Appendix A
Notation and Formulae

In this Appendix we list our notion and formulae used in this thesis. Since we work on $1 + 1$ and $3 + 1$ dimensions, here we generalize the space-time dimensionality to $d + 1$ dimensions, and we use the signature $(-, +, +, \cdots, +)$ for a Lorentzian metric $g_{\mu\nu}$. Greek indices run from 0 to d in the $d + 1$-dimensional space-time, and we also use Latin indices for the d-dimensional Euclidean space. Indices of all $d + 1$-dimensional tensors, pseudo-tensors and densities are lowered and raised by the metric $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$, respectively. In addition, we apply the Einstein summation convention. The covariant derivative compatible with $g_{\mu\nu}$ is denoted by ∇_λ, and we use Δ as the Laplacian, $g^{\mu\nu}\nabla_\mu\nabla_\nu$. We write the cosmological constant and Newton constant as Λ and G_{d+1}, respectively. Then here we go.

Christoffel symbol:

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right). \tag{A.1}$$

Covariant derivative acting on covariant and contravariant vectors:

$$\nabla_\mu t^\lambda = \partial_\mu t^\lambda + \Gamma^\lambda_{\mu\nu} t^\nu, \quad \nabla_\mu w_\nu = \partial_\mu w_\nu - \Gamma^\lambda_{\mu\nu} w_\lambda. \tag{A.2}$$

Covariant derivative acting on density:

$$\nabla_\sigma p^{\mu\nu} = \partial_\sigma p^{\mu\nu} + \Gamma^\mu_{\sigma\tau} p^{\tau\nu} + \Gamma^\nu_{\sigma\tau} p^{\mu\tau} - \Gamma^\tau_{\tau\sigma} p^{\mu\nu}. \tag{A.3}$$

Riemann tensor:

$$R^\lambda{}_{\mu\nu\sigma} = \partial_\nu \Gamma^\lambda_{\mu\sigma} - \partial_\sigma \Gamma^\lambda_{\mu\nu} + \Gamma^\eta_{\mu\sigma} \Gamma^\lambda_{\nu\eta} - \Gamma^\eta_{\mu\nu} \Gamma^\lambda_{\sigma\eta}. \tag{A.4}$$

Ricci tensor and Ricci scalar:

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}, \quad R = g^{\mu\nu} R_{\mu\nu}. \tag{A.5}$$

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Commutators of covariant derivatives:

$$[\nabla_\mu, \nabla_\nu]t^\lambda = R^\lambda{}_{\sigma\mu\nu}t^\sigma, \tag{A.6}$$

$$[\nabla_\mu, \nabla_\nu]w_\lambda = -R^\sigma{}_{\lambda\mu\nu}w_\sigma. \tag{A.7}$$

Variation of metric:

$$\delta g^{\mu\nu} = -g^{\mu\sigma}g^{\nu\tau}\delta g_{\sigma\tau}. \tag{A.8}$$

Variation of Christoffel symbol:

$$\delta\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(\nabla_\mu\delta g_{\nu\sigma} + \nabla_\nu\delta g_{\mu\sigma} - \nabla_\sigma\delta g_{\mu\nu}\right). \tag{A.9}$$

Variation of Ricci tensor:

$$\delta R_{\mu\nu} = \nabla_\lambda\delta\Gamma^\lambda_{\mu\nu} - \nabla_\nu\delta\Gamma^\lambda_{\lambda\mu}. \tag{A.10}$$

Variation of Ricci scalar:

$$\delta R = -R^{\mu\nu}\delta g_{\mu\nu} + (\nabla^\mu\nabla^\nu - g^{\mu\nu}\Delta)\delta g_{\mu\nu}. \tag{A.11}$$

Variation of Laplacian acting on scalar:

$$\delta(\Delta e) = \Delta(\delta e) - \nabla^\mu\nabla^\nu e\delta g_{\mu\nu} - \nabla^\mu e\nabla^\nu\delta g_{\mu\nu} + \frac{1}{2}g^{\mu\nu}\nabla^\tau e\nabla_\tau\delta g_{\mu\nu}. \tag{A.12}$$

Einstein-Hilbert action with Gibbons-Hawking-York boundary term:

$$S = -\frac{1}{16\pi G_{d+1}}\int\limits_{\mathscr{M}} d^{d+1}x\sqrt{-g}\,(R - 2\Lambda + \mathscr{K}) \tag{A.13}$$

$$= -\frac{1}{16\pi G_{d+1}}\int\limits_{\mathscr{M}} d^{d+1}x\sqrt{-g}\,(R - 2\Lambda) + \frac{1}{8\pi G_{d+1}}\int\limits_{\partial\mathscr{M}} d^dx\sqrt{h}K. \tag{A.14}$$

Appendix B
Computations in *n*-DBI Gravity

B.1 Derivation of the Solutions

Taking the ansatz

$$ds^2 = -N^2(r)dt^2 + e^{2f(r)}\left(dr + e^{2g(r)}dt\right)^2 + r^2 d\Omega_2, \tag{B.1}$$

it follows that ($' \equiv d/dr$):

$$\begin{aligned} K_{ij} &= -\frac{e^{2g}}{N}\,\mathrm{diag}\left\{e^{2f}(f+2g)', r, r\sin^2\theta\right\}, \\ K &= -\frac{e^{2g}}{N}\left(\frac{2}{r} + (f+2g)'\right), \end{aligned} \tag{B.2}$$

$$R_{ij} = \mathrm{diag}\left\{\frac{2f'}{r}, \frac{rf' + e^{2f} - 1}{e^{2f}}, \sin^2\theta\,\frac{rf' + e^{2f} - 1}{e^{2f}}\right\}, \tag{B.3}$$

$$R = \frac{2e^{-2f}}{r^2}\left(2rf' + e^{2f} - 1\right), \tag{B.4}$$

Thus,

$$\mathscr{R} = \frac{2}{r^2}\left[1 - \left(re^{-2f}\right)' - \frac{\left(re^{4g}\right)'}{N^2} - \frac{2rf'e^{4g}}{N^2} - \frac{e^{-f}}{N}\left(r^2N'e^{-f}\right)'\right]. \tag{B.5}$$

From the effective Lagrangian (3.16), the A equation of motion can be solved straightaway to yield

$$A' = Q\frac{Ne^f}{r^2}, \tag{B.6}$$

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where Q is an integration constant. The g equation of motion yields the compact relation

$$\left(\frac{1}{\sqrt{1+\frac{G_4}{6\lambda}\mathscr{R}}}\right)' = \frac{(f+\ln N)'}{\sqrt{1+\frac{G_4}{6\lambda}\mathscr{R}}}, \tag{B.7}$$

and can be integrated to yield

$$\left(1+\frac{G_4}{6\lambda}\mathscr{R}\right)^{-1/2} = \frac{Ne^f}{C}, \tag{B.8}$$

where C is an integration constant. More explicitly, this equation may be written as

$$\left(re^{4g+2f}\right)' = N^2e^{2f}\left(1-(re^{-2f})'\right) - Ne^f\left(r^2e^{-f}N'\right)' + \frac{3\lambda}{G_4}r^2\left(N^2e^{2f}-C^2\right). \tag{B.9}$$

The f and N equations of motion, upon using (B.6) and (B.8), read, respectively

$$\left(re^{4g+2f}\right)' = re^{-2f}\left(e^{2f}N^2\right)' - \frac{Ne^f}{2}\left(r^2e^{-f}N'\right)' + \frac{r^2e^{-f}N'}{2}(Ne^f)' - \frac{3\lambda}{G_4}r^2\left(C^2-CqNe^f\right) + \frac{G_4CQ^2Ne^f}{2r^2}, \tag{B.10}$$

$$\left(re^{4g+2f}\right)' = -\frac{1}{4}\left(r^2\left(e^{-2f}\right)'N^2e^{2f}\right)' - \frac{3\lambda}{G_4}r^2\left(C^2-CqNe^f\right) + \frac{G_4CQ^2Ne^f}{2r^2}. \tag{B.11}$$

Equation (B.11) is the Hamiltonian constraint (3.3), after using (B.8) and (B.9).

B.1.1 Solutions with Constant $\mathscr{R}$

To proceed we take the combination $Ne^f = \tilde{C} =$ constant which implies that $\mathscr{R}$ is constant. We shall address the general solution in the next subsection, but it turns out that the most interesting solution are found in this subset. With this choice, we observe from Eq. (B.8) that $\mathscr{R} =$ constant. From the resulting equations of motion, equating (B.9) with either (B.10) or (B.11) (which become identical), we find the ODE:

$$Y'' + \frac{6}{r}Y' + \frac{4}{r^2}Y = \frac{12\lambda}{G_4}\left(1-\frac{qC}{\tilde{C}}\right) - \frac{2G_4CQ^2}{\tilde{C}r^4}, \tag{B.12}$$

where $Y \equiv e^{-2f} - 1$. It is now straightforward to obtain the exact solution. It reads (3.17), where $\tilde{C}$ has been eliminated by rescaling C and the time coordinate.

B.1.2 Generic Solution

Since the set of equations we are solving is a second order ODE with three unknowns, we expect a total of six integration constants. The constant $\mathscr{R}$ solution exhibited below has only five integration constants and thus it is not the most general one. The latter can be obtained observing that the Eqs. (B.10) and (B.11) imply

$$-\left(r^{-2}(\log N)'\right)' = \left(r^{-2}f'\right)' \qquad \Longrightarrow \qquad Ne^{f} = \tilde{C}e^{\frac{1}{3}C_4 r^3}\,. \tag{B.13}$$

C_4 is the sixth integration constant, which was absent in the constant $\mathscr{R}$ solution. Similarly to the constant $\mathscr{R}$ case, it is straightforward to find a second order ODE:

$$W'' + \frac{6}{r}W' + \frac{4}{r^2}W = \frac{4e^{\frac{2C_4r^3}{3}}}{r^2} + \frac{12\lambda}{G_4}\left(e^{\frac{2C_4r^3}{3}} - \frac{qCe^{\frac{C_4r^3}{3}}}{\tilde{C}}\right) - \frac{2G_4CQ^2e^{\frac{C_4r^3}{3}}}{\tilde{C}r^4}\,, \tag{B.14}$$

where we defined $W \equiv e^{-2\left(f - \frac{C_4r^3}{3}\right)}$. This can be integrated to give explicit solutions. As they are not very illuminating, however, we will not present them here. Indeed, the solutions with $C_4 \neq 0$ seem rather exotic, since (B.8) and (B.13) imply that their asymptotic behavior at $r = +\infty$ is very different from that of Einstein gravity: the $C_4 < 0$ solutions have a curvature singularity at $r = +\infty$ and thus we regard these solutions as unphysical; the $C_4 > 0$ solutions have the maximal negative curvature $\mathscr{R} = -6\lambda/G_4$ at $r = +\infty$. Although they are interesting in their own right, we shall not discuss these solutions further herein.

B.2 Computations of Constraint Algebra

In this Appendix, we give an explicit computation of the constraint algebra and classification of the constraints presented in Sect. 3.2.1. To facilitate the computation, we introduce smooth test vector fields, $\xi^\mu = (\xi^0, \xi^i)$ and $\eta^\mu = (\eta^0, \eta^i)$, which fall off fast enough to suppress all the boundary contributions [1]. Henceforth, we define the smeared constraints:

$$\hat{\Phi}_1(\xi^0) = \int d^3x \xi^0(x)\Phi_1(x), \tag{B.15}$$

$$\hat{\Phi}_2(\xi^i) = \int d^3x \xi^i(x)\Phi_{2i}(x), \tag{B.16}$$

$$\hat{\Phi}_3(\xi^0) = \int d^3x \xi^0(x)\Phi_3(x), \tag{B.17}$$

$$\hat{\Phi}_4(\xi^0) = \int d^3x \xi^0(x)\Phi_4(x), \tag{B.18}$$

$$\hat{\Phi}_5(\xi^i) = \int d^3x \xi^i(x)\Phi_{5i}(x), \tag{B.19}$$

$$\hat{\Phi}_6(\xi^0) = \int d^3x \xi^0(x)\Phi_6(x), \tag{B.20}$$

$$\hat{\Phi}_N^{(G)}(\xi^i) = \int d^3x\xi^i(x)\Phi_{Ni}^{(G)}(x) = \int d^3x\xi^i(x)(-\Phi_1\partial_i N)(x), \tag{B.21}$$

$$\hat{\Phi}_e^{(G)}(\xi^i) = \int d^3x\xi^i(x)\Phi_{ei}^{(G)}(x) = \int d^3x\xi^i(x)(-\Phi_3\partial_i e)(x), \tag{B.22}$$

$$\hat{\Phi}_{\mathbf{N}}^{(G)}(\xi^i) = \int d^3x\xi^i(x)\Phi_{\mathbf{N}i}^{(G)}(x) = \int d^3x\xi^i(x)\Big[-\left(\Phi_2^i\nabla_i N^j + \nabla_j\left(\Phi_{2i}N^j\right)\right)(x)\Big], \tag{B.23}$$

$$\hat{\Phi}_5'(\xi^i) = \hat{\Phi}_5(\xi^i) + \hat{\Phi}_N^{(G)}(\xi^i) + \hat{\Phi}_e^{(G)}(\xi^i) = \int d^3x\xi^i(x)\tilde{\Phi}_{5i}(x), \tag{B.24}$$

where Φ_1, Φ_{2i}, Φ_3, Φ_4, Φ_{5i}, Φ_6 and $\tilde{\Phi}_{5i}$ are the constraints defined in Sect. 3.2.1. The idea is to compute the commutators like

$$\left\{\hat{\Phi}_4(\xi^0), \hat{\Phi}_5(\eta^j)\right\} = \int d^3yd^3x\xi^0(y)\eta^j(x)\left\{\Phi_4(y), \Phi_{5j}(x)\right\}, \tag{B.25}$$

and read off the algebra of local constraints from the R.H.S. The basic non-vanishing Poisson brackets are given by

$$\left\{p^{ij}(y), h_{kl}(x)\right\} = \frac{1}{2}\left(\delta_k^i\delta_l^j + \delta_l^i\delta_k^j\right)\delta(y-x), \tag{B.26}$$

$$\{p_N(y), N(x)\} = \delta(y-x), \tag{B.27}$$

$$\left\{p_{\mathbf{N}}^i(y), N_j(x)\right\} = \delta_j^i\delta(y-x), \tag{B.28}$$

$$\{p_e(y), e(x)\} = \delta(y-x). \tag{B.29}$$

To compute the Poisson brackets of constraints, we take the variations of the smeared constraints:

$$\frac{\delta\hat{\Phi}_1(\xi^0)}{\delta p_N} = \xi^0, \tag{B.30}$$

$$\frac{\delta\hat{\Phi}_2(\xi^i)}{\delta p_{\mathbf{N}i}} = \xi^i, \tag{B.31}$$

$$\frac{\delta\hat{\Phi}_3(\xi^0)}{\delta p_e} = \xi^0, \tag{B.32}$$

$$\begin{aligned}\frac{\delta\hat{\Phi}_4(\xi^0)}{\delta h_{mn}} = \xi^0\Big[&\frac{1}{2}h^{mn}\Phi_4 - \frac{\kappa}{e\sqrt{h}}h^{mn}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right)\\ &+ \frac{2\kappa}{e\sqrt{h}}\left(p^{ml}p_l^n - \frac{1}{2}p^{mn}p\right)\\ &+ \frac{e}{\kappa}\sqrt{h}R^{mn} - \frac{\sqrt{h}}{\kappa}(\nabla^m\nabla^n e)\Big]\\ &+ \frac{\sqrt{h}}{\kappa}\Big[-(\nabla^m\nabla^n\xi^0)e + h^{mn}(\nabla^a e)(\nabla_a\xi^0) + h^{mn}e(\Delta\xi^0)\Big],\end{aligned} \tag{B.33}$$

$$\frac{\delta\hat{\Phi}_4(\xi^0)}{\delta p^{mn}} = \xi^0\left[\frac{2\kappa}{e\sqrt{h}}\left(p_{mn} - \frac{1}{2}h_{mn}p\right)\right], \tag{B.34}$$

$$\frac{\delta\hat{\Phi}_4(\xi^0)}{\delta e} = -\xi^0\left[\frac{\kappa}{e^2\sqrt{h}}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right) + \frac{\sqrt{h}}{\kappa}R + \frac{6\lambda\sqrt{h}}{\kappa G_4}\left(1 - \frac{1}{e^2}\right)\right] + \frac{2\sqrt{h}}{\kappa}\Delta\xi^0, \tag{B.35}$$

$$\frac{\delta\hat{\Phi}_5(\xi^i)}{\delta h_{mn}} = -(\nabla_l\xi^i)p^{lj}(\delta_i^m\delta_j^n + \delta_j^m\delta_i^n) + \nabla_l(p^{mn}\xi^l) = \pounds_\xi p^{mn}, \tag{B.36}$$

$$\frac{\delta\hat{\Phi}_5(\xi^i)}{\delta p^{mn}} = -h_{li}(\nabla_j\xi^l)(\delta_m^i\delta_n^j + \delta_n^i\delta_m^j) = -\pounds_\xi h_{mn}, \tag{B.37}$$

$$\begin{aligned}\frac{\delta\hat{\Phi}_6(\xi^0)}{\delta h_{mn}} = \xi^0\Bigg[&\frac{1}{2}h^{mn}\Phi_6 + \frac{N\kappa}{e^2\sqrt{h}}h^{mn}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right)\\ &- \frac{2\kappa N}{e^2\sqrt{h}}\left(p^{ml}p_l^n - \frac{1}{2}p^{mn}p\right)\\ &+ \frac{N\sqrt{h}}{\kappa}R^{mn} - \frac{\sqrt{h}}{\kappa}(\nabla^m\nabla^n N)\Bigg]\\ &+ \frac{\sqrt{h}}{\kappa}\left[-N(\nabla^m\nabla^n - h^{mn}\Delta)\xi^0 + h^{mn}(\nabla^l N)(\nabla_l\xi^0)\right],\end{aligned} \tag{B.38}$$

$$\frac{\delta\hat{\Phi}_6(\xi^0)}{\delta p^{mn}} = -\xi^0\left[\frac{2N\kappa}{e^2\sqrt{h}}\left(p_{mn} - \frac{1}{2}h_{mn}p\right)\right], \tag{B.39}$$

$$\frac{\delta\hat{\Phi}_6(\xi^0)}{\delta N} = -\xi^0\left[\frac{\kappa}{e^2\sqrt{h}}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right) + \frac{\sqrt{h}}{\kappa}R + \frac{6\lambda\sqrt{h}}{\kappa G_4}\left(1 - \frac{1}{e^2}\right)\right] + 2\frac{\sqrt{h}}{\kappa}\Delta(\xi^0), \tag{B.40}$$

$$\frac{\delta\hat{\Phi}_6(\xi^0)}{\delta e} = \xi^0\left[\frac{2N\kappa}{e^3\sqrt{h}}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right) - \frac{12\lambda N\sqrt{h}}{\kappa e^3 G_4}\right], \tag{B.41}$$

$$\frac{\delta\hat{\Phi}_N^{(G)}(\xi^i)}{\delta N} = \partial_i(\xi^i p_N), \tag{B.42}$$

$$\frac{\delta\hat{\Phi}_N^{(G)}(\xi^i)}{\delta p_N} = -\xi^i\partial_i N = -\pounds_\xi N, \tag{B.43}$$

$$\frac{\delta\hat{\Phi}_e^{(G)}(\xi^i)}{\delta e} = \partial_i(\xi^i p_e), \tag{B.44}$$

$$\frac{\delta\hat{\Phi}_e^{(G)}(\xi^i)}{\delta p_e} = -\xi^i\partial_i e = -\pounds_\xi e, \tag{B.45}$$

$$\frac{\delta\hat{\Phi}_{\mathbf{N}}^{(G)}(\xi^i)}{\delta N^j} = \nabla_i(\xi^i p_{\mathbf{N}j}) + (\nabla_j\xi^i)p_{\mathbf{N}i}, \tag{B.46}$$

$$\frac{\delta\hat{\Phi}_{\mathbf{N}}^{(G)}(\xi^i)}{\delta p_{\mathbf{N}}} = -\xi^i\nabla_i N^j + N^i\nabla_i\xi^j = -\pounds_\xi N^j. \tag{B.47}$$

Using these, it is tedious but straightforward to compute the constraint algebra:

$$\{\Phi_1(y), \Phi_6(x)\} = -\left[\frac{\kappa}{e^2\sqrt{h}}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right) + \frac{\sqrt{h}}{\kappa}R + \frac{6\lambda\sqrt{h}}{\kappa G_4}\left(1 - \frac{1}{e^2}\right)\right] \delta(y-x) + 2\frac{\sqrt{h}}{\kappa}\Delta\delta(y-x), \tag{B.48}$$

$$\left\{\Phi_1(y), \Phi_{Nj}^{(G)}(x)\right\} = \Phi_1(x)\partial_{y^j}\delta(y-x), \tag{B.49}$$

$$\{\Phi_3(y), \Phi_4(x)\} = -\left[\frac{\kappa}{e^2\sqrt{h}}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right) + \frac{\sqrt{h}}{\kappa}R + \frac{6\lambda\sqrt{h}}{\kappa G_4}\left(1 - \frac{1}{e^2}\right)\right] \delta(y-x) + \frac{2\sqrt{h}}{\kappa}\Delta\delta(y-x), \tag{B.50}$$

$$\{\Phi_3(y), \Phi_6(x)\} = \left[\frac{2N\kappa}{e^3\sqrt{h}}\left(p^{ab}p_{ab} - \frac{1}{2}p^2\right) - \frac{12\lambda N\sqrt{h}}{\kappa e^3 G_4}\right]\delta(y-x), \tag{B.51}$$

$$\left\{\Phi_3(y), \Phi_{ej}^{(G)}(x)\right\} = \Phi_3(x)\partial_{y^j}\delta(y-x), \tag{B.52}$$

$$\{\Phi_4(y), \Phi_4(x)\} = \Phi_5^k(y)\partial_{y^k}\delta(y-x) - \Phi_5^k(x)\partial_{x^k}\delta(y-x) - \frac{p}{e}\partial_{y^k}e\partial_{y_k}\delta(y-x) + \frac{p}{e}\partial_{x^k}e\partial_{x_k}\delta(y-x), \tag{B.53}$$

$$\left\{\Phi_4(y), \Phi_{5j}(x)\right\} = \Phi_4(x)\partial_{y^j}\delta(y-x) - \frac{\partial_{x^j}e}{N}\Phi_6(x)\delta(y-x) - 2\frac{\sqrt{h}}{\kappa}\partial_{x^j}e\left(\Delta - \frac{\Delta N}{N}\right)\delta(y-x), \tag{B.54}$$

$$\begin{aligned}\{\Phi_4(y), \Phi_6(x)\}\} = {} & \frac{2Np_{mn}}{e}\left[2R^{mn} - \left(\frac{\nabla^m\nabla^m e}{e} + \frac{\nabla^m\nabla^n N}{N}\right)\right]\delta(y-x) \\ & - \frac{Np}{e}\left[R - \frac{6\lambda}{G_4}\left(1 - \frac{q}{e}\right)\right]\delta(y-x) \\ & - \frac{2Np_{mn}(y)}{e}\nabla^m_{(y)}\partial_{y_n}\delta(y-x) - \frac{2Np_{mn}(x)}{e}\nabla^m_{(x)}\partial_{x_n}\delta(y-x) \\ & - \left(\frac{Np}{e}\right)\frac{\partial_{y^m}N}{N}\partial_{y_m}\delta(y-x) - \left(\frac{Np}{e}\right)\frac{\partial_{x^m}e}{e}\partial_{x_m}\delta(y-x)\,,\end{aligned} \tag{B.55}$$

$$\left\{\Phi_4(y), \Phi_{ej}^{(G)}(x)\right\} = \frac{\partial_{x^j}e}{N}\Phi_6(x)\delta(y-x) + 2\frac{\sqrt{h}}{\kappa}\partial_{x^j}e\left(\Delta - \frac{\Delta N}{N}\right)\delta(y-x), \tag{B.56}$$

$$\left\{\Phi_{5j}(y), \Phi_{5i}(x)\right\} = \Phi_{5j}(x)\partial_{y^i}\delta(y-x) - \Phi_{5i}(y)\partial_{x^j}\delta(y-x), \tag{B.57}$$

$$\begin{aligned}\left\{\Phi_6(y), \Phi_{5j}(x)\right\} = {} & \Phi_6(x)\partial_{y^j}\delta(y-x) - \frac{\partial_{x^j}N}{N}\Phi_6(x)\delta(y-x) \\ & - 2\frac{\sqrt{h}}{\kappa}\partial_{x^j}N\left(\Delta - \frac{\Delta N}{N}\right)\delta(y-x) \\ & - \frac{2N\sqrt{h}}{\kappa}\partial_{y^j}e\left(\frac{B(h,p)}{e^3}\right)\delta(y-x),\end{aligned} \tag{B.58}$$

$$\begin{aligned}\{\Phi_6(y),\Phi_6(x)\} &= -\partial_{y_m}\left(\frac{2N^2}{e^2}\right)p_{mn}(y)\partial_{y_n}\delta(y-x)+\partial_{x_m}\left(\frac{2N^2}{e^2}\right)\\ &\quad p_{mn}(x)\partial_{x_n}\delta(y-x)\\ &\quad -\left(\frac{N}{e}\right)^2\Phi_{5n}(y)\partial_{y_n}\delta(y-x)+\left(\frac{N}{e}\right)^2\Phi_{5n}(x)\partial_{x_n}\delta(y-x)\\ &\quad +\frac{Np\partial_{y^m}N}{e^2}\partial_{y_m}\delta(y-x)-\frac{Np\partial_{x^m}N}{e^2}\partial_{x_m}\delta(y-x),\end{aligned} \tag{B.59}$$

$$\left\{\Phi_6(y),\Phi^{(G)}_{Nj}(x)\right\} = \frac{\partial_{x^j}N}{N}\Phi_6(x)\delta(y-x)+2\frac{\sqrt{h}}{\kappa}\partial_{x^j}N\left(\Delta-\frac{\Delta N}{N}\right)\delta(y-x), \tag{B.60}$$

$$\left\{\Phi_6(y),\Phi^{(G)}_{ej}(x)\right\} = \frac{2N\sqrt{h}}{\kappa}\partial_{y^j}e\left(\frac{B(h,p)}{e^3}\right)\delta(y-x), \tag{B.61}$$

$$\left\{\Phi^{(G)}_{Nj}(y),\Phi^{(G)}_{Ni}(x)\right\} = \Phi^{(G)}_{Nj}(x)\partial_{y^i}\delta(y-x)-\Phi^{(G)}_{Ni}(y)\partial_{x^j}\delta(y-x), \tag{B.62}$$

$$\left\{\Phi^{(G)}_{ej}(y),\Phi^{(G)}_{ei}(x)\right\} = \Phi^{(G)}_{ej}(x)\partial_{y^i}\delta(y-x)-\Phi^{(G)}_{ei}(y)\partial_{x^j}\delta(y-x), \tag{B.63}$$

$$\begin{aligned}\left\{\Phi_1(y),\Phi_{2j}(x)\right\} &= \left\{\Phi_{2i}(y),\Phi_{2j}(x)\right\}=\left\{\Phi_3(y),\Phi_{2j}(x)\right\}=\left\{\Phi_4(y),\Phi_{2j}(x)\right\}\\ &= \left\{\Phi_{5i}(y),\Phi_{2j}(x)\right\}=\left\{\Phi_6(y),\Phi_{2j}(x)\right\}=\left\{\Phi^{(G)}_{Ni}(y),\Phi_{2j}(x)\right\}\\ &= \left\{\Phi^{(G)}_{ei}(y),\Phi_{2j}(x)\right\}=0.\end{aligned} \tag{B.64}$$

In order to classify the class of the constraints, it is more appropriate to choose

$$(\Phi_1,\Phi_{2j},\Phi_3,\Phi_4,\tilde{\Phi}_{5j},\Phi_6) \tag{B.65}$$

as a set of independent constraints. $\tilde{\Phi}_{5j}$ is defined in (B.24) and given by a linear combination $\Phi_{5i}-\partial_i N\Phi_1-\partial_i e\Phi_3$. As is clear from the above computation, Φ_{2j} and $\tilde{\Phi}_{5j}$ commute with all the constraints:

$$\begin{aligned}\left\{\Phi_1(y),\Phi_{2j}(x)\right\} &= \left\{\Phi_{2i}(y),\Phi_{2j}(x)\right\}=\left\{\Phi_3(y),\Phi_{2j}(x)\right\}=\left\{\Phi_4(y),\Phi_{2j}(x)\right\}\\ &= \left\{\tilde{\Phi}_{5i}(y),\Phi_{2j}(x)\right\}=\left\{\Phi_6(y),\Phi_{2j}(x)\right\}=0.\end{aligned} \tag{B.66}$$

$$\left\{\Phi_1(y),\tilde{\Phi}_{5j}(x)\right\} = \Phi_1(x)\partial_{y^j}\delta(y-x)\approx 0, \tag{B.67}$$

$$\left\{\Phi_3(y),\tilde{\Phi}_{5j}(x)\right\} = \Phi_3(x)\partial_{y^j}\delta(y-x)\approx 0, \tag{B.68}$$

$$\left\{\Phi_4(y),\tilde{\Phi}_{5j}(x)\right\} = \Phi_4(x)\partial_{y^j}\delta(y-x)\approx 0, \tag{B.69}$$

$$\left\{\tilde{\Phi}_{5i}(y),\tilde{\Phi}_{5j}(x)\right\} = \tilde{\Phi}_{5i}(x)\partial_{y^j}\delta(y-x)-\tilde{\Phi}_{5j}(y)\partial_{x^i}\delta(y-x)\approx 0, \tag{B.70}$$

$$\left\{\Phi_6(y),\tilde{\Phi}_{5j}(x)\right\} = \Phi_6(x)\partial_{y^j}\delta(y-x)\approx 0. \tag{B.71}$$

It is easy to show that the set (B.65) is complete. Namely, the time flows of the secondary constraints do not give rise to any new constraints:

$$\dot{\Phi}_4(x) = \int d^3y \left\{ \mathscr{H}^{e(0)}_{\text{nDBI}}(y), \Phi_4(x) \right\} + \sum_{a=1,3} \int d^3y \left\{ \Phi_a(y), \Phi_4(x) \right\} \lambda_a + \int d^3y \left\{ \Phi_2^i(y), \Phi_4(x) \right\} \lambda_{2i} \approx 0, \tag{B.72}$$

$$\dot{\tilde{\Phi}}_{5j}(x) = \int d^3y \left\{ \mathscr{H}^{e(0)}_{\text{nDBI}}(y), \tilde{\Phi}_{5j}(x) \right\} + \sum_{a=1,3} \int d^3y \left\{ \Phi_a(y), \tilde{\Phi}_{5j}(x) \right\} \lambda_a + \int d^3y \left\{ \Phi_2^i(y), \tilde{\Phi}_{5j}(x) \right\} \lambda_{2i} \approx 0, \tag{B.73}$$

$$\dot{\Phi}_6(x) = \int d^3y \left\{ \mathscr{H}^{e(0)}_{\text{nDBI}}(y), \Phi_6(x) \right\} + \sum_{a=1,3} \int d^3y \left\{ \Phi_a(y), \Phi_6(x) \right\} \lambda_a + \int d^3y \left\{ \Phi_2^i(y), \Phi_6(x) \right\} \lambda_{2i} \approx 0. \tag{B.74}$$

Since the Hamiltonian density takes the form

$$\mathscr{H}^{e(0)}_{\text{nDBI}} = -\left(N\Phi_4 + N^j \Phi_{5j} \right) - \frac{2}{\kappa}\sqrt{h}\,(e\Delta N - N\Delta e)\,, \tag{B.75}$$

one can see that (B.72) and (B.74) determine λ_1 and λ_3, while (B.73) is trivially satisfied. Hence, there are no additional constraints from these time flows. Having established the completeness of the set (B.65), we conclude that Φ_{2j} and $\tilde{\Phi}_{5j}$ are first class, and the rest are second class.

Finally, let us end this Appendix with a comment on the generator of the spatial diffeomorphism. The generator $\mathscr{G}(\xi^i)$ of the spatial diffeomorphism acts on a phase-space variable A as

$$\left\{ A(y), \mathscr{G}(\xi^i) \right\} = \pounds_\xi A(y). \tag{B.76}$$

Since p_N, $p^i_{\mathbf{N}}$, and p_e are primary constraints, we only need to consider the reduced set of phase-space variables, $(h_{ij}, p^{ij}, N, \mathbf{N}, e)$. The spatial diffeomorphisms for this set are generated by $(\Phi_5, -\Phi_5, \Phi^{(G)}_N, \Phi^{(G)}_{\mathbf{N}}, \Phi^{(G)}_e)$. They are indeed all generated by the first class constraints, as can be seen from (B.21)–(B.24).

B.3 Computational Details of Perturbations

In this Appendix, we show some details of the computations in the perturbative analysis of the scalar mode in Sects. 3.2.1 and 3.2.2.

B.3.1 Perturbation of the Equations of Motion

The linearised version of Eqs. (3.72)–(3.75) and (3.76) can be obtained in a fashion similar to [2]. To the approximation explained in Sect. 3.2.3.2, we find, in the gauge $N^i = 0$,

$$\dot{\gamma}_{ij} - 2\bar{N}\kappa_{ij} - 2\bar{K}_{ij}n = 0, \tag{B.77}$$

$$2\bar{K}_{ij}\kappa^{ij} - 2\kappa\bar{K} - \nabla^i\nabla^j\gamma_{ij} + \Delta\gamma = \frac{G_4}{6\lambda}\Delta\alpha, \tag{B.78}$$

$$\nabla^j\kappa_{ij} - \nabla_i\kappa - \frac{3}{2}\bar{K}^{jk}\nabla_i\gamma_{jk} + \bar{K}^{jk}\nabla_k\gamma_{ij} + \frac{1}{2}\bar{K}_{ij}\nabla^j\gamma = \frac{G_4}{12\lambda}\left(\bar{K}_{ij} - \bar{h}_{ij}\bar{K}\right)\nabla^j\alpha, \tag{B.79}$$

$$\dot{\kappa}^{ij} + \dot{\gamma}^{ij}\bar{K} - \bar{h}^{ij}\dot{\kappa} - \dot{\gamma}_{kl}\bar{h}^{ij}\bar{K}^{kl} - \nabla^i\nabla^j n + \bar{h}^{ij}\Delta n + \frac{N}{2}\left(\nabla_k\nabla^j\gamma^{ik} + \nabla_k\nabla^i\gamma^{jk} - \Delta\gamma^{ij} - \nabla^i\nabla^j\gamma\right) = -\frac{G_4}{12\lambda}\bar{N}\nabla^i\nabla^j\alpha, \tag{B.80}$$

and

$$\Delta\dot{\alpha} = \Delta\partial_t\left[\nabla^i\nabla^j\gamma_{ij} - \Delta\gamma + 2\bar{K}_{ij}\kappa^{ij} - 2\kappa\bar{K} - 2\bar{N}^{-1}\left(\Delta n - \nabla_i\gamma^{ij}\nabla_j\bar{N} + \frac{1}{2}\nabla^i\gamma\nabla_i\bar{N}\right)\right] = 0\,. \tag{B.81}$$

The L.H.S. of (B.77)–(B.80) are the same as those in GR and agree with the $\lambda = \xi = 1$ case of [2]. Equation (B.81), however, is very different.

We are interested in perturbations of the modes with wavelengths much shorter than the characteristic scale L of the background, *i.e.*, ω, $p \gg 1/L$, where the space-time is virtually flat. First, (B.81) enforces α to the constant. Thus the R.H.S. of (B.77)–(B.80) are always negligible and thus we have in Fourier space

$$i\omega\gamma^{ij} + 2(\bar{N}\kappa^{ij} + \bar{K}^{ij}n) = 0, \tag{B.82}$$

$$(p_i p_j - p^2\delta_{ij})\gamma^{ij} - 2(\bar{K}\kappa - \bar{K}_{ij}\kappa^{ij}) = 0, \tag{B.83}$$

$$p_j\kappa^{ij} - p^i\kappa - \frac{3}{2}\bar{K}_{jk}\gamma^{jk}p^i + \bar{K}_{jk}p^k\gamma^{ij} + \frac{1}{2}\bar{K}^{ij}p_j\gamma = 0, \tag{B.84}$$

$$i\omega(\delta^{ij}\kappa - \kappa^{ij}) - (p^2\delta^{ij} - p^ip^j)n - i\omega(\bar{K}\gamma^{ij} - \delta^{ij}\bar{K}_{kl}\gamma^{kl}) - \frac{N}{2}(p_kp^j\gamma^{ik} + p_kp^i\gamma^{jk} - p^2\gamma^{ij} - p^ip^j\gamma) = 0. \tag{B.85}$$

Plugging (B.82) into (B.85) and discarding sub-leading terms linear in $\bar{K}_{ij} \sim \mathcal{O}(1/L)$, we simply obtain the fluctuation equation of GR in flat space-time:

$$\frac{\omega^2}{p^2}(\delta^{ij}\kappa - \kappa^{ij}) + i\omega\left(\delta^{ij} - \frac{p^ip^j}{p^2}\right)n + \bar{N}^2\left(\kappa^{ij} + \frac{1}{p^2}\left(p^ip^j\kappa - p_kp^j\kappa^{ik} - p_kp^i\kappa^{jk}\right)\right) = 0. \tag{B.86}$$

Contraction with δ_{ij} and p_j, respectively, yields

$$i\omega\kappa - p^2 n = 0, \tag{B.87}$$

$$p^i\kappa - p_j\kappa^{ij} = 0. \tag{B.88}$$

Using these relations, one can find that (B.86) gives the massless dispersion relation

$$\omega^2 = \bar{N}^2 p^2. \tag{B.89}$$

In this approximation, the rest of the equations, the Hamiltonian and momentum constraints (B.83) and (B.84), are automatically satisfied. In GR, we can set $n = 0$ by using the residual gauge symmetry. Thus we are left with a massless transverse traceless tensor mode, *i.e.*, the usual graviton.

In n-DBI gravity, we do not have liberty to gauge away n. However, we have (B.81) to take into account. Using (B.82), it becomes

$$\kappa^{ij}\left[i\omega\bar{N}\left(\bar{K}_{ij} - \bar{K}\delta_{ij}\right) + \bar{N}^2\left(p^2\delta_{ij} - p_i p_j + \delta_{ij} i p^k \partial_k \ln\bar{N} - 2ip_i\partial_j \ln\bar{N}\right)\right]$$
$$+ n\left[-i\omega p^2 + \bar{N}\bar{K}_{ij}\left(p^2\delta_{ij} - p_i p_j + ip^k\partial_k \ln\bar{N}\delta_{ij} - 2ip_i\partial_j\ln\bar{N}\right)\right] = 0. \tag{B.90}$$

To leading order, this yields

$$\bar{N}^2\kappa^{ij}\left(p^2\delta_{ij} - p_i p_j\right) - in\omega p^2 = 0 \quad \overset{(B.88)}{\Longrightarrow} \quad n\omega p^2 = 0. \tag{B.91}$$

Since our approximation is valid only for $\omega, p \gg 1/L$, this implies that $n = 0$ and we are again left with a graviton. To the next order, however, this reads

$$\kappa^{ij}\left[i\omega\bar{N}\left(\bar{K}_{ij} - \bar{K}\delta_{ij}\right) + \bar{N}^2\left(\delta_{ij} i p^k\partial_k \ln\bar{N} - 2ip_i\partial_j\ln\bar{N}\right)\right]$$
$$+ n\left[-i\omega p^2 + \bar{N}\bar{K}_{ij}\left(p^2\delta_{ij} - p_i p_j\right)\right] = 0. \tag{B.92}$$

Due to (B.87), the κ^{ij} terms contribute. For n to be non-vanishing, we must have the relation

$$\omega \sim \bar{N}p \sim i\left(\bar{N}\bar{K} + \bar{N}\partial\ln\bar{N}\right) \sim \frac{i}{L}. \tag{B.93}$$

However, once again, this cannot be satisfied. Hence there is no scalar mode of the type $(n, \gamma) \sim (n(\omega, p), \gamma(\omega, p))\, e^{i\omega t + ip\cdot x}$ with $|\omega|, |p| \gg 1/L$, including the one with imaginary ω.

B.3.2 Perturbation of the Stückelberg Field

To obtain the equation of motion for the Stückelberg field ϕ, we vary the action (1.140) or (1.141)

$$\delta S \sim e\delta\mathscr{K} \sim e\delta\left(D_\alpha\left(n^\alpha D_\beta n^\beta\right)\right) \sim \left(D_\alpha\left(n^\beta D_\beta e\right) - KD_\alpha e\right)\delta n^\alpha, \tag{B.94}$$

where $e = \left(1 + \frac{G_4}{6\lambda}\left({}^{(4)}R + \mathscr{K}\right)\right)^{-\frac{1}{2}}$ and $\mathscr{K} = -2D_\alpha(n^\alpha D_\beta n^\beta)$. Using

$$\delta n^\alpha = \frac{\partial^\alpha \delta\phi + n^\alpha n^\beta \partial_\beta \delta\phi}{\sqrt{-X}}, \tag{B.95}$$

we find

$$D^\alpha \left(\frac{(\partial_\alpha + n_\alpha n^\sigma \partial_\sigma)(n^\beta \partial_\beta e) - D_\beta n^\beta (\partial_\alpha + n_\alpha n^\sigma \partial_\sigma) e}{\sqrt{-X}} \right) = 0. \tag{B.96}$$

It can be shown that this becomes (3.76) when $n_\mu = (-N, 0, 0, 0)$, *i.e.*, $\phi = t$. Note that this equation is invariant under $\phi \to f(\phi)$, as it should be, owing to the fact that $n^\alpha(\partial_\alpha + n_\alpha n^\sigma \partial_\sigma) = 0$. This also implies that the quantity in the parenthesis is proportional to the space-like vector $n^\beta D_\beta n_\alpha$ tangential to the hypersurface.

Now we consider perturbations $\phi = \bar{\phi} + \varphi$, expanding (B.96) to linear order in φ. We work in unitary gauge, $\bar{\phi} = t$ and $N^i = 0$ and use

$$\delta n^\alpha = (0, \bar{N}\partial^i \varphi), \qquad \delta\left(\sqrt{-X}\right)^{-1/2} = -\bar{N}\dot{\varphi}, \qquad \delta K = \bar{N}^{-1}\nabla_i(\bar{N}^2 \partial^i \varphi), \tag{B.97}$$

as well as

$$\delta e = -\frac{G_4}{12\lambda}\bar{e}^3 \delta\mathscr{K},$$
$$-\frac{1}{2}\delta\mathscr{K} = \left(2\bar{K} - \bar{N}^{-1}\partial_t\right)\left(\bar{N}^{-1}\nabla_i\left(\bar{N}^2\partial^i\varphi\right)\right) + \bar{N}\partial^i\varphi\partial_i\bar{K}. \tag{B.98}$$

The terms coming from perturbing n^α and $1/\sqrt{-X}$ in (B.96) contain at most 3 φ-derivatives, while those from perturbing e contain 6 and 5 φ-derivatives. Thus we only need to consider the latter. It also suffices to keep 3 and 2 δe-derivatives. Then we find

$$\Delta\delta\dot{e} + \bar{K}\bar{N}\Delta\delta e = 0. \tag{B.99}$$

In terms of φ the leading terms (with 6 and 5 derivatives) yield (3.83).

B.3.3 Perturbation of the Stückelberg Field-2nd Order

To find the cubic action of the fluctuation φ in flat space-time (with $q = 1$), we expand the action as

$$S = -\frac{1}{16\pi G_4}\int d^4x \left[\mathscr{K} - \frac{1}{4}\left(\frac{G_N}{6\lambda}\right)\mathscr{K}^2 - \frac{5}{8}\left(\frac{G_N}{6\lambda}\right)^2\mathscr{K}^3 + \cdots\right]. \tag{B.100}$$

The first term is a surface term and does not contribute to the equation of motion. In unitary gauge, the time-like vector takes the form

$$n_0 = -\frac{1+\dot{\varphi}}{\sqrt{(1+\dot{\varphi})^2 - (\partial_i\varphi)^2}}, \qquad n_i = -\frac{\partial_i\varphi}{\sqrt{(1+\dot{\varphi})^2 - (\partial_i\varphi)^2}}. \tag{B.101}$$

This can be expanded as

$$-n_0 = 1 + \frac{1}{2}(\partial_i\varphi)^2 - \dot{\varphi}(\partial_i\varphi)^2 + \mathscr{O}(\varphi^4), \tag{B.102}$$

$$-n_i = \partial_i\varphi - \dot{\varphi}\partial_i\varphi + \dot{\varphi}^2\partial_i\varphi + \frac{1}{2}(\partial_i\varphi)^3 + \mathscr{O}(\varphi^4). \tag{B.103}$$

In the flat background we have

$$\mathscr{K} = -2\left[\partial_0\left(n^0\partial_0 n^0\right) + \partial_i\left(n^i\partial_j n^j\right) + \partial_0\left(n^0\partial_i n^i\right) + \partial_i\left(n^i\partial_0 n^0\right)\right], \tag{B.104}$$

and we find to quadratic order

$$\mathscr{K} = 2\left[\Delta\dot{\varphi} + \partial_0(\dot{\varphi}\Delta\varphi) - \partial_i(\partial_i\varphi\Delta\varphi)\right]. \tag{B.105}$$

Hence the cubic scalar field action is given by

$$S_3 = \frac{1}{192\pi\lambda}\int d^4x\left[\frac{1}{2}(\Delta\dot{\varphi})^2 - (\dot{\varphi}\Delta\ddot{\varphi} - \partial_i\varphi\Delta\partial_i\dot{\varphi})\,\Delta\varphi + \frac{5G_4}{2\lambda}\frac{1}{3!}(\Delta\dot{\varphi})^3\right]. \tag{B.106}$$

B.3.4 A Nonlinear Analysis of the Stückelberg Field

The scalar mode φ obeys the equation of motion (B.96). To determine whether the scalar mode leads to an instability or not, we need to study (B.96) beyond linear order approximation. As we discussed, to linear order, the equation of motion is simply

$$\Delta^2\ddot{\varphi} = 0, \tag{B.107}$$

and the solution is

$$\varphi(t,x) = \varepsilon\left[\varphi_0(x) + \varphi_1(x)t\right], \tag{B.108}$$

where $\varphi_0(x)$ and $\varphi_1(x)$ are arbitrary functions of space. We have included the factor of $\varepsilon \ll 1$ for later convenience. The fully nonlinear solution can in principle be found systematically order by order in ε expansions:

$$\varphi(t,x) = \varepsilon\left[\varphi_0(x) + \varphi_1(x)t\right] + \sum_{n=2}^{\infty}\varepsilon^n\varphi_n(t,x). \tag{B.109}$$

The higher order fluctuations $\varphi_n(t,x)$'s are determined in terms of the initial data $(\varphi_0(x), \varphi_1(x))$ and of order n in powers of (spatial derivatives of) $\varphi_0(x)$ and $\varphi_1(x)$ and polynomial in time t. Using (B.106), for example, the next-to-leading order fluctuation can be found as

$$\varphi_2(t,x) = \frac{1}{2}\varphi_2^{(2)}(x)t^2 + \frac{1}{3!}\varphi_2^{(3)}(x)t^3, \tag{B.110}$$

where

$$\begin{aligned}\Delta^2\varphi_2^{(2)} &= \Delta\left[2\Delta\varphi_0\Delta\varphi_1 + 2\partial_i\varphi_0\Delta\partial_i\varphi_1 + \Delta\partial_i\varphi_0\partial_i\varphi_1\right] - \left(\Delta\varphi_0\Delta^2\varphi_1 + \Delta\partial_i\varphi_0\Delta\partial_i\varphi_1\right),\\ \Delta^2\varphi_2^{(3)} &= \Delta\left[2(\Delta\varphi_1)^2 + 2(\partial_i\varphi_1)^2 + \partial_i\varphi_1\Delta\partial_i\varphi_1\right] - \left(\Delta^2\varphi_1\Delta\varphi_1 + \partial_i\varphi_1\Delta\partial_i\varphi_1\right).\end{aligned} \tag{B.111}$$

Clearly, the late time behaviour of the scalar mode requires the knowledge of all order fluctuations. Thus, to see whether the scalar mode yields an instability or not, we need to re-sum the infinite series (B.109). However, this seems to be out of our reach and we will instead resort to an alternative analysis working in the Einstein frame.

B.4 Linear Perturbations in the Einstein Frame

We consider linear perturbations of the scalar fields around flat space-time in the Einstein frame (1.135). The scalar field Lagrangian density reads

$$\begin{aligned}4\pi G_4\mathscr{L}^E_{\text{scalar}} &= 2\dot{\psi}^2 + 4\dot{\psi}(\dot{E} + \Delta B) - (4n - 2\psi)\Delta\psi\\ &\quad + 4\dot{\chi}\left(\dot{E} + \Delta B + 2\dot{\psi}\right) + 6\dot{\chi}^2 - 6\chi\Delta\chi - \frac{24\lambda}{G_4}\chi^2.\end{aligned} \tag{B.112}$$

This is the Einstein frame counterpart of (3.50). The equations of motion are given by

$$\ddot{\psi} = -\ddot{\chi}, \tag{B.113}$$

$$\Delta\dot{\psi} = -\Delta\dot{\chi}, \tag{B.114}$$

$$\ddot{E} + \Delta\dot{B} + \Delta n = -\ddot{\chi}, \tag{B.115}$$

$$\Delta\psi = 0, \tag{B.116}$$

which clearly reduce to those of GR for constant χ, plus

$$\ddot{\chi} + \Delta\chi + \frac{1}{3}(\ddot{E} + \Delta\dot{B} + 2\ddot{\psi}) + \frac{4\lambda}{G_4}\chi = 0. \tag{B.117}$$

This is the linearization of (3.87). In the $E = 0$ gauge the general solution can be found as

$$B = B_0(x) + B_1(x)t, \tag{B.118}$$

$$n = -B_1(x), \tag{B.119}$$

$$\psi = 0, \tag{B.120}$$

$$\chi = \chi_0(x), \tag{B.121}$$

with $\chi_0(x)$ related to $B_1(x)$ by

$$\Delta B_1 = -3\Delta\chi_0 - \frac{12\lambda}{G_N}\chi_0. \tag{B.122}$$

Here we have again imposed the boundary condition that all the fields fall off at spatial infinity. Note that $\chi_0(x)$ is essentially $B_1(x)$ which is the degree of freedom responsible for the linear time growth. This suggests the identification $\chi(t,x) \sim \dot{T}(t,x)$, that is, the auxiliary field χ can be regarded as the time derivative of the scalar mode.

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