha H_i, \tag{A.12}$$

where e_i^α $(i = 1, 2)$ are the i-th component of the root vector $\hat{e}^\alpha$ in two-dimensional root space and $c_{\alpha\beta}$ are normalization constants. Here, H_i is the Hermitian operator $H_i^\dagger = H_i$ and $E_{-\alpha}$ is the Hermitian conjugate of E_α, namely $E_\alpha^\dagger = E_{-\alpha}$.

Normalizing the root vectors such that

$$\sum_\alpha e_i^\alpha e_j^\alpha = \delta_{ij}, \tag{A.13}$$

one can choose the root vectors

$$\hat{e}^1 = -\hat{e}^{-1} = \left(\frac{1}{\sqrt{3}}, 0\right),$$
$$\hat{e}^2 = -\hat{e}^{-2} = \left(\frac{1}{2\sqrt{3}}, \frac{1}{2}\right),$$
$$\hat{e}^3 = -\hat{e}^{-3} = \left(-\frac{1}{2\sqrt{3}}, \frac{1}{2}\right), \tag{A.14}$$

and one has two simple roots $\hat{e}^2$ and $\hat{e}^{-3}$ of the equal length separated by angle $\frac{2\pi}{3}$ so that one can obtain the Dynkin diagram [296, 297] for the SU(3) Lie algebra given by the group theoretical symbol ⚬–⚬. Next, $c_{\alpha\beta}$ satisfy the identities

$$c_{\alpha\beta} = -c_{\beta\alpha} = -c_{-\alpha,-\beta} = c_{\beta,-\gamma} = c_{-\gamma,\alpha},$$
$$c_{\alpha\beta}c_{\alpha+\beta,\rho} + c_{\rho\alpha}c_{\alpha+\rho,\beta} + c_{\beta\rho}c_{\beta+\rho,\alpha} = 0. \tag{A.15}$$

Explicitly, we have $c_{13} = c_{3,-2} = c_{-1,2} = \frac{1}{\sqrt{6}}$ [303]. Moreover, the matrix representations of the SU(3) generators H_i and E_α can be given in terms of the Gell-Mann matrices,

$$H_1 = \frac{1}{2\sqrt{3}}\lambda_3,\ H_2 = \frac{1}{2\sqrt{3}}\lambda_8,\ E_{\pm 1} = \frac{1}{2\sqrt{3}}(\lambda_1 \pm i\lambda_2),$$
$$E_{\pm 2} = \frac{1}{2\sqrt{3}}(\lambda_4 \pm i\lambda_5),\ E_{\pm 3} = \frac{1}{2\sqrt{3}}(\lambda_6 \pm i\lambda_7). \tag{A.16}$$

Substituting the root vectors normalized as in Eq. (A.14) into the relations (A.11) and (A.12) one can readily derive the commutator relations [299]

$$\begin{aligned}
&[H_1,\ H_2] = 0, && [H_1,\ E_1] = \frac{1}{\sqrt{3}}E_1,\\
&[H_1,\ E_2] = \frac{1}{2\sqrt{3}}E_2, && [H_1,\ E_3] = -\frac{1}{2\sqrt{3}}E_3,\\
&[H_2,\ E_1] = 0, && [H_2,\ E_2] = \frac{1}{2}E_2,\\
&[H_2,\ E_3] = \frac{1}{2}E_3, && [E_1,\ E_{-1}] = \frac{1}{\sqrt{3}}H_1,\\
&[E_2,\ E_{-2}] = \frac{1}{2\sqrt{3}}H_1 + \frac{1}{2}H_2, &&\\
&[E_3,\ E_{-3}] = -\frac{1}{2\sqrt{3}}H_1 + \frac{1}{2}H_2, && [E_1,\ E_3] = \frac{1}{\sqrt{6}}E_2,\\
&[E_2,\ E_{-3}] = -\frac{1}{2\sqrt{3}}H_1 + \frac{1}{2}H_2, && [E_1,\ E_3] = \frac{1}{\sqrt{6}}E_2,\\
&[E_2,\ E_{-3}] = \frac{1}{\sqrt{6}}E_1, && [E_{-1},\ E_2] = \frac{1}{\sqrt{6}}E_3.
\end{aligned} \tag{A.17}$$

Associating the root vectors H_i ($i = 1, 2$) and E_α ($\alpha = \pm 1, \pm 2, \pm 3$) with the physical operators Y, I_3, $I_\pm$, $U_\pm$ and $V_\pm$ through the definitions

$$\begin{aligned} H_1 &= \frac{1}{\sqrt{3}} I_3, \quad H_2 = \frac{1}{2} Y, \\ E_{\pm 1} &= \frac{1}{\sqrt{6}} I_\pm,\ E_{\pm 2} = \frac{1}{\sqrt{6}} V_\pm,\ E_{\pm 3} = \frac{1}{\sqrt{6}} U_\pm, \end{aligned} \tag{A.18}$$

we can use the commutator relations (A.17) to yield the explicit expressions for the eigenvalue equations of the spin operators in the SU(3) group [299]

$$\begin{aligned} I_+|Y, I, I_3\rangle &= [(I - I_3)(I + I_3 + 1)]^{\frac{1}{2}} |Y, I, I_3 + 1\rangle, \\ I_-|Y, I, I_3\rangle &= [(I + I_3)(I - I_3 + 1)]^{\frac{1}{2}} |Y, I, I_3 - 1\rangle, \\ U_+|Y, I, I_3\rangle &= [a_+(I - I_3 + 1)]^{\frac{1}{2}} |Y + 1, I + \frac{1}{2}, I_3 - \frac{1}{2}\rangle \\ &\quad - [a_-(I + I_3)]^{\frac{1}{2}} |Y + 1, I - \frac{1}{2}, I_3 - \frac{1}{2}\rangle, \\ U_-|Y, I, I_3\rangle &= -[b_+(I + I_3 + 1)]^{\frac{1}{2}} |Y - 1, I + \frac{1}{2}, I_3 + \frac{1}{2}\rangle \\ &\quad + [b_-(I - I_3)]^{\frac{1}{2}} |Y - 1, I - \frac{1}{2}, I_3 + \frac{1}{2}\rangle, \\ V_+|Y, I, I_3\rangle &= [a_+(I + I_3 + 1)]^{\frac{1}{2}} |Y + 1, I + \frac{1}{2}, I_3 + \frac{1}{2}\rangle \\ &\quad + [a_-(I - I_3)]^{\frac{1}{2}} |Y + 1, I - \frac{1}{2}, I_3 + \frac{1}{2}\rangle, \\ V_-|Y, I, I_3\rangle &= [b_+(I - I_3 + 1)]^{\frac{1}{2}} |Y - 1, I + \frac{1}{2}, I_3 - \frac{1}{2}\rangle \\ &\quad + [b_-(I + I_3)]^{\frac{1}{2}} |Y - 1, I - \frac{1}{2}, I_3 - \frac{1}{2}\rangle. \end{aligned} \tag{A.19}$$

In Fig. A.1 is depicted the $I_\pm$-, $U_\pm$- and $V_\pm$-spin symmetry operation diagram in the case of the decuplet baryons. In Eq. (A.19), we have used the de Swart phase convention [299] and

$$\begin{aligned} a_+ &= \frac{(Y_+ + 1)(Y_+ + q + 2)(-Y_+ + p)}{2(I + 1)(2I + 1)}, \\ a_- &= \frac{Y_-(Y_- + q + 1)(Y_- - p - 1)}{2I(2I + 1)}, \\ b_+ &= \frac{(Y_- - 1)(Y_- + q)(Y_- - p - 2)}{2(I + 1)(2I + 1)}, \end{aligned}$$

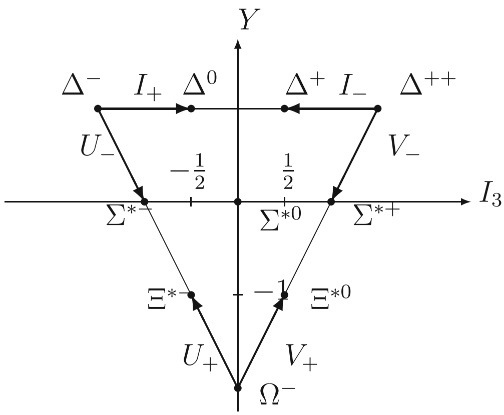

Fig. A.1 $I_\pm$-, $U_\pm$- and $V_\pm$-spin symmetry operations in the baryon decuplet

$$b_- = \frac{Y_+(Y_+ + q + 1)(-Y_+ + p + 1)}{2I(2I+1)}, \tag{A.20}$$

with $Y_\pm = \frac{1}{2}Y \pm I + \frac{1}{3}(p-q)$. Here, p and q are nonnegative coefficients needed to construct bases for the irreducible representations $D(p,q)$ of SU(3) group. The dimension $\mathbf{n}$ of $D(p,q)$, namely the number of the basis vectors, is then given by

$$\mathbf{n} = \frac{(p+1)(q+1)(p+q+2)}{2}, \tag{A.21}$$

to yield the irreducible representations $\mathbf{1} = D(0,0)$, $\mathbf{3} = D(1,0)$, $\bar{\mathbf{3}} = D(0,1)$, $\mathbf{8} = D(1,1)$, $\mathbf{10} = D(3,0)$, $\overline{\mathbf{10}} = D(0,3)$, $\overline{\mathbf{27}} = D(2,2)$, $\mathbf{35} = D(4,1)$, $\overline{\mathbf{35}} = D(1,4)$, $\mathbf{28} = D(6,0)$, $\mathbf{64} = D(3,3)$, $\overline{\mathbf{81}} = D(5,2)$ and $\overline{\mathbf{81}} = D(2,5)$ [299, 300].

Now, we first investigate the isoscalar factors for **8⊗35**. To this end we consider Fig. A.2 in which are depicted the eigenvalue diagrams for the lowest irreducible representations. For the dimension $\mathbf{n} = D(p,q)$, we have the highest eigenvalue e_H and its corresponding integer hypercharge Y_H defined as [299]

$$e_H = \left(\frac{p+q}{2}, \frac{p-q}{2\sqrt{3}}\right), \; Y_H = \frac{p-q}{3}, \tag{A.22}$$

in the (I_3, Y) coordinates. For instances, e_H are denoted by the solid disks at $(1,0)$ and $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ in the diagrams in Fig. A.2 for the **8** and **10**, respectively.

Starting from the solid disk e_H for a given dimension, and applying to the solid disk the spin operators $U_\pm$ and $V_\pm$ and the relations (A.19), we construct effectively the irreducible representations $(I_3,\; Y)$ denoted by the points along the lines indicated in Fig. A.2. Similarly, we act the spin operator $I_\pm$ on the solid disks and points and use the relations (A.19) to yield the remnant irreducible representations $(I_3,\; Y)$ denoted by the points in Fig. A.2, so that we can derive

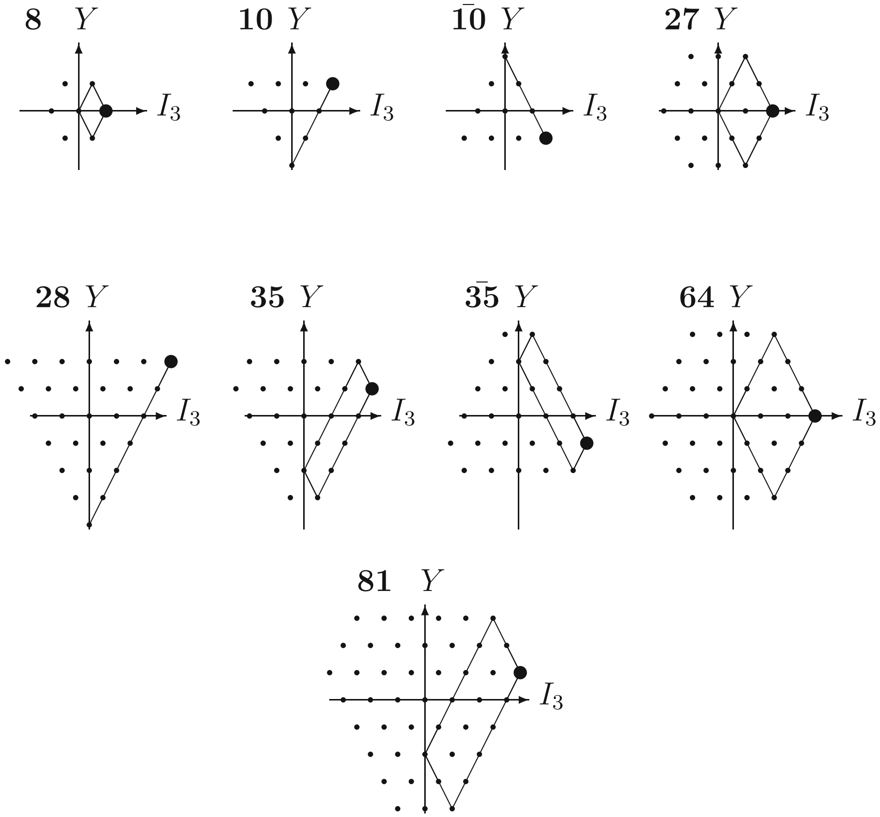

Fig. A.2 Eigenvalue diagrams for the lowest irreducible representations

the isoscalar factors of the SU(3) group for the Clebsch-Gordan series as shown in Table A.1. The Clebsch-Gordan coefficients of SU(3) group are given by [299, 304]

$$\begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix} = C^{I_{1z} I_{2z} I_z}_{I_1 I_2 I} \left(\begin{matrix} \mu_1 & \mu_2 \\ Y_1 I_1 & Y_2 I_2 \end{matrix} \middle| \begin{matrix} \mu_\gamma \\ Y I \end{matrix} \right), \tag{A.23}$$

where the first part of the right hand side is the Clebsch-Gordan coefficient of SU(2) group and the second one is the isoscalar factor of the SU(3) group. In order to evaluate uniquely the SU(3) Clebsch-Gordan coefficients, it suffices to give the SU(3) isoscalar factors, since the SU(2) Clebsch-Gordan coefficients are well known. In Table A.1, we list the isoscalar factors of the SU(3) group for the Clebsch-Gordan series **8**⊗**35** with $\mu_1 = \mathbf{8}$ and $\mu_2 = \mathbf{35}$. In the first row of each table in Table A.1, we have (Y, I) for μ_γ being given by the right hand side of the Clebsch-Gordan series **8**⊗**35** = **81** ⊕**64**⊕**35**⊕**35**⊕**28**⊕**27**⊕ **10**. In the following rows, we have two pairs for (Y_1, I_1) of **8** and (Y_2, I_2) of **35**, and the corresponding SU(3)

Table A.1 The isoscalar factors for **8⊗35**. In the first row of each table, we have (Y, I) for μ_γ being given by the right hand side of the Clebsch-Gordon series **8⊗35** = **81⊕64⊕35⊕35⊕28⊕27⊕10**. In the following rows, two pairs for (Y_1, I_1) of **8** and (Y_2, I_2) of **35** are given together with the corresponding SU(3) isoscalar factor values under the dimensions μ_γ

$Y = 3$	$I = \frac{5}{2}$	**81**
$1, \frac{1}{2}$	2,2	1

$Y = 3$	$I = \frac{3}{2}$	**64**
$1, \frac{1}{2}$	2, 2	−1

$Y = 2$	$I = 3$	**81**	**28**
$1, \frac{1}{2}$	$1, \frac{5}{2}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$
0, 1	2, 2	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$

$Y = 2$	$I = 2$	**81**	**64**	**35**$_S$	**35**$_A$
$1, \frac{1}{2}$	$1, \frac{5}{2}$	$\sqrt{\frac{1}{200}}$	$-\sqrt{\frac{8}{25}}$	$\sqrt{\frac{81}{180}}$	$\sqrt{\frac{81}{360}}$
$1, \frac{1}{2}$	$1, \frac{5}{2}$	$\sqrt{\frac{144}{200}}$	$\sqrt{\frac{2}{25}}$	$-\sqrt{\frac{4}{180}}$	$\sqrt{\frac{64}{360}}$
0, 1	2, 2	$-\sqrt{\frac{10}{200}}$	$-\sqrt{\frac{5}{25}}$	$-\sqrt{\frac{90}{180}}$	$\sqrt{\frac{90}{360}}$
0, 0	2, 2	$\sqrt{\frac{45}{200}}$	$-\sqrt{\frac{10}{25}}$	$-\sqrt{\frac{5}{180}}$	$-\sqrt{\frac{125}{360}}$

$Y = 2$	$I = 1$	**64**	**27**
$1, \frac{1}{2}$	$1, \frac{3}{2}$	$-\sqrt{\frac{6}{7}}$	$-\sqrt{\frac{1}{7}}$
0, 1	2, 2	$\sqrt{\frac{1}{7}}$	$-\sqrt{\frac{6}{7}}$

$Y = 1$	$I = \frac{7}{2}$	**81**
0, 1	$1, \frac{5}{2}$	1

$Y = 1$	$I = \frac{5}{2}$	**81**	**64**	**35**$_S$	**35**$_A$	**28**
$1, \frac{1}{2}$	0, 2	$\sqrt{\frac{280}{800}}$	$\sqrt{\frac{5}{75}}$	$-\sqrt{\frac{120}{720}}$	$\sqrt{\frac{120}{1{,}440}}$	$-\sqrt{\frac{40}{120}}$
0, 1	$1, \frac{5}{2}$	$-\sqrt{\frac{9}{800}}$	$-\sqrt{\frac{14}{75}}$	$-\sqrt{\frac{21}{720}}$	$\sqrt{\frac{1{,}029}{1{,}440}}$	$\sqrt{\frac{7}{120}}$
0, 1	$1, \frac{3}{2}$	$\sqrt{\frac{336}{800}}$	$\sqrt{\frac{6}{75}}$	$\sqrt{\frac{64}{720}}$	$\sqrt{\frac{16}{1{,}440}}$	$\sqrt{\frac{48}{120}}$
0, 0	$1, \frac{5}{2}$	$\sqrt{\frac{105}{800}}$	$-\sqrt{\frac{30}{75}}$	$\sqrt{\frac{245}{720}}$	$-\sqrt{\frac{5}{1{,}440}}$	$-\sqrt{\frac{15}{120}}$
$-1, \frac{1}{2}$	2, 2	$\sqrt{\frac{70}{800}}$	$-\sqrt{\frac{20}{75}}$	$-\sqrt{\frac{270}{720}}$	$-\sqrt{\frac{270}{1{,}440}}$	$\sqrt{\frac{10}{120}}$

(continued)

Table A.1 (continued)

$Y=1$	$I=\frac{3}{2}$	**81**	**64**	**35_S**	**35_A**	**27**	**10**
$1, \frac{1}{2}$	$0, 2$	$\sqrt{\frac{15}{1,400}}$	$-\sqrt{\frac{45}{175}}$	$\sqrt{\frac{245}{720}}$	$\sqrt{\frac{80}{360}}$	$-\sqrt{\frac{45}{560}}$	$\sqrt{\frac{5}{56}}$
$1, \frac{1}{2}$	$0, 1$	$\sqrt{\frac{675}{1,400}}$	$\sqrt{\frac{25}{175}}$	$-\sqrt{\frac{25}{720}}$	$\sqrt{\frac{100}{360}}$	$\sqrt{\frac{25}{560}}$	$\sqrt{\frac{1}{56}}$
$0, 1$	$1, \frac{5}{2}$	$-\sqrt{\frac{2}{1,400}}$	$\sqrt{\frac{6}{175}}$	$-\sqrt{\frac{96}{720}}$	$-\sqrt{\frac{6}{360}}$	$-\sqrt{\frac{216}{560}}$	$\sqrt{\frac{24}{56}}$
$0, 1$	$1, \frac{3}{2}$	$-\sqrt{\frac{108}{1,400}}$	$-\sqrt{\frac{49}{175}}$	$-\sqrt{\frac{289}{720}}$	$\sqrt{\frac{49}{360}}$	$\sqrt{\frac{49}{560}}$	$\sqrt{\frac{1}{56}}$
$0, 0$	$1, \frac{3}{2}$	$\sqrt{\frac{540}{1,400}}$	$-\sqrt{\frac{45}{175}}$	$-\sqrt{\frac{45}{720}}$	$-\sqrt{\frac{45}{360}}$	$-\sqrt{\frac{45}{560}}$	$-\sqrt{\frac{5}{56}}$
$-1, \frac{1}{2}$	$2, 2$	$-\sqrt{\frac{60}{1,400}}$	$\sqrt{\frac{5}{175}}$	$-\sqrt{\frac{20}{720}}$	$\sqrt{\frac{80}{360}}$	$-\sqrt{\frac{180}{560}}$	$-\sqrt{\frac{20}{56}}$

$Y=1$	$I=\frac{1}{2}$	**64**	**27**
$1, \frac{1}{2}$	$0, 1$	$-\sqrt{\frac{5}{7}}$	$-\sqrt{\frac{2}{7}}$
$0, 1$	$1, \frac{3}{2}$	$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{5}{7}}$

$Y=0$	$I=3$	**81**	**64**
$0, 1$	$0, 2$	$\sqrt{\frac{4}{5}}$	$\sqrt{\frac{1}{5}}$
$-1, \frac{1}{2}$	$1, \frac{5}{2}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{4}{5}}$

$Y=0$	$I=2$	**81**	**64**	**35_S**	**35_A**	**28**	**27**
$1, \frac{1}{2}$	$-1, \frac{3}{2}$	$\sqrt{\frac{90}{400}}$	$\sqrt{\frac{20}{175}}$	$-\sqrt{\frac{45}{180}}$	$\sqrt{\frac{90}{720}}$	$-\sqrt{\frac{10}{50}}$	$\sqrt{\frac{120}{1,400}}$
$0, 1$	$0, 2$	$-\sqrt{\frac{5}{400}}$	$-\sqrt{\frac{40}{175}}$	0	$\sqrt{\frac{405}{720}}$	$\sqrt{\frac{5}{50}}$	$\sqrt{\frac{135}{1,400}}$
$0, 1$	$0, 1$	$\sqrt{\frac{135}{400}}$	$\sqrt{\frac{30}{175}}$	$-\sqrt{\frac{30}{180}}$	$\sqrt{\frac{15}{720}}$	$\sqrt{\frac{15}{50}}$	$-\sqrt{\frac{5}{1,400}}$
$0, 0$	$0, 2$	$\sqrt{\frac{90}{400}}$	$-\sqrt{\frac{45}{175}}$	$\sqrt{\frac{20}{180}}$	$\sqrt{\frac{10}{720}}$	$-\sqrt{\frac{10}{50}}$	$-\sqrt{\frac{270}{1,400}}$
$-1, \frac{1}{2}$	$1, \frac{5}{2}$	$-\sqrt{\frac{8}{400}}$	$\sqrt{\frac{4}{175}}$	$-\sqrt{\frac{36}{180}}$	$\sqrt{\frac{72}{720}}$	$\sqrt{\frac{2}{50}}$	$-\sqrt{\frac{864}{1,400}}$
$-1, \frac{1}{2}$	$1, \frac{3}{2}$	$\sqrt{\frac{72}{400}}$	$-\sqrt{\frac{36}{175}}$	$-\sqrt{\frac{49}{180}}$	$-\sqrt{\frac{128}{720}}$	$\sqrt{\frac{8}{50}}$	$\sqrt{\frac{6}{1,400}}$

$Y=0$	$I=1$	**81**	**64**	**35_S**	**35_A**	**27**	**10**
$1, \frac{1}{2}$	$-1, \frac{3}{2}$	$\sqrt{\frac{10}{560}}$	$-\sqrt{\frac{20}{105}}$	$\sqrt{\frac{25}{108}}$	$\sqrt{\frac{98}{432}}$	$-\sqrt{\frac{8}{56}}$	$\sqrt{\frac{8}{42}}$
$1, \frac{1}{2}$	$-1, \frac{1}{2}$	$\sqrt{\frac{160}{560}}$	$\sqrt{\frac{20}{105}}$	$-\sqrt{\frac{4}{108}}$	$\sqrt{\frac{128}{432}}$	$\sqrt{\frac{8}{56}}$	$\sqrt{\frac{2}{42}}$
$0, 1$	$0, 2$	$-\sqrt{\frac{3}{560}}$	$\sqrt{\frac{6}{105}}$	$-\sqrt{\frac{30}{108}}$	$-\sqrt{\frac{15}{432}}$	$-\sqrt{\frac{15}{56}}$	$\sqrt{\frac{15}{42}}$
$0, 1$	$0, 1$	$-\sqrt{\frac{45}{560}}$	$-\sqrt{\frac{40}{105}}$	$-\sqrt{\frac{32}{108}}$	$\sqrt{\frac{25}{432}}$	$\sqrt{\frac{9}{56}}$	$\sqrt{\frac{1}{42}}$
$0, 0$	$0, 1$	$\sqrt{\frac{270}{560}}$	$-\sqrt{\frac{15}{105}}$	$-\sqrt{\frac{12}{108}}$	$-\sqrt{\frac{6}{432}}$	$-\sqrt{\frac{6}{56}}$	$-\sqrt{\frac{6}{42}}$
$-1, \frac{1}{2}$	$1, \frac{3}{2}$	$-\sqrt{\frac{72}{560}}$	$\sqrt{\frac{4}{105}}$	$-\sqrt{\frac{5}{108}}$	$\sqrt{\frac{160}{432}}$	$-\sqrt{\frac{10}{56}}$	$-\sqrt{\frac{10}{42}}$

(continued)

Table A.1 (continued)

$Y = 0$	$I = 0$	**64**	**27**
$1, \frac{1}{2}$	$-1, \frac{1}{2}$	$-\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$
$0, 1$	$0, 1$	$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{4}{7}}$

$Y = -1$	$I = \frac{5}{2}$	**81**	**64**
$0, 1$	$-1, \frac{3}{2}$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$
$-1, \frac{1}{2}$	$0, 2$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$

$Y = -1$	$I = \frac{3}{2}$	**81**	**64**	$\mathbf{35}_S$	$\mathbf{35}_A$	**28**	**27**
$1, \frac{1}{2}$	$-2, 1$	$\sqrt{\frac{20}{160}}$	$\sqrt{\frac{5}{35}}$	$-\sqrt{\frac{36}{144}}$	$\sqrt{\frac{36}{288}}$	$-\sqrt{\frac{4}{40}}$	$\sqrt{\frac{36}{140}}$
$0, 1$	$-1, \frac{3}{2}$	$-\sqrt{\frac{1}{160}}$	$-\sqrt{\frac{9}{35}}$	$\sqrt{\frac{5}{144}}$	$\sqrt{\frac{125}{288}}$	$\sqrt{\frac{5}{40}}$	$\sqrt{\frac{20}{140}}$
$0, 1$	$-1, \frac{1}{2}$	$\sqrt{\frac{40}{160}}$	$\sqrt{\frac{10}{35}}$	$\sqrt{\frac{32}{144}}$	$\sqrt{\frac{8}{288}}$	$\sqrt{\frac{8}{40}}$	$-\sqrt{\frac{2}{140}}$
$0, 0$	$-1, \frac{3}{2}$	$\sqrt{\frac{45}{160}}$	$-\sqrt{\frac{5}{35}}$	$\sqrt{\frac{1}{144}}$	$\sqrt{\frac{25}{288}}$	$-\sqrt{\frac{9}{40}}$	$-\sqrt{\frac{36}{140}}$
$-1, \frac{1}{2}$	$0, 2$	$-\sqrt{\frac{9}{160}}$	$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{45}{144}}$	$\sqrt{\frac{45}{288}}$	$\sqrt{\frac{5}{40}}$	$-\sqrt{\frac{45}{140}}$
$-1, \frac{1}{2}$	$0, 1$	$\sqrt{\frac{45}{160}}$	$-\sqrt{\frac{5}{35}}$	$-\sqrt{\frac{25}{144}}$	$-\sqrt{\frac{49}{288}}$	$\sqrt{\frac{9}{40}}$	$\sqrt{\frac{1}{140}}$

$Y = -1$	$I = \frac{1}{2}$	**81**	**64**	$\mathbf{35}_S$	$\mathbf{35}_A$	**27**	**10**
$1, \frac{1}{2}$	$-2, 1$	$\sqrt{\frac{2}{70}}$	$-\sqrt{\frac{4}{35}}$	$\sqrt{\frac{18}{144}}$	$\sqrt{\frac{18}{72}}$	$-\sqrt{\frac{18}{112}}$	$\sqrt{\frac{18}{56}}$
$1, \frac{1}{2}$	$-2, 0$	$\sqrt{\frac{9}{70}}$	$\sqrt{\frac{8}{35}}$	$-\sqrt{\frac{4}{144}}$	$\sqrt{\frac{16}{72}}$	$\sqrt{\frac{36}{112}}$	$\sqrt{\frac{4}{56}}$
$0, 1$	$-1, \frac{3}{2}$	$-\sqrt{\frac{1}{70}}$	$\sqrt{\frac{2}{35}}$	$-\sqrt{\frac{64}{144}}$	$-\sqrt{\frac{4}{72}}$	$-\sqrt{\frac{16}{112}}$	$\sqrt{\frac{16}{56}}$
$0, 1$	$-1, \frac{1}{2}$	$-\sqrt{\frac{4}{70}}$	$-\sqrt{\frac{18}{35}}$	$-\sqrt{\frac{25}{144}}$	$\sqrt{\frac{1}{72}}$	$\sqrt{\frac{25}{112}}$	$\sqrt{\frac{1}{56}}$
$0, 0$	$-1, \frac{1}{2}$	$\sqrt{\frac{36}{70}}$	$-\sqrt{\frac{2}{35}}$	$-\sqrt{\frac{25}{144}}$	$\sqrt{\frac{1}{72}}$	$-\sqrt{\frac{9}{112}}$	$-\sqrt{\frac{9}{56}}$
$-1, \frac{1}{2}$	$0, 1$	$-\sqrt{\frac{18}{70}}$	$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{8}{144}}$	$\sqrt{\frac{32}{72}}$	$-\sqrt{\frac{8}{112}}$	$-\sqrt{\frac{8}{56}}$

$Y = -2$	$I = 2$	**81**	**64**
$0, 1$	$-2, 1$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$
$-1, \frac{1}{2}$	$-1, \frac{3}{2}$	$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$

(continued)

Table A.1 (continued)

$Y=-2$	$I=1$	**81**	**64**	$\mathbf{35}_S$	$\mathbf{35}_A$	**28**	**27**
$1, \frac{1}{2}$	$-3, \frac{1}{2}$	$\sqrt{\frac{1}{20}}$	$\sqrt{\frac{16}{105}}$	$-\sqrt{\frac{6}{36}}$	$\sqrt{\frac{3}{36}}$	$-\sqrt{\frac{1}{30}}$	$\sqrt{\frac{72}{140}}$
$0, 1$	$-2, 1$	0	$-\sqrt{\frac{25}{105}}$	$\sqrt{\frac{6}{36}}$	$\sqrt{\frac{12}{36}}$	$\sqrt{\frac{4}{30}}$	$\sqrt{\frac{18}{140}}$
$0, 1$	$-2, 0$	$\sqrt{\frac{3}{20}}$	$\sqrt{\frac{48}{105}}$	$\sqrt{\frac{8}{36}}$	$\sqrt{\frac{1}{36}}$	$\sqrt{\frac{3}{30}}$	$-\sqrt{\frac{6}{140}}$
$0, 0$	$-2, 1$	$\sqrt{\frac{6}{20}}$	$-\sqrt{\frac{6}{105}}$	$-\sqrt{\frac{1}{36}}$	$\sqrt{\frac{8}{36}}$	$-\sqrt{\frac{6}{30}}$	$-\sqrt{\frac{27}{140}}$
$-1, \frac{1}{2}$	$-1, \frac{3}{2}$	$-\sqrt{\frac{2}{20}}$	$\sqrt{\frac{2}{105}}$	$-\sqrt{\frac{12}{36}}$	$\sqrt{\frac{6}{36}}$	$\sqrt{\frac{8}{30}}$	$-\sqrt{\frac{16}{140}}$
$-1, \frac{1}{2}$	$-1, \frac{1}{2}$	$\sqrt{\frac{8}{20}}$	$-\sqrt{\frac{8}{105}}$	$-\sqrt{\frac{3}{36}}$	$-\sqrt{\frac{6}{36}}$	$\sqrt{\frac{8}{30}}$	$\sqrt{\frac{1}{140}}$

$Y=-2$	$I=0$	**81**	$\mathbf{35}_S$	$\mathbf{35}_A$	**10**
$1, \frac{1}{2}$	$-3, \frac{1}{2}$	$\sqrt{\frac{3}{56}}$	$\sqrt{\frac{1}{36}}$	$\sqrt{\frac{25}{72}}$	$\sqrt{\frac{8}{14}}$
$0, 1$	$-2, 1$	$-\sqrt{\frac{2}{56}}$	$-\sqrt{\frac{24}{36}}$	$-\sqrt{\frac{6}{72}}$	$\sqrt{\frac{3}{14}}$
$0, 0$	$-2, 0$	$\sqrt{\frac{27}{56}}$	$-\sqrt{\frac{9}{36}}$	$\sqrt{\frac{9}{72}}$	$-\sqrt{\frac{2}{14}}$
$-1, \frac{1}{2}$	$-1, \frac{1}{2}$	$-\sqrt{\frac{24}{56}}$	$-\sqrt{\frac{2}{36}}$	$\sqrt{\frac{32}{72}}$	$-\sqrt{\frac{1}{14}}$

$Y=-3$	$I=\frac{3}{2}$	**81**	**64**
$0, 1$	$-3, \frac{1}{2}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$
$-1, \frac{1}{2}$	$-2, 1$	$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$

$Y=-3$	$I=\frac{1}{2}$	**81**	$\mathbf{35}_S$	$\mathbf{35}_A$	**28**
$0, 1$	$-3, \frac{1}{2}$	$\sqrt{\frac{1}{32}}$	$\sqrt{\frac{81}{144}}$	$\sqrt{\frac{81}{288}}$	$\sqrt{\frac{1}{8}}$
$0, 0$	$-3, \frac{1}{2}$	$\sqrt{\frac{9}{32}}$	$-\sqrt{\frac{25}{144}}$	$\sqrt{\frac{121}{288}}$	$-\sqrt{\frac{1}{8}}$
$-1, \frac{1}{2}$	$-2, 1$	$-\sqrt{\frac{4}{32}}$	$-\sqrt{\frac{36}{144}}$	$\sqrt{\frac{36}{288}}$	$\sqrt{\frac{4}{8}}$
$-1, \frac{1}{2}$	$-2, 0$	$\sqrt{\frac{18}{32}}$	$-\sqrt{\frac{2}{144}}$	$-\sqrt{\frac{50}{288}}$	$\sqrt{\frac{2}{8}}$

$Y=-4$	$I=1$	**81**
$-1, \frac{1}{2}$	$-3, \frac{1}{2}$	1

$Y=-4$	$I=0$	**28**
$-1, \frac{1}{2}$	$-3, \frac{1}{2}$	1

isoscalar factor values under the dimensions μ_γ. The global signs in Table A.1 are fixed to be consistent with those in the previous works [299, 300], by checking the fact that each submatrix is unitary.

Exploiting the isoscalar factors obtained in Table A.1, we evaluate in Table A.2 explicit expectation values of Wigner D functions such as

$$\begin{aligned} D^8_{33} &= \langle 010|D^8|010\rangle, \quad D^8_{38} = \langle 010|D^8|000\rangle, \\ D^8_{83} &= \langle 000|D^8|010\rangle, \quad D^8_{88} = \langle 000|D^8|000\rangle, \\ D^{10}_{33} &= \langle 010|D^{10}|010\rangle, \; D^{\overline{10}}_{33} = \langle 010|D^{\overline{10}}|010\rangle, \\ D^{27}_{33} &= \langle 010|D^{27}|010\rangle, \; D^{27}_{83} = \langle 000|D^{27}|010\rangle. \end{aligned} \tag{A.24}$$

In the SU(3) strange hadron physics, the expectation value of D_{ab} in the transition $B_1 \to B_2$ is given by

$$\langle \lambda_2 B_2 | D_{ab} | \lambda_1 B_1 \rangle = \int \mathrm{d}A \; \Phi^{\lambda_2 *}_{B_2} D_{ab}(A) \Phi^{\lambda_1}_{B_1}, \tag{A.25}$$

where

$$D_{ab}(A) = \frac{1}{3}\mathrm{tr}(A^\dagger \lambda_a A \lambda_b). \tag{A.26}$$

Here, one notes that the wavefunction Φ^λ_B for the baryon B with quantum numbers $(\alpha) = (Y, I, I_3)$ and $(\beta) = (Y_R, S, -S_3)$ are given in irreducible representations λ by

$$\Phi^\lambda_{(\alpha)(\beta)}(A) = \sqrt{\lambda}\langle Y, I, I_3 | D^\lambda(A) | Y_R, S, -S_3\rangle, \tag{A.27}$$

where Y, I and S are the hypercharge, isospin and spin of the hyperon B, and the right hypercharge Y_R is given by $Y_R = \frac{1}{3}N_c$ due to the Wess-Zumino constraint to yield $Y_R = 1$ for the $N_c = 3$ case. Next, we have

$$\begin{aligned} &\int \mathrm{d}A \; \Phi^{\lambda_2 *}_{(\alpha_2)(\beta_2)}(A) \langle \alpha | D^\lambda(A) | \beta \rangle \Phi^{\lambda_1}_{(\alpha_1)(\beta_1)}(A) \\ &\quad = \sqrt{\frac{\lambda_1}{\lambda_2}} \sum_\gamma \begin{pmatrix} \lambda_1 & \lambda & \lambda_{2\gamma} \\ \alpha_1 & \alpha & \alpha_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & \lambda & \lambda_{2\gamma} \\ \beta_1 & \beta & \beta_2 \end{pmatrix}, \end{aligned} \tag{A.28}$$

where the summation runs over the independent irreducible representations in the process $\lambda_1 \otimes \lambda \to \lambda_2$.

Since the coefficients in the sum rules for the baryon magnetic moments and form factors are solely given by the SU(3) group structure of the chiral models, these Wigner D functions can be practically referred in the strange flavor hadron phenomenology researches using the hedgehog ansatz solution corresponding to the little group $\mathrm{SU}(2) \times Z_2$. In this kind of task, it is also powerful to use the

Table A.2 The Wigner D functions

D^{λ}_{ab}	p	n	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-	Δ^{++}
D^8_{33}	$-\frac{7}{30}$	$\frac{7}{30}$	0	$-\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{15}$	$-\frac{1}{15}$	$-\frac{3}{8}$
D^8_{38}	$\frac{\sqrt{3}}{30}$	$-\frac{\sqrt{3}}{30}$	0	$\frac{\sqrt{3}}{6}$	0	$-\frac{\sqrt{3}}{6}$	$\frac{2\sqrt{3}}{15}$	$-\frac{2\sqrt{3}}{15}$	$\frac{\sqrt{3}}{8}$
D^8_{83}	$-\frac{\sqrt{3}}{30}$	$-\frac{\sqrt{3}}{30}$	$\frac{\sqrt{3}}{10}$	$-\frac{\sqrt{3}}{10}$	$-\frac{\sqrt{3}}{10}$	$-\frac{\sqrt{3}}{10}$	$\frac{2\sqrt{3}}{15}$	$\frac{2\sqrt{3}}{15}$	$-\frac{\sqrt{3}}{8}$
D^8_{88}	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{1}{10}$	$-\frac{1}{10}$	$-\frac{1}{10}$	$-\frac{1}{10}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{8}$
D^{10}_{33}	$-\frac{1}{15}$	$\frac{1}{15}$	0	$\frac{1}{15}$	0	$-\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{15}$	0
$D^{\overline{10}}_{33}$	$-\frac{1}{15}$	$\frac{1}{15}$	0	$\frac{1}{15}$	0	$-\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{15}$	0
D^{27}_{33}	$-\frac{4}{135}$	$\frac{4}{135}$	0	0	0	0	$\frac{4}{135}$	$-\frac{4}{135}$	$-\frac{1}{21}$
D^{27}_{83}	$-\frac{2\sqrt{3}}{135}$	$-\frac{2\sqrt{3}}{135}$	$\frac{6\sqrt{3}}{135}$	$\frac{2\sqrt{3}}{405}$	$\frac{2\sqrt{3}}{405}$	$\frac{2\sqrt{3}}{405}$	$-\frac{2\sqrt{3}}{135}$	$-\frac{2\sqrt{3}}{135}$	$-\frac{\sqrt{3}}{63}$

D^{λ}_{ab}	Δ^+	Δ^0	Δ^-	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^-
D^8_{33}	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{8}$	0
D^8_{38}	$\frac{\sqrt{3}}{24}$	$-\frac{\sqrt{3}}{24}$	$-\frac{\sqrt{3}}{8}$	$\frac{\sqrt{3}}{12}$	0	$-\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{24}$	$-\frac{\sqrt{3}}{24}$	0
D^8_{83}	$-\frac{\sqrt{3}}{8}$	$-\frac{\sqrt{3}}{8}$	$-\frac{\sqrt{3}}{8}$	0	0	0	$\frac{\sqrt{3}}{8}$	$\frac{\sqrt{3}}{8}$	$\frac{\sqrt{3}}{4}$
D^8_{88}	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	0	0	0	$-\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{4}$
D^{10}_{33}	0	0	0	0	0	0	0	0	0
$D^{\overline{10}}_{33}$	0	0	0	0	0	0	0	0	0
D^{27}_{33}	$-\frac{1}{63}$	$\frac{1}{63}$	$\frac{1}{21}$	$\frac{1}{21}$	0	$-\frac{1}{21}$	$\frac{4}{63}$	$-\frac{4}{63}$	0
D^{27}_{83}	$-\frac{\sqrt{3}}{63}$	$-\frac{\sqrt{3}}{63}$	$-\frac{\sqrt{3}}{63}$	$\frac{5\sqrt{3}}{189}$	$\frac{5\sqrt{3}}{189}$	$\frac{5\sqrt{3}}{189}$	$\frac{\sqrt{3}}{63}$	$\frac{\sqrt{3}}{63}$	$-\frac{\sqrt{3}}{21}$

D^{λ}_{ab}	$\Lambda\Sigma^0$	$p\Delta^+$	$n\Delta^0$	$\Lambda\Sigma^{*0}$	$\Sigma^+\Sigma^{*+}$	$\Sigma^0\Sigma^{*0}$	$\Sigma^-\Sigma^{*-}$	$\Xi^0\Xi^{*0}$	$\Xi^-\Xi^{*-}$
D^8_{33}	$-\frac{\sqrt{3}}{10}$	$\frac{2\sqrt{5}}{15}$	$\frac{2\sqrt{5}}{15}$	$\frac{\sqrt{15}}{15}$	$-\frac{\sqrt{5}}{15}$	0	$\frac{\sqrt{5}}{15}$	$-\frac{\sqrt{5}}{15}$	$\frac{\sqrt{5}}{15}$
D^8_{38}	$-\frac{1}{10}$	0	0	0	0	0	0	0	0
D^8_{83}	0	0	0	0	$-\frac{\sqrt{15}}{15}$	$-\frac{\sqrt{15}}{15}$	$-\frac{\sqrt{15}}{15}$	$-\frac{\sqrt{15}}{15}$	$-\frac{\sqrt{15}}{15}$
D^8_{88}	0	0	0	0	0	0	0	0	0
D^{10}_{33}	$\frac{\sqrt{3}}{15}$	0	0	0	0	0	0	0	0
$D^{\overline{10}}_{33}$	$-\frac{\sqrt{3}}{15}$	$\frac{\sqrt{5}}{15}$	$\frac{\sqrt{5}}{15}$	0	$\frac{\sqrt{5}}{15}$	0	$-\frac{\sqrt{5}}{15}$	$\frac{\sqrt{5}}{15}$	$-\frac{\sqrt{5}}{15}$
D^{27}_{33}	$\frac{4\sqrt{3}}{135}$	$\frac{\sqrt{5}}{270}$	$\frac{\sqrt{5}}{270}$	$-\frac{\sqrt{15}}{135}$	$-\frac{\sqrt{5}}{90}$	0	$\frac{\sqrt{5}}{90}$	$\frac{\sqrt{5}}{135}$	$-\frac{\sqrt{5}}{135}$
D^{27}_{83}	0	0	0	0	$-\frac{2\sqrt{15}}{405}$	$-\frac{2\sqrt{15}}{405}$	$-\frac{2\sqrt{15}}{405}$	$\frac{\sqrt{15}}{135}$	$\frac{\sqrt{15}}{135}$

mathematical theorem that the tensor product of the Wigner D functions can be decomposed into sum of the single D functions [299],

$$D^{\mu_1}_{\nu_1\lambda_1} D^{\mu_2}_{\nu_2\lambda_2} = \sum_{\mu\nu\lambda\gamma} \begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix} \begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \lambda_1 & \lambda_2 & \lambda \end{pmatrix} D^{\mu}_{\nu\lambda}. \tag{A.29}$$

As applications of the above SU(3) group theoretical properties associated with the spin symmetries of our interest, we investigate Coleman-Glashow type sum rules, in the SU(3) flavor symmetric limit with the chiral symmetry breaking masses $m_u = m_d = m_s, m_K = m_\pi$ and decay constants $f_K = f_\pi$. To do end in this limit, for instance we introduce the topological Skyrmion model [58, 158, 252] which is one of the chiral models used in the nuclear phenomenology. The Skyrmion soliton Lagrangian with SU(3) flavor group is given by the equation of the form

$$\mathcal{L} = -\frac{1}{4} f_\pi^2 \mathrm{tr}(l_\mu l^\mu) + \frac{1}{32e^2}\mathrm{tr}[l_\mu, l_\nu]^2 + \mathcal{L}_{WZW}, \tag{A.30}$$

where f_π and e are the pion decay constant and the Skyrmion parameter and $l_\mu = U^\dagger \partial_\mu U$. The chiral field $U = e^{i\lambda_a \pi_a / f_\pi} \in$ SU(3) is described by the pseudoscalar meson fields π_a $(a = 1, \ldots, 8)$ and Gell-Mann matrices λ_a with $\lambda_a\lambda_b = \frac{2}{3}\delta_{ab} + (if_{abc} + d_{abc})\lambda_c$. The Wess-Zumino-Witten term [59, 115, 116] is already described in the main context above.

The Noether theorem then yields the flavor octet vector currents $J_V^{\mu a}$ $(a = 1, \ldots, 8)$ from the derivative terms in the above Skyrmion Lagrangian as follows

$$\begin{aligned} J_V^{\mu a} &= -\frac{i}{2} f_\pi^2 \mathrm{tr}\left[\frac{\lambda_a}{2} l^\mu + (U \leftrightarrow U^\dagger)\right] + \frac{i}{8e^2}\mathrm{tr}\left[\left[\frac{\lambda_a}{2}, l^\nu\right][l^\mu, l^\nu] + (U \leftrightarrow U^\dagger)\right] \\ &\quad + \frac{N_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left[\frac{\lambda_a}{2} l_\nu l_\alpha l_\beta - (U \leftrightarrow U^\dagger)\right], \end{aligned} \tag{A.31}$$

with $\epsilon^{0123} = 1$. Exploiting the above flavor octet vector currents, we next calculate the electromagnetic currents J^μ as follows

$$J^\mu = J_V^{\mu 3} + \frac{1}{\sqrt{3}} J_V^{\mu 8}, \tag{A.32}$$

from which we can construct the magnetic moment operators defined by

$$\hat{\mu}^i = \frac{1}{2}\int \mathrm{d}^3x\, \epsilon^{ijk} x^j J^k. \tag{A.33}$$

For given operators, we can evaluate the matrix elements of the form factors or the transition magnetic moments for the diagonal, or off-diagonal cases, respectively. For instance, with the spinning chiral model ansatz in the SU(3) chiral models, the magnetic moment operators in Eq. (A.33) become the following form

$$\hat{\mu}^{i} = \hat{\mu}^{i(3)} + \frac{1}{\sqrt{3}}\hat{\mu}^{i(8)}. \tag{A.34}$$

Here, $\hat{\mu}^{i(a)}$ ($a = 1, 2, \ldots, 8$) are given by

$$\hat{\mu}^{i(a)} = \frac{N_c}{2\sqrt{3}}\mathcal{M}D^{8}_{a8}\hat{J}_i - \mathcal{N}D^{8}_{ai} + \cdots, \tag{A.35}$$

where $\hat{J}_i = -\hat{T}^R_i$ are the SU(2) spin operators and $\hat{T}^R_i$ are the right SU(3) isospin operators along the isospin direction, and the inertia parameters $\mathcal{M}$ and $\mathcal{N}$ depend on the properties of the given SU(3) chiral model. Here, the ellipsis stands for other contributions to the baryon magnetic moments μ_B of the baryon B, for instance, in addition to those of the chiral symmetric limit [58]. In the Yabu-Ando scheme [121], we need also some additional terms in μ_B. The Wigner D functions in the operators in Eq. (A.35) can be used in evaluating their matrix elements or expectation elements of the form factors, or the transition magnetic moments via Eq. (A.25).

Specifically, exploiting this operator (A.34) together with the baryon wave function (A.27), we can evaluate the decuplet-to-octet transition magnetic moments for $\mathbf{10}(S_3 = 1/2) \to \mathbf{8}(S_3 = 1/2) + \gamma$ to yield the V-spin symmetry sum rules

$$\mu_{\Sigma^+\Sigma^{*+}} = \mu_{\Xi^0\Xi^{*0}}, \tag{A.36}$$

the U-spin symmetry ones

$$\mu_{p\Delta^+} = -\mu_{\Sigma^+\Sigma^{*+}}, \quad \mu_{\Sigma^-\Sigma^{*-}} = \mu_{\Xi^-\Xi^{*-}}, \tag{A.37}$$

the I-spin symmetry ones

$$\mu_{p\Delta^+} = \mu_{n\Delta^0}, \quad 2\mu_{\Sigma^0\Sigma^{*0}} = \mu_{\Sigma^+\Sigma^{*+}} + \mu_{\Sigma^-\Sigma^{*-}}, \tag{A.38}$$

and the other ones,

$$\mu_{\Sigma^+\Sigma^{*+}} + \mu_{\Sigma^-\Sigma^{*-}} = \mu_{\Xi^0\Xi^{*0}} + \mu_{\Xi^-\Xi^{*-}}, \quad \mu_{\Sigma^0\Sigma^{*0}} = -\sqrt{3}\mu_{\Lambda\Sigma^{*0}}. \tag{A.39}$$

In the strange flavor channel of the decuplet-to-octet transition magnetic moments, we construct the s-flavor currents $J^{\mu(s)}$ by substituting the electromagnetic charge operator $\hat{Q}_{EM}$ in Eq. (A.3) with the s-flavor electromagnetic charge operator $\hat{Q}_s$. Here, one notes that by defining the q-flavor projection operators

$$\hat{P}_u = \frac{1}{3} + \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8,$$

$$\hat{P}_d = \frac{1}{3} - \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8,$$
$$\hat{P}_s = \frac{1}{3} - \frac{1}{2\sqrt{3}}\lambda_8, \tag{A.40}$$

satisfying

$$\hat{P}_q^2 = \hat{P}_q, \quad \sum_q \hat{P}_q = 1, \tag{A.41}$$

we can readily obtain the q-flavor electromagnetic charge operators

$$\hat{Q}_q = \hat{Q}_{EM}\hat{P}_q = \hat{Q}_q\hat{P}_q. \tag{A.42}$$

The electromagnetic currents are then split into three pieces

$$J^\mu = J^{\mu(u)} + J^{\mu(d)} + J^{\mu(s)}. \tag{A.43}$$

Exploiting the s-flavor electromagnetic currents $J^{\mu(s)}$ in the SU(3) flavor symmetric limit, we find the symmetry identities

$$\mu^{(s)}_{N\Delta} = \mu^{(s)}_{\Lambda\Sigma^{*0}}, \quad \mu^{(s)}_{\Sigma\Sigma^*} = \mu^{(s)}_{\Xi\Xi^*}, \tag{A.44}$$

and their sum rules

$$\mu^{(s)}_{N\Delta} + \mu^{(s)}_{\Sigma\Sigma^*} = \mu^{(s)}_{\Lambda\Sigma^{*0}} + \mu^{(s)}_{\Xi\Xi^*}. \tag{A.45}$$

Next, we construct the octet magnetic moments to yield the V-spin symmetry sum rule,

$$\mu_p + \mu_{\Sigma_-} = -2\mu_\Lambda, \tag{A.46}$$

the U-spin symmetry ones,

$$\mu_{\Sigma^+} = \mu_p, \quad \mu_{\Xi^0} = \mu_n, \quad \mu_{\Xi^-} = \mu_{\Sigma^-}, \tag{A.47}$$

and the I-spin symmetry ones,

$$2\mu_{\Sigma^0} = \mu_{\Sigma^+} + \mu_{\Sigma^-}. \tag{A.48}$$

Finally, exploiting the decuplet baryon magnetic moments, we find the V-spin symmetry sum rules

$$\mu_{\Delta^+} + \mu_{\Sigma^{*0}} + \mu_{\Xi^{*-}} = 0, \quad \mu_{\Sigma^{*+}} + \mu_{\Xi^{*0}} + \mu_{\Omega^-} = 0, \tag{A.49}$$

and their other sum rules

$$\mu_{\Delta^{++}} + \mu_{\Xi^{*0}} + \mu_{\Omega^-} - \mu_{\Sigma^{*+}} = 0, \quad \mu_{\Delta^{++}} + 2\mu_{\Omega^-} = 0. \tag{A.50}$$

We also obtain the U-spin symmetry sum rules

$$\mu_{\Delta^-} = \mu_{\Sigma^{*-}} = \mu_{\Xi^{*-}} = \mu_{\Omega^-}, \quad \mu_{\Delta^0} = \mu_{\Sigma^{*0}} = \mu_{\Xi^{*0}}, \quad \mu_{\Delta^+} = \mu_{\Sigma^+}, \tag{A.51}$$

and the I-spin symmetry ones

$$2\mu_{\Sigma^{*0}} = \mu_{\Sigma^{*+}} + \mu_{\Sigma^{*-}}, \quad \mu_{\Delta^-} + \mu_{\Delta^{++}} = \mu_{\Delta^0} + \mu_{\Delta^+}. \tag{A.52}$$

Here, we have included Eqs. (A.47) and (A.48) in Refs. [119, 222], and Eqs. (A.51) and (A.52) in Ref. [120] for the sake of completeness.

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