Appendix
Fundamentals of Backstepping Control

This Appendix provides the fundamentals of the recursive backstepping control method. The presented material is a summary of more detailed descriptions that may be found in [43, 49]. Lyapunov-based controller design may be systematically tackled by a recursive design procedure called *backstepping*. Backstepping is suitable for strict-feedback systems that are also known as "lower triangular". An example of a strict-feedback systems is:

$$\begin{aligned}
\dot{\xi}_1 &= f_1(\xi_1) + g_1(\xi_1)\xi_2 \\
\dot{\xi}_2 &= f_2(\xi_1, \xi_2) + g_2(\xi_1, \xi_2)\xi_3 \\
&\vdots \\
\dot{\xi}_{r-1} &= f_{r-1}(\xi_1, \xi_2, \ldots, \xi_{r-1}) + g_{r-1}(\xi_1, \xi_2, \ldots, \xi_{r-1})\xi_r \\
\dot{\xi}_r &= f_r(\xi_1, \xi_2, \ldots, \xi_r) + g_r(\xi_1, \xi_2, \ldots, \xi_r)u
\end{aligned} \tag{A.1}$$

where $\xi_1, \ldots, \xi_r \in \mathbb{R}$, $u \in \mathbb{R}$ is the control input and f_i, g_i for $i = 1, \ldots, r$ are known functions. A typical feedback linearization approach in most cases leads to cancellation of useful nonlinearities. The backstepping design exhibits more flexibility compared to feedback linearization since it does not require that the resulting input–output dynamics be linear. Cancellation of potentially useful nonlinearities can be avoided resulting in less complex controllers. The main idea is to use some of the state variables of (A.1) as "virtual controls" or "pseudo controls", and depending on the dynamics of each state, design intermediate control laws. The backstepping design is a recursive procedure where a Lyapunov function is derived for the entire system. The recursive procedure can be easily expanded from the nominal case of a system augmented by an integrator. This is also referred to as *integrator backstepping*. Based on the design principles of the integrator backstepping, the control design can be easily expanded for the case of strict-feedback systems given by (A.1).

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A.1 Integrator Backstepping

The baseline design of the recursive procedure is the *integrator backstepping*. Consider the system:

$$\dot{\eta} = f(\eta) + g(\eta)\sigma \tag{A.2}$$

$$\dot{\sigma} = u \tag{A.3}$$

where $[\eta \quad \sigma]^T \in \mathbb{R}^{n+1}$ is the state vector and $u \in \mathbb{R}$ is the control input. The objective is to design a state feedback control law such that $\eta, \sigma \to 0$ as $t \to \infty$. It is assumed that both f and g are known. This system can be viewed as a cascade connection of two subsystems. The first subsystem is (A.2) with σ as input and the second subsystem is the integrator (A.3). The main design idea is to treat σ as a virtual control input for the stabilization of η. Assume that there exists a smooth state feedback control law $\sigma = \phi(\eta)$, with $\phi(0) = 0$, such that the origin of:

$$\dot{\eta} = f(\eta) + g(\eta)\phi(\eta) \tag{A.4}$$

is asymptotically stable. Consider that for the choice of $\phi(\eta)$ a Lyapunov function $V(\eta)$ is known such that:

$$\frac{\partial V}{\partial \eta}[f(\eta) + g(\eta)\phi(\eta)] \leq -W(\eta), \quad \forall \eta \in \mathbb{R}^n \tag{A.5}$$

where $W(\eta)$ is positive definite. By adding and subtracting $g(\eta)\phi(\eta)$ on the right hand side of (A.2), one has:

$$\dot{\eta} = f(\eta) + g(\eta)\phi(\eta) + g(\eta)[\sigma - \phi(\eta)] \tag{A.6}$$

$$\dot{\sigma} = u \tag{A.7}$$

Denote by e_σ the error between the state σ and the pseudo control $\phi(\eta)$, that is:

$$e_\sigma = \sigma - \phi(\eta) \tag{A.8}$$

Writing the initial system in the (η, e_σ) coordinates, one has:

$$\dot{\eta} = [f(\eta) + g(\eta)\phi(\eta)] + g(\eta)e_\sigma \tag{A.9}$$

$$\dot{e}_\sigma = u - \dot{\phi}(\eta) \tag{A.10}$$

Since f, g and ϕ are known, one of the advantages of the backstepping design is that it does not require a differentiator. In particular, the derivative $\dot{\phi}$ can be computed by using the expression:

$$\dot{\phi} = \frac{\partial \phi}{\partial \eta}[f(\eta) + g(\eta)\sigma] \tag{A.11}$$

Setting $u = v + \dot{\phi}$, where $v \in \mathbb{R}$ is a nominal control input, the transformed system takes the form:

$$\dot{\eta} = [f(\eta) + g(\eta)\phi(\eta)] + g(\eta)e_\sigma \tag{A.12}$$

$$\dot{e}_\sigma = v \tag{A.13}$$

which is similar to the initial system, except that now the first component has an asymptotically stable origin when the input is zero. Using this procedure, the pseudo control $\phi(\eta)$ has been “back stepped” through the integrator from $u = v + \phi(\eta)$. The knowledge of $V(\eta)$ is exploited in the design of v for the stabilization of the overall system. Using:

$$V_c(\eta, \sigma) = V(\eta) + \frac{1}{2}e_\sigma^2 \tag{A.14}$$

as a Lyapunov function candidate, one obtains:

$$\begin{aligned}\dot{V}_c &= \frac{\partial V}{\partial \eta}[f(\eta) + g(\eta)\phi(\eta)] + \frac{\partial V}{\partial \eta}g(\eta)e_\sigma + e_\sigma v \\ &\leq -W(\eta) + \frac{\partial V}{\partial \eta}g(\eta)e_\sigma + e_\sigma v\end{aligned} \tag{A.15}$$

The control input v is chosen as:

$$v = -\frac{\partial V}{\partial \eta}g(\eta) - ke_\sigma, \quad k > 0 \tag{A.16}$$

Substituting the above choice of v to (A.15), one has:

$$\dot{V}_c \leq -W(\eta) - ke_\sigma^2 \tag{A.17}$$

which shows that the origin ($\eta = 0, e_\sigma = 0$) is asymptotically stable. Since $\phi(0) = 0$, and $e_\sigma \to 0$ as $t \to \infty$; then, the, origin ($\eta = 0, \sigma = 0$) is asymptotically stable as well. Substituting for v, e_σ, and $\dot{\phi}$, the final form of the control law is:

$$u = \frac{\partial \phi}{\partial \eta}[f(\eta) + g(\eta)\sigma] - \frac{\partial V}{\partial \eta}g(\eta) - k[\sigma - \phi(\eta)] \tag{A.18}$$

A.2 Example of a Recursive Backstepping Design

This Section illustrates the implementation of the backstepping methodology to a strict feedback system of high order. The construction of the controller for high order systems is based on the recursive implementation of the integrator backstepping methodology. An illustration of the backstepping procedure based on the generic formulation of the strict feedback systems given in (A.1), would result in the derivation of tedious recursive formulas which are difficult to follow. In this Section, a simple third order strict feedback system is used instead. This approach provides a better insight to the key features and potentials of the backstepping design. Consider the following system:

$$\begin{aligned}\dot{\xi}_1 &= f_1(\xi_1) + \xi_2 \\ \dot{\xi}_2 &= f_2(\xi_2) + \xi_3 \\ \dot{\xi}_3 &= u\end{aligned} \tag{A.19}$$

where $\xi_i \in \mathbb{R}$ for $i = 1, 2, 3$ are the system states, $u \in \mathbb{R}$ is the control input and $f_i(\xi_i) : \mathbb{R} \to \mathbb{R}$ are known functions. The objective is to design a state feedback control law such that $\xi_1, \xi_2, \xi_3 \to 0$ as $t \to 0$. Similarly to the integrator backstepping case, the idea is to use the state variable ξ_2 as an input for the stabilization of ξ_1. Consider the Lyapunov function $V_1 = \frac{1}{2}\xi_1^2$. The derivative of V_1 along the trajectory of ξ_1 is computed as:

$$\dot{V}_1 = \xi_1(f_1(\xi_1) + \xi_2) \tag{A.20}$$

The objective of this step is to find a control law $\phi_2(\xi_1)$ with $\phi_2(0) = 0$, such that when $\xi_2 = \phi_2(\xi_1)$ then $\dot{V}_1(\xi_1) \leq -W_1(\xi_1)$ where W_1 is a positive definite function for every $\xi_1 \in \mathbb{R}$. An obvious choice would be to remove the effect of the function $f_1(\xi_1)$ and inject a stabilizing feedback term. Thus, we pick:

$$\phi_2(\xi_1) = -f_1(\xi_1) - \kappa_1\xi_1 \tag{A.21}$$

where k_1 is a positive gain. This choice yields $\dot{V}_1 \leq -k_1\xi_1^2$. Denote the error $e_2 = \xi_2 - \phi_2(\xi_1)$. Using the new coordinate e_2 the system given in (A.19) can be written as:

$$\begin{aligned} \dot{\xi}_1 &= -k_1\xi_1 + e_2 \\ \dot{e}_2 &= -\dot{\phi}_2(\xi_1) + f_2(\xi_1, e_2) + \xi_3 \\ \dot{\xi}_3 &= u \end{aligned} \tag{A.22}$$

Similarly to Sect. A.1, the implementation of the derivative $\phi_2(\xi_1)$ does not require a differentiator since:

$$\dot{\phi}_2 = \frac{\partial \phi_2}{\partial \xi_1}[f_1(\xi_1) + \xi_2] \tag{A.23}$$

Let $V_2(\xi_1, e_2) = \frac{1}{2}\xi_1 + \frac{1}{2}e_2^2$. The goal of the second design step is to determine a pseudo control $\phi_3(\xi_1, e_2)$ with $\phi_3(0,0) = 0$ such that when $\xi_3 = \phi_3(\xi_1, e_2)$ then $\dot{V}_2(\xi_1, e_2) \leq -W_2(\xi_1, e_2)$ where W_2 is a positive definite function for every ξ_1, e_2. Consequently, the derivative of V_2 along the solutions of ξ_1, e_2 is:

$$\dot{V}_2 = -k_1\xi_1^2 + e_2(\xi_1 - \dot{\phi}_2(\xi_1) + f_2(\xi_1, e_2) + \phi_3(\xi_1, e_2)) \tag{A.24}$$

An obvious choice would be:

$$\phi_3(\xi_1, e_2) = -\xi_1 + \dot{\phi}_2(\xi_1) - f_2(\xi_1, e_2) - k_2e_2 \tag{A.25}$$

where k_2 is a positive constant. In this case $\dot{V}_2 = -k_1\xi_1^2 - k_2e_2^2$. Using the change of variables $e_3 = \xi_3 - \phi_3(\xi_1, e_2)$ the system dynamics become:

$$\begin{aligned} \dot{\xi}_1 &= -k_1\xi_1 + e_2 \\ \dot{e}_2 &= -\xi_1 - k_2e_2 + e_3 \\ \dot{e}_3 &= -\dot{\phi}_3(\xi_1, e_2) + u \end{aligned} \tag{A.26}$$

Similarly to $\dot{\phi}_2$, the computation of $\dot{\phi}_3$ does not require a differentiator. Using $V_3 = V_2 + \frac{1}{2}e_3^2$ as a candidate Lyapunov function one has:

$$\dot{V}_3 = -k_1\xi_1^2 - k_2e_2^2 + e_3(e_2 - \dot{\phi}_3(\xi_1, e_2) + u) \tag{A.27}$$

The choice of u is:

$$u = -e_2 + \dot{\phi}_3(\xi_1, e_2) - k_3 e_3 \tag{A.28}$$

where k_3 is a positive constant. This choice yields:

$$\dot{V}_3 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{A.29}$$

therefore the origin of the error system is globally asymptotically stable. Since $\phi_2(0), \phi_3(0,0) = 0$ then $\xi_1, \xi_2, \xi_3 \to 0$ as $t \to \infty$. The final system dynamics have the form:

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 & 0 \\ -1 & -k_2 & 1 \\ 0 & -1 & -k_3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{A.30}$$

As indicated by [49], an important structural property of the above system is that the system matrix is composed by the sum of a negative diagonal and a skew-symmetric matrix. This is a typical structural pattern when the backstepping design is based on a sequential construction of Lyapunov functions. The key feature of the backstepping methodology is the fact that it provides significant design freedom. The choice of the pseudo controls ϕ_2, ϕ_3 and the control input u is not unique. For example, we could have picked:

$$\begin{aligned} \phi_2(\xi_1) &= -f_1(\xi_1) - \kappa_1 \xi_1 \\ \phi_3(\xi_1, e_2) &= \dot{\phi}_2(\xi_1) - f_2(\xi_2) - k_2 e_2 \\ u &= \dot{\phi}_3(\xi_1, e_2) - k_3 e_3 \end{aligned}$$

resulting to the system:

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 & 0 \\ 0 & -k_2 & 1 \\ 0 & 0 & -k_3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{A.31}$$

which is obviously asymptotically stable. Thus, the stabilization of the same system can be achieved with a much more simpler design. This potential constitutes the backstepping methodology as a powerful design tool for the development of simplistic controllers for nonlinear systems.

References

1. P.P. Angelov, D.P. Filev, An approach to online identification of Takagi–Sugeno fuzzy models. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics **34**, 484–498 (2004)
2. N. Antequera, M. Santos, J.M. De la Cruz, A helicopter control based on eigenstructure assignment, in *IEEE Conference Emerging Technologies and Factory Automation*, 2006
3. J.S. Bay, *Linear State Space Systems* (McGraw-Hill, New York, 1999)
4. M. Bejar, A. Isidori, L. Marconi, R. Naldi, Robust vertical/lateral/longitudinal control of a helicopter with constant yaw-attitude, in *44th IEEE Conference on Decision and Control, and 2005 European Control Conference, CDC-ECC*, 2005
5. J.S. Bendat, A.J. Piersol, *Random Data: Analysis & Measurement Procedures* (Wiley–Interscience, New York, 1971)
6. P. Bendotti, J.C. Morris, Robust hover control for a model helicopter, in *Proceedings of the American Control Conference*, 1995
7. A.R.S. Bramwell, G. Done, D. Balmford, *Bramwell's Helicopter Dynamics* (Butterworth Heinemann, Stoneham, 2001)
8. A. Budiyonoa, S.S. Wibowob, Optimal tracking controller design for a small scale helicopter. Journal of Bionic Engineering **4**(4), 271–280 (2007)
9. C.I. Byrnes, F.D. Priscoli, A. Isidori (eds.), *Output regulation of uncertain nonlinear systems* (Birkhäuser, Basel, 1997)
10. G. Cai, B.M. Chen, K. Peng, M. Dong, T.H. Lee, Modeling and control system design for a UAV helicopter, in *14th Mediterranean Conference on Control and Automation*, 2006
11. P. Castillo, R. Lozano, A.E. Dzul, *Modelling and Control of Mini-Flying Machines* (Springer, Berlin, 2005)
12. P.C. Chandrasekharan, *Robust Control of Linear Dynamical Systems* (Academic Press, San Diego, 1996)
13. R. Chen, Effects of primary rotor parameters on flapping dynamics, Technical report, TP-1431, NASA, 1980
14. A.J. Prasad, J.V.R. Corban, J.E. Calise, Implementation of adaptive nonlinear control for flight test on an unmanned helicopter, in *Proceedings of the 37th IEEE Conference on Decision and Control*, vol. 4, December 1998, pp. 3641–3646
15. J.E. Corban, A.J. Calise, J.V.R. Prasad, J. Hur, N. Kim, Flight evaluation of adaptive high-bandwidth control methods for unmanned helicopters, in *AIAA Guidance, Navigation, and Control Conference*, 2003
16. J.J. Craig, *Introduction to Robotics: Mechanics and Control* (Prentice Hall, New York, 2004)
17. J.C. Doyle, B.A. Francis, A. Tannenbaum, *Feedback Control Theory* (Macmillan, New York, 1992)

I.A. Raptis, K.P. Valavanis, *Linear and Nonlinear Control of Small-Scale Unmanned Helicopters*, Intelligent Systems, Control and Automation: Science and Engineering 45,
DOI 10.1007/978-94-007-0023-9,

18. M.E. Dreier, *Introduction to Helicopter and Tiltrotor Flight Simulation*. AIAA Education Series (AIAA, Washington, 2007)
19. D. Ernst, K. Valavanis, J. Craighead, Automated process for unmanned aerial systems controller implementation using MATLAB, in *14th Mediterranean Conference on Control and Automation, 2006, MED '06*, 2006
20. B. Etkin, *Dynamics of Flight: Stability and Control* (Wiley, New York, 1959)
21. I. Fantoni, R. Lozano, *Nonlinear Control for Underactuated Mechanical Systems* (Springer, New York, 2001)
22. M. Fliess, J. Levine, P. Martin, P. Rouchon, Flatness and defect of nonlinear systems: Introductory theory and applications. International Journal of Control **61**, 1327–1361 (1995)
23. G.F. Franklin, J.D. Powell, A. Emami-Naeini, *Feedback Control of Dynamic Systems* (Prentice Hall, New York, 2002)
24. E. Frazzoli, M.A. Dahleh, E. Feron, Trajectory tracking control design for autonomous helicopters using a backstepping algorithm, in *Proceedings of the American Control Conference*, vol. 6, 2000, pp. 4102–4107
25. D. Fujiwara, J. Shin, K. Hazawa, K. Nonami, $\mathcal{H}_\infty$ hovering and guidance control for an autonomous small-scale unmanned helicopter, in *International Conference on Intelligent Robots and Systems*, 2004
26. J. Gadewadikar, F. Lewis, K. Subbarao, B. Chen, $\mathcal{H}_\infty$ static output-feedback control for rotorcraft. Journal of Intelligent and Robotic Systems, **54**, 629–646 (2008)
27. J. Gadewadikar, F. Lewis, K. Subbarao, B. Chen, Structured $\mathcal{H}_\infty$ command and control-loop design for unmanned helicopters. Journal of Guidance, Control and Dynamics **31**, 1093–1102 (2008)
28. J. Gadewadikar, F.L. Lewis, K. Subbarao, K. Peng, B.M. Chen, $\mathcal{H}_\infty$ static output-feedback control for rotorcraft, in *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2006
29. V. Gavrilets, B. Mettler, E. Feron, Dynamical model for a miniature aerobatic helicopter, Technical report, Massachusetts Institute of Technology, 2001
30. V. Gavrilets, B. Mettler, E. Feron, Nonlinear model for a small-size acrobatic helicopter, in *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2001
31. D.T. Greenwood, *Principles of Dynamics* (Prentice Hall, New York, 1965)
32. N. Guenard, T. Hamel, V. Moreau, Dynamic modeling and intuitive control strategy for an "X4-flyer", in *International Conference on Control and Automation*, 2005, pp. 141–146
33. T. Hamel, R. Mahony, R. Lozano, J. Ostrowski, Dynamic modeling and configuration stabilization for an X4-flyer, in *15th Triennial World Congress of IFAC*, 2002
34. N. Hovakimyan, N. Kim, A.J. Calise, J.V.R. Prasad, N. Corban, Adaptive output feedback for high-bandwidth control of an unmanned helicopter, in *AIAA Guidance, Navigation, and Control Conference*, 2001
35. I.I. Hussein, M. Leok, A.K. Sanyal, A.M. Bloch, A discrete variational integrator for optimal control problems on SO(3), in *45th IEEE Conference on Decision and Control*, 2006
36. A. Isidori, L. Marconi, A. Serrani, *Robust Autonomous Guidance* (Springer, Berlin, 2003)
37. A. Isidori, L. Marconi, A. Serrani, Robust nonlinear motion control of a helicopter. IEEE Transactions on Automatic Control **48**, 413–426 (2003)
38. E.N. Johnson, S.K. Kannan, Adaptive flight control for an autonomous unmanned helicopter, in *AIAA Guidance, Navigation and Control Conference*, 2002
39. E.N. Johnson, S.K. Kannan, Adaptive trajectory control for autonomous helicopters. Journal of Guidance, Control and Dynamics **28**, 524–538 (2005)
40. W. Johnson, *Helicopter Theory* (Princeton University Press, Princeton, 1980)
41. R.E. Kalman, R.S. Bucy, New results in linear filtering and prediction theory. Journal of Basic Engineering **83**, 95–108 (1961)
42. F. Kendoul, D. Lara, I. Fantoni-Coichot, R. Lozano, Real-time nonlinear embedded control for an autonomous quadrotor helicopter. Journal of Guidance, Control and Dynamics **30**, 1049–1061 (2007)
43. H.K. Khalil, *Nonlinear Systems* (Prentice Hall, New York, 2002)

44. H.J. Kim, D.H. Shim, A flight control system for aerial robots: algorithms and experiments. Control Engineering Practice **11**, 1389–1400 (2003)
45. N. Kim, A.J. Calise, N. Hovakimyan, J.V.R. Prasad, E. Corban, Adaptive output feedback for high-bandwidth flight control. Journal of Guidance, Control and Dynamics **25**, 993–1002 (2002)
46. V. Klein, E.A. Moreli, *Aircraft System Identification Theory and Practice*, AIAA Education Series (AIAA, Washington, 2006)
47. T.J. Koo, S. Sastry, Output tracking control design of a helicopter model based on approximate linearization, in *Proceedings of the 37th IEEE Conference on Decision and Control*, vol. 4, 1998, pp. 3635–3640
48. T.J. Koo, S. Sastry, Differential flatness based full authority helicopter control design, in *Proceedings of the 38th IEEE Conference on Decision and Control*, 1999
49. M. Krstic, I. Kanellakopoulos, P.V. Kokotovic, *Nonlinear and Adaptive Control Design* (Wiley–Interscience, New York, 1995)
50. R. Kureemun, D.J. Walker, B. Manimala, M. Voskuijl, Helicopter flight control law design using $\mathcal{H}_\infty$ techniques, in *IEEE Conference on Decision and Control*, 2005
51. M. La Civita, Integrated modeling and robust control for full envelope flight of robotic helicopters, PhD thesis, Carnegie Mellon University, 2002
52. M. La Civita, W.C. Messner, T. Kanade, Modeling of small-scale helicopters with integrated first-principles and system identification techniques, in *Proceedings of the 58th Forum of the American Helicopter Society*, vol. 2, 2002, pp. 2505–2516
53. M. La Civita, G. Papageorgiou, W. Messner, T. Kanade, Design and flight testing of a high-bandwidth $\mathcal{H}_\infty$ loop shaping controller for a robotic helicopter, in *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2002
54. M. La Civita, G. Papageorgiou, W.C. Messner, T. Kanade, Integrated modeling and robust control for full-envelope flight of robotic helicopters, in *Proceedings of IEEE International Conference on Robotics and Automation*, 2003, pp. 552–557
55. M. La Civita, G. Papageorgiou, W.C. Messner, T. Kanade, Design and flight testing of an $\mathcal{H}_\infty$ controller for a robotic helicopter. Journal of Guidance, Control, and Dynamics, 485–494 (2006)
56. E.H. Lee, H. Shim, L. Park, K. Lee, Design of hovering attitude controller for a model helicopter, in *Proceedings of Society of Instrument and Control Engineers*, 1993, pp. 1385–1390
57. T. Lee, N.H. McClamroch, M. Leok, Optimal control of a rigid body using geometrically exact computations on SE(3), in *45th IEEE Conference on Decision and Control*, 2006
58. J.G. Leishman, *Principles of Helicopter Aerodynamics* (Cambridge University Press, Cambridge, 2000)
59. W. Levine, M. Athans, On the determination of the optimal constant output feedback gains for linear multivariable systems. IEEE Transactions on Automatic Control **15**, 44–48 (1970)
60. F.L. Lewis, V.L. Syrmos, *Optimal Control* (Wiley–Interscience, New York, 1995)
61. L. Ljung, *System Identification* (Prentice Hall, New York, 1987)
62. L. Ljung, *System Identification: Theory for the User* (Prentice Hall, New York, 1999)
63. A. Loria, E. Panteley, *Advanced Topics in Control Systems Theory: Lecture Notes from FAP 2004*, chapter 2 (Springer, Berlin, 2005), pp. 23–64
64. R. Mahony, T. Hamel, Robust trajectory tracking for a scale model autonomous helicopter. International Journal of Robust and Nonlinear Control **14**(12), 1035–1059 (2004)
65. R. Mahony, T. Hamel, A. Dzul, Hover control via Lyapunov control for an autonomous model helicopter, in *Proceedings of the 38th IEEE Conference on Decision and Control*, vol. 4, 1999, pp. 3490–3495
66. L. Marconi, R. Naldi, Robust full degree-of-freedom tracking control of a helicopter. Automatica **43**, 1909–1920 (2007)
67. L. Marconi, R. Naldi, Aggressive control of helicopters in presence of parametric and dynamical uncertainties. Mechatronics **1**, 381–389 (2008)
68. D. McFarlane, K. Glover, A loop-shaping design procedure using $\mathcal{H}_\infty$ synthesis. IEEE Transactions on Automatic Control **37**, 759–769 (1992)

69. J.M. Mendel, *Lessons in estimation theory for signal processing, communications, and control* (Prentice Hall PTR, New York, 1995)
70. B. Mettler, *Identification Modeling and Characteristics of Miniature Rotorcraft* (Kluwer Academic Publishers, Norwell, 2003)
71. B. Mettler, T. Kanade, M.B. Tischler, System identification modeling of a model-scale helicopter, Technical report, Carnegie Mellon University, 2000
72. B. Mettler, M.B. Tischler, T. Kanade, System identification of small-size unmanned helicopter dynamics, in *Presented at the American Helicopter Society 55th Forum*, May 1999
73. S.K. Mitra, *Digital Signal Processing: A Computer-Based Approach* (McGraw-Hill, New York, 2006)
74. A. Moerder, D. Calise, Convergence of a numerical algorithm for calculating optimal output feedback gains. IEEE Transactions on Automatic Control **30**(9), 900–903 (1985)
75. R.M. Murray, L. Zexiang, S. Sastry, *A Mathematical Introduction to Robotic Manipulation* (CRC Press, Boca Raton, 1994)
76. A.V. Oppenheim, R.W. Shafer, J.R. Buck, *Discrete-Time Signal Processing* (Prentice Hall, New York, 1999)
77. A.V. Oppenheim, A.S. Willsky, I.T. Young, *Signals and Systems* (Prentice Hall, New York, 1983)
78. H. Ozbay, *Introduction to Feedback Control Theory* (CRC Press, Boca Raton, 1999)
79. G.D. Padfield, *Helicopter Flight Dynamics: The Theory and Application of Flying Qualities and Simulation Modeling*, AIAA Education Series (Blackwell Science, Oxford, 1996)
80. G. Papageorgiou, K. Glover, $\mathcal{H}_\infty$ loop-shaping: Why is it a sensible procedure for designing robust flight controllers, in *AIAA Conference on Guidance, Navigation and Control*, 1999
81. K.M. Passino, S. Yurkovich, *Fuzzy Control* (Prentice Hall, New York, 1998)
82. S. Pieper, J.K. Baillie, K.R. Goheen, Linear-quadratic optimal model-following control of a helicopter in hover, in *American Control Conference*, 1994
83. E. Prempain, I. Postlethwaite, Static $\mathcal{H}_\infty$ loop shaping control of a fly-by-wire helicopter. Automatica **41**, 1517–1528 (2005)
84. R.W. Prouty, *Helicopter Performance, Stability and Control* (Krieger Publishing Company, Melbourne, 1995)
85. I.A. Raptis, K.P. Valavanis, A. Kandel, W.A. Moreno, System identification for a miniature helicopter at hover using fuzzy models. Journal of Intelligent and Robotic Systems **56**, 345–362 (2009)
86. E. Seckel, *Stability and Control of Airplanes and Helicopters* (Academic Press, San Diego, 1964)
87. J.S. Shamma, M. Athans, Analysis of gain scheduled control for nonlinear plants. IEEE Transactions on Automatic Control **35**, 898–907 (1990)
88. H. Shim, T.J. Koo, F. Hoffmann, S. Sastry, A comprehensive study of control design for an autonomous helicopter, in *Proceedings of the 37th IEEE Conference on Decision and Control*, vol. 4, 1998, pp. 3653–3658
89. H.D. Shim, H.J. Kim, S. Sastry, Control system design for rotorcraft-based unmanned aerial vehicles using time-domain system identification, in *Proceedings of the 2000 IEEE International Conference on Control Applications*, 2000, pp. 808–813
90. J. Shin, K. Nonami, D. Fujiwara, K. Hazawa, Model-based optimal attitude and positioning control of small-scale unmanned helicopter. Robotica **23**, 51–63 (2005)
91. H. Sira-Ramirez, M. Zribi, S. Ahmad, Dynamical sliding mode control approach for vertical flight regulation in helicopters, Control Theory and Applications **141**, 19–24 (1994)
92. S. Skogestad, I. Postlethwaite, *Multivariable Feedback Control* (Wiley, New York, 1996)
93. T. Soderstrom, P. Stoica, *System Identification* (Prentice Hall, New York, 1989)
94. E.D. Sontag, Remarks on stabilization and input-to-state stability, in *Proceedings of the 28th IEEE Conference on Decision and Control*, vol. 2, 1989, pp. 1376–1378
95. M.W. Spong, S. Hutchinson, M. Vidyasagar, *Robot Modeling and Control* (Wiley, New York, 2005)
96. J.T. Spooner, K.M. Passino, Stable adaptive control using fuzzy systems and neural networks. IEEE Transactions on Fuzzy Systems **4**, 339–359 (1996)

97. X.D. Sun, T. Clarke, Application of hybrid $\mu/\mathcal{H}_\infty$ control to modern helicopters, in *International Conference on Control*, 1994
98. H.J. Sussmann, P.V. Kokotovic, The peaking phenomenon and the global stabilization of nonlinear systems. IEEE Transactions on Automatic Control **36**, 424–440 (1991)
99. V.L. Syrmos, C. Abdallah, P. Dorato, Static output feedback: A survey, in *33rd Conference on Decision and Control*, 1994
100. V.L. Syrmos, C. Abdallah, P. Dorato, K. Grigoriadis, Static output feedback: A survey, Technical report, University of New Mexico, 1995
101. T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics **15**, 116–132 (1985)
102. A.R. Teel, Global stabilization and restricted tracking for multiple integrators with bounded controls. Systems & Control Letters **18**, 165–171 (1992)
103. A.R. Teel, Using saturation to stabilize a class of single-input partially linear composite systems, in *IFAC NOLCOS'92 Symposium*, 1992, pp. 379–384
104. M.B. Tischler, M.G. Cauffman, Frequency-response method for rotorcraft system identification: Flight applications to BO-105 coupled fuselage/rotor dynamics. Journal of the American Helicopter Society **3**, 3–17 (1992)
105. M.B. Tischler, R.K. Remple, *Aircraft and Rotorcraft System Identification*, AIAA Education Series (AIAA, Washington, 2006)
106. K.P. Valavanis (ed.), *Advances in Unmanned Aerial Vehicles State of the Art and the Road to Autonomy*, Intelligent Systems, Control and Automation: Science and Engineering, vol. 33 (Springer, Berlin, 2007)
107. M.J. Van Nieuwstadt, Trajectory generation for nonlinear control systems, PhD thesis, California Institute of Technology, 1997
108. D.J. Walker, I. Postlethwaite, Advanced helicopter flight control using two-degree-of-freedom $\mathcal{H}_\infty$ optimization. Journal of Guidance, Control and Dynamics **19**, 461–468 (1996)
109. M.F. Weilenmann, U. Christen, H.P. Geering, Robust helicopter position control at hover, in *American Control Conference*, 1999
110. M.F. Weilenmann, P. Hans, A test bench for rotorcraft hover control, in *AIAA Guidance, Navigation and Control Conference*, 1993, pp. 1371–1382
111. J.H. Williams, *Fundamentals of Applied Dynamics* (Wiley, New York, 1996)
112. J. Zhao, I. Kanellakopoulos, Adaptive control of discrete-time strict-feedback nonlinear systems. Proceedings of the American Control Conference **1**, 828–832 (1997)
113. K. Zhou, J.C. Doyle, K. Glover, *Robust and Optimal Control* (Prentice Hall, New York, 1996)