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Symbols and Notations

Below we list the symbol that either appear in the text more than just locally, or are important for other reasons. The more relevant conventions and notations in use throughout the book are found in Sections 1.1 and 2.1.

Logical Operators

$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$	p. 3	(propositional operators)
\perp	p. 3	(falsity)
\top	p. 3	(truth)
\Box, \Diamond	p. 39	(modal operators)
\Rightarrow	p. 4	(sequent arrow)
\bigwedge, \bigvee	p. 4	(iterated conjunction and disjunction)

Languages and Well-Formed Formulas

\mathcal{L}^c	p. 3	(classical propositional language)
\mathcal{L}^\Box	p. 39	(modal propositional language)
$\mathcal{L}_{\neg, \wedge, \Box}^\Box$	p. 40	(modal propositional language restricted to the connectives \neg, \wedge, \Box)
WF	p. 3	(set of well-formed formulas)
PL	p. 3	(propositional letters)
WMF	p. 39	(set of well-formed modal formulas)

Measures on Formulas and Derivations

d, d'	p. 4	(formal derivations)
$cmp(\alpha)$	p. 40	(complexity of a formula)
$h(d)$	p. 6	(height of a derivation)
$s(G)$	p. 172	(size of a tree-hypersequent)
$sf(G)$	p. 172	(set of subformulas of a tree-hypersequent)

Turnstiles Symbols

$\vdash_{\mathbf{H}}$	p. 4	(deducibility in a Hilbert system)
$\vdash_{\mathbf{G}}$	p. 4	(deducibility in a Gentzen system)
$d \vdash_{\mathbf{G}}^n$	p. 6	(deducibility with height $\leq n$ in a Gentzen system)
$i \models_{\mathfrak{M}}$	p. 41	(satisfiability)
$\models_{\mathfrak{c}}$	p. 41	(validity in a class of frames)

Formalisms**- general formalisms**

S	p. 4	(formal system)
NMS	p. 40	(normal modal system)
SLH	p. 44	(Scott-Lemmon Hilbert system)
H	p. 4	(Hilbert system)
G	p. 4	(Gentzen system)
N	p. 4	(natural deduction system)

- non modal formalisms

Gcl	p. 5	(Gentzen system for classical logic)
Gcl_L	p. 9	(logical variant of the Gentzen system for classical logic)
Gcl_S	p. 10	(structural variant of the Gentzen system for classical logic)
Gil	p. 11	(Gentzen system for intuitionistic logic)
Gll	p. 11	(Gentzen system for linear logic without exponentials)
Hcl	p. 7	(Hilbert system for classical logic)

- Hilbert modal formalisms

K	p. 40	KD	p. 44
KT	p. 44	K4	p. 44
KB	p. 44	S4	p. 44
S5	p. 44	GL	p. 44

- Gentzen modal formalisms

Msk*	p. 59	(multiple sequent calculi)
H-ask*	p. 65	(higher-arity sequent calculi)
Dsk*	p. 71	(display sequent calculi)

Ssk*	p. 83	(semantic sequent calculi)
Isk*	p. 91	(indexed sequent calculi)
Ifsk*	p. 98	(internalised forcing sequent calculi)
Thsk*	p. 127	(tree-hypersequent calculi)

Syntactic Tools

Let $\star \in \{\wedge, \vee, \rightarrow, W, C\}$

\mathcal{R}	p. 6	(rule)
\mathcal{R}^*	p. 74	(repeated running application of the same rule)
$\mathcal{R}^* + \mathcal{R}'^* + \mathcal{R}''^*$	p. 163	(repeated running application of different rules)
$\star A, \star K$	p. 5	(left and right introduction rules)
$Ax, A\perp$	p. 5	(axioms)
cut_α	p. 5	(cut-rule)

Semantic Tools

i, j, \dots	p. 40	(variables for possible worlds)
R	p. 40	(accessibility relation)
\mathcal{F}	p. 40	(frame)
\mathfrak{M}	p. 41	(model)
\mathcal{T}	p. 40	(tree-frame)

Other Notations

\square	p. 7	(end-of-proof symbol)
δ, τ	p. 4	(translations and embedding)
\rightsquigarrow	p. 141	(application of the inductive hypothesis)
\dashrightarrow	p. 141	(application of the invertibility of the logical rules)
$prove(G, (\mathbf{Y})^+)$	p. 172	(procedure for building derivation trees in tree-hypersequent calculi)

Index

μετά-βασις εἰς ἄλλο γένος, 13

A

Accessibility relation, 40, 76, 85

Adequacy, 44

Admissibility, 6

height-preserving, 6

Alternative, 7

multiset, 7

sequence, 7

set, 7

structural databases, 34

Aristotle, 12, 13

Arnauld, 13

Avron, 3, 56, 73, 121

Axioms, 4

Löb's, 44

B

Belnap, 19, 66, 71, 72, 74

Blamey, 51, 62, 64–66

Bolzano, 13

Boolos, 23

Bounded morphic image, 42

Bounded morphism, 42

Brünnler, 121, 170, 207

Braüner, 47

C

Calculus of structures, 56

Cerrato, 74, 76, 77, 83

Chellas, 58

Closure

euclidean, 81

reflexive, 80

reflexive-transitive, 41, 171

serial, 171

symmetric, 80, 171

transitive, 41, 81, 171

Common knowledge, 207

Completeness, 44, 170

Concatenation, 85

Conservativeness criterion

for anti-realistic definitions, 23

for realistic definitions, 17

Cresswell, 85

Curry, 46, 56, 72

Cut-rule, 6

admissibility, 14, 22, 143, 181,
196

analytic, 24, 49

eliminability, 6, 14, 22

lack of, 58, 81

mixcut, 91

Cyclic, 172

D

D'agostino, 23

Decidability, 98, 156, 159, 183

Definition

anti-realistic, 16

realistic, 16

Derivation

in Gentzen systems, 4

height of, 6

in Hilbert systems, 4

in natural deduction systems, 4

Descartes, 13, 23

Display

calculi, 55, 104, 113

equivalence, 70

logic, 66

proper display calculus, 72

property, 71

sequent, 67

structure, 67

theorem, 70

Došen, 10, 14, 29, 32, 33, 55

Došen's principle, 31
 Došen's principle redefined, 34, 62, 66, 73, 99
 Dragalin, 9
 Dummett, 19

E

Eigenvariable, 97
 Eliminability, 6
 height-preserving, 6
 Eliminability criterion
 for anti-realistic definitions, 25
 for realistic definitions, 17
 Equivalent position, 124
 Explicit bound, 160
 Explicitness, 28, 62

F

Feys, 46
 Fine, 42
 Finished
 sequent, 172
 tree-hypersequent, 172
 First-order logic, 96, 127
 first-order constructive logics with
 negation, 207
 Fitting, 58, 59
 Formal system, 4
 Gentzen systems, 4
 Gentzen calculi, 4
 sequent calculi, 4
 Hilbert systems, 4
 normal modal Hilbert systems, 40
 natural deduction systems, 4, 76, 207
 tableaux systems, 207
 Formulas
 auxiliary formulas, 6
 B-formulas, 56
 complexity of a formula, 40
 principal formula, 6
 side formulas, 6
 well-formed formulas, 4
 well-formed modal formulas, 39
 Frame, 40
 frame correspondence, 43
 first-order correspondent, 43
 second-order correspondent, 43
 frame definability, 42, 43
 frame languages, 43
 S5 Kripke frame, 176
 transitive frame without infinite ascending
 R-chains, 44, 187
 tree-frame, 40, 75, 119
 Francez, 19

G

Galileo, 13
 Gentzen, 3, 14
 Goble, 45
 Goré, 51, 53, 74
 Gottwald, 62
 Grefe, 72

H

Hacking, 21
 Harmony, 18
 as conservativeness, 19
 as deductive equilibrium, 19
 General Elimination, 19
 as reduction, 19
 Hein, 31
 Higher-arity sequent, 62
 Higher-arity sequent calculi, 55, 104
 Higher-dimensional sequent calculi, 55
 Higher-level sequent calculi, 55
 Hill, 207
 Hippocrates of Chius, 13
 Hjortland, 19, 27
 Hughes, 85
 Humberstone, 51, 62, 64–66
 Hypersequent, 121, 176
 negative part of a, 185
 positive part of a, 185
 Hypersequent calculi, 55

I

Indexed sequent calculi, 85, 107, 112
 Indices, 85
 Indrzejczak, 56, 59, 62
 Inference rules, 4
 additive rules, 8
 conclusion of a rule, 5
 duality rules, 74
 inference rules of the sequent calculus, 5
 inverse of a rule, 7
 logical rules, 5
 multiplicative rules, 8
 premises of a rule, 5
 structural rules, 5
 Inferentialism, 16, 20
 Internalised forcing sequent, 93
 Internalised forcing sequent calculi, 94, 107,
 113
 Interpolation, 58, 207
 Intuitionistic logic, 11
 Invertibility, 188
 height-preserving, 7, 126, 177
 Ishigaki, 207

K

Kanger, 76
 Kashima, 121, 207
 Kikuchi, 207
 Kracht, 72
 Kremer, 35
 Kripke, 47, 84
 Kripke semantics, 40
 Kushida, 76

L

Labelled deductive systems, 76
 Leśniewski, 17
 Leivant, 45, 51, 187
 Lemmon, 43, 44
 Linear logic, 11
 Logicality, 14
 Lukasiewicz n -valued logics, 62

M

Masini, 55
 Matsumoto, 45–47, 50
 Meaning
 global, 26
 operational, 26
 Minimal derivations, 156, 183
 Mints, 45, 47, 48, 84, 85, 94, 108, 116
 Modal sequent, 56
 Modal tree-sequent, 76
 Model, 41
 tree-model property, 42, 75
 Modularity, 34, 62, 96, 126
 Moen, 51
 Mondadori, 23
 Multiple sequent calculi, 55, 107

N

Negri, 92, 96–99, 113, 187
 Newton, 13

O

Ohnishi, 45–47, 50
 Okada, 76
 Olbach, 85

P

Paoli, 13, 21, 26, 27, 71
 Pappus, 13
 Pascal, 13
 Peano arithmetic, 187
 Plato, 12
 Pliuškevičiene, 76
 Postulates, 4
 Pottinger, 121

Prawitz, 19
 Prior, 18
 Proclus, 12
 Product
 of two tableaux, 88
 of two tree-hypersequents, 124
 Proof
 analytic, 12
 synthetic, 12
 Proof-nets, 207
 Proof-theoretic semantics, 16
 Proper variable, 97
 Property
 irreflexivity, 43, 99
 Propositional dynamic logic, 207
 Pure derivation, 88, 98

Q

Quine, 26, 28

R

Ramanayake, 51
 Range, 98
 Rautenberg, 47, 53
 Read, 19–21
 Restall, 56, 71, 84, 99
 Rousseau, 62

S

Sahlqvist, 44
 Sambin, 21, 45, 46, 51, 71
 Sasaki, 51
 Satisfiability relation, 41, 92
 Sato, 47, 50, 62
 Schroeder-Heister, 16, 19
 Schröter, 62
 Scott, 43, 44
 Segregation, 71
 Semantic element, 31
 algebraic elements, 75
 Kripke semantics elements, 75, 76
 Semantic modal sequent, 77
 Semantic modal sequent calculi, 77, 112
 Separation, 20, 28
 Sequent, 4
 antecedent of, 4
 auxiliary, 128
 consequent of, 4
 Shvarts, 51
 Smullyan, 59
 Soundness, 44, 165
 Stewart, 31
 Stouppa, 31
 Structural deduction, 15

- Stüder, 207
 Subformula property, 14, 22, 99
 Substitution
 substitution in zoom tree-hypersequents, 123
 substitution lemma, 97
 Subintuitionistic predicate logics, 207
 Substructure property, 74
 Subterm property, 99
 Subtree, 173
 immediate, 173
 proper, 173
 Symmetry, 28
 Syntactic purity
 strong, 29
 weak, 29
- T**
- Tableau, 85
 pure, 88
 Takano, 46
 Tennant, 19
 Theorem, 4
 in a Gentzen system, 4
 in a Hilbert system, 4
 Thomason, 42
 Tragesser, 29
 Tree-hypersequent, 122
 leaf of, 172
 length of, 197
 negative part of, 160
 position of a sequent in, 197
 positive part of, 160
 set, 171
 size of, 172
 zoom, 122
- U**
- Undecidability, 72
 Uniqueness, 29
 Unraveling, 42
- V**
- Valentini, 45, 46, 51
 Variant, 7
 general, 9
 logical, 9, 24, 66, 73, 126, 186, 201
 structural, 10, 28, 61, 84, 99
 Visibility, 71
- W**
- Wallen, 85
 Wansing, 21, 29, 47, 55, 66, 71–74, 101, 107, 116
- Z**
- Zeman, 51, 56
 Zucker, 29