Appendix A
Avatamsaka Stochastic Process*

A.1 Interactions in Traditional Game Theory and Their Problems

Even in traditional game theory, noncooperative agents usually encounter various kinds of interactions in generating a set of complicated behaviors. In the course of finding curious results, experimental economists are often involved in giving game theorists new clues for solving the games. Their concerns are limited to informational structures outside the essential game structure, e.g., using an auxiliary apparatus such as "cheap talks", i.e., negotiations without costs. Traditional game theory is allowed to argue actual interactions extensively, but may run into difficulties, because the treatment of the information is merely intuitive and not systemic.

We exemplify one limitation in the case of a Nash equilibrium. If we are faced with multiple Nash equilibria, the concept of correlated equilibria may be activated. As Kono (2008, 2009) explored, however, this concept really requires the restrictive assumption that all players' mixed strategies are assumed to be stochastically independent. Without this, a selection of equilibria may be inconsistent.[1] Plentiful realistic devices for strengthening traditional game theory cannot necessarily guarantee the assumption of stochastic independence of mixed strategies. So traditional theory may often be linked to evolutionary theory to argue realistic interactions.

A general framework for encompassing various contexts/stages can be systematically proposed by focusing on the concept of information partition, often used in statistical physics or population dynamics. It is easy to find such an application in the field of socio- and/or econo-physics. We can then argue that a player changes into another as the situation alters.

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*Appendix A first appeared in Aruka and Akiyama (2009).

[1]See Kono (2008, 2009).

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Table A.1 Coordination game

Player A	Player B Strategy 1	Strategy 2
Strategy 1	(R,R)	(S,T)
Strategy 2	(T,S)	(P,P)

Table A.2 Avatamsaka game

Player A	Player B Cooperation	Defection
Cooperation	(1,1)	(0,1)
Defection	(1,0)	(0,0)

The exchangeable agents change with the situation/stage.
The cluster dynamics change with the exchangeable agents.

The exchangeable agents emerge from the use of a random partition vector in statistical physics or population genetics. The partition vector provides us with information about the state. We can therefore argue the size-distribution of the components and their cluster dynamics with the exchangeable agents. We can link the cluster dynamics with the exchangeable agents. We then define a maximum countable set, in which the probability density of transitions from state i to state j is given. In this setting, dynamics of the heterogeneous interacting agents give the field where an agent can become another. This way of thinking easily incorporates **the unknown agents**, as Fig. 1.9 shows (Aoki and Yoshikawa 2006).

A.1.1 A Two-Person Game of Heterogeneous Interaction: Avatamsaka Game

Aruka (2001) applied a famous tale from Mahayana Buddhism Sutra called *Avatamsaka*. Now I would like to illustrate the Avatamsaka game. Suppose that two men sat down face to face at a table, tied up except for one arm, then each was given an over-long spoon. They cannot serve themselves, because of the length of the spoon. There is food enough for both on the table. If they cooperate to feed each other, they can both be happy. This defines **Paradise**. Alternatively, one may provide the other with food but the second might not reciprocate, which the first would hate, leading to **Hell**. The gain structure will not depend on the altruistic willingness to cooperate.[2] The payoff matrix of the traditional form will be (Tables A.1 and A.2):

The properties of the two games may be demonstrated by the relationships between R, S, T, P: Here we call

$$D_r = P - S \tag{A.1}$$

[2]Recently, this tale has been cited in *Chicken Soup* (Canfield and Hansen 2001), a best-selling book in the United States.

the "Risk Aversion Dilemma" and

$$D_g = T - R \tag{A.2}$$

the "Risk Seeking Dilemma".

Selfishness cannot be defined *without* interactions between agents. The direction of the strategy will depend on the situation of the community as a whole. One agent's selfishness depends on the other cooperating. A gain from defection can never be assured independently. The sanction for defection, as a reaction of the rival agent, never implies selfishness of the rival.[3]

A.1.2 Dilemmas Geometrically Depicted: Tanimoto's Diagram

Tanimoto's (2007) geometrics for the two-person game neatly describe the game's geometrical structure. Given the payoff of Table A.1, his geometrics define the next equation system:

$$P = 1 - 0.5 r_1 \cos\left(\frac{\pi}{4}\right) \tag{A.3}$$

$$R = 1 + 0.5 r_1 \cos\left(\frac{\pi}{4}\right) \tag{A.4}$$

$$S = 1 + r r_1 \cos\left(\frac{\pi}{4} + \theta\right) \tag{A.5}$$

$$T = 1 + r r_1 \sin\left(\frac{\pi}{4} + \theta\right) \tag{A.6}$$

Here $r = \frac{r_2}{r_1}; r_1 = PS, r_2 = SM$.

Thus it is easily verified that spillovers of the Avatamsaka game are positive:

$$\begin{aligned} \text{Spillover} &= R - S \\ &= T - P > 0 \end{aligned}$$

Each player's situation can be improved by the other player's strategy switching from D to C, whether he/she employs D or C (see Aruka 2001, p. 118) (Fig. A.1).

[3]The same payoff was used as the "mutual fate control" game by the psychologists Thibaut and Kelley (1959), revived by Mitropoulos (2004). However, the expected value of the gain for any agent could be reinforced if the average rate of cooperation were improved, in a macroscopically weak control mechanism, which cannot always guarantee a mutual fate. Aruka and Akiyama (2009) introduced different spillovers or payoff matrices, so that each agent may then be faced with a different payoff matrix.

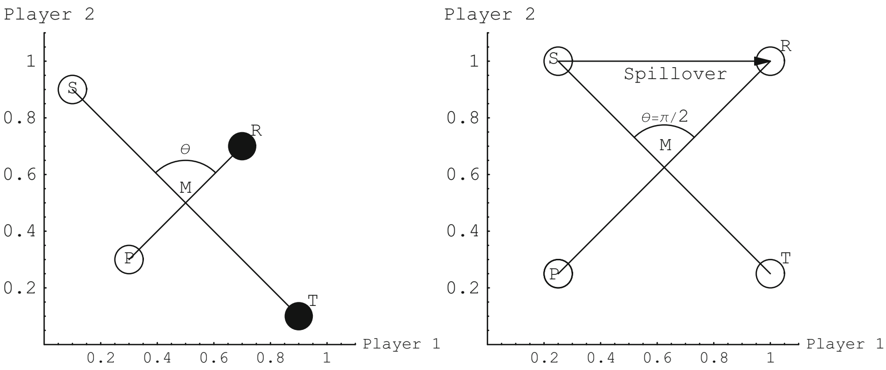

Fig. A.1 Tanimoto's geometrics and the positive spillover. See Tanimoto (2007)

A.1.2.1 The Path-Dependent Property of Polya's Urn Process

The original viewpoint focuses on an emerging/evolving environment, i.e., **path dependency**. We focus on two kinds of averaging:

Self-averaging: Eventually, players' behavior could be independent from others'.
Non-self-averaging: The invariance of the random partition vectors under the properties of exchangeability and size-biased permutation does not hold.

In the original stage, consider an urn containing a white ball and a red ball only. Draw out one ball, and return it with another of the same color to the urn. Repeat over and over again. The number of balls increases by one each time, so after the completion of two draws,

$$\text{The total number of balls after the second trial} = 2 + 1 + 1 = 4$$
$$\text{The total number of balls after the } n\text{-th trial} = n + 2$$

After the completion of n trials, what is the probability that the urn contains just one white ball? This must be equivalent to the probability that we can have n successive draws of red balls. It is therefore easy to prove that

$$P(n,k) = \frac{1}{n+1} \tag{A.7}$$

This result shows that any event (n,k) at the n-th trial can emerge, i.e., any number of white balls can go everywhere at the ratio of $\frac{1}{n+1}$.

$$\frac{1}{2}\frac{1+1}{2+1}\frac{1+1}{2+1+1}\cdots\frac{n}{n+1} = \frac{1}{n+1} \tag{A.8}$$

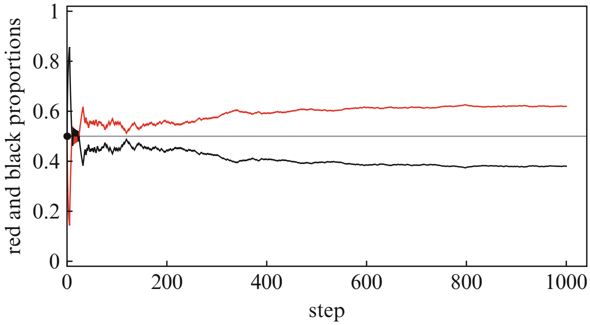

Fig. A.2 The elementary Polya urn process. http://demonstrations.wolfram.com/PolyaUrnSimulation/

Comparing this process with the market, it is clear that in the stock or commodity exchanges, for instance, any sequence of trades must be settled at the end of the session. Any trade, once started, must have an end within a definite period, even in the futures market. The environment in the market must be reset each time. In the market game, a distribution of types of trader can affect the trading results, but not vice versa. On the other hand, the Avatamsaka game in a repeated form must change its own environment each round. A distribution of types of agents can affect the results of the game, and vice versa. Agents must inherit their previous results. This situation describes the path dependency of a repeated game (Fig. A.2).

A.2 Avatamsaka Stochastic Process Under a Given Payoff Matrix

We apply the Polya urn stochastic process to our Avatamsaka game experiment to predict an asymptotic structure of the game. We impose the next assumptions for a while Aruka (2011, Part III).

Assumption 1 There are a finite number of players, who join the game as pairs.
Assumption 2 The ratio of cooperation, or C-ratio for each player, is in proportion to the total possible gains for each player.

As Aoki and Yoshikawa (2006) dealt with a product innovation and a process innovation, they criticized Lucas's representative method and the idea that players face micro-shocks drawn from the same unchanged probability distribution. In light of their findings, I show the same argument in the Avatamsaka game with different payoffs. In this setting, innovations occurring in urns may be regarded as **increases in the number of cooperators** in urns whose payoffs are different. Moving on from a classical Polya urn process with a given payoff matrix (Aruka 2011, Part III), we then need the following formulae:

Ewens sampling formula (Ewens 1972) A K-dimensional Polya urn process with multiple payoff matrices and new agents allowed to arrive.

Self-averaging Eventually, however, players' behavior can be independent of others'.
Pitman's sampling formula Two-parameter Poisson-Dirichlet distribution.
Non-self-averaging The invariance of the random partition vectors under the properties of exchangeability and size-biased permutation does not hold.

According to Aoki and Yoshikawa (2007, p. 6), the economic meaning of non-self-averaging is important because such models are sample-dependent, and some degree of impreciseness or dispersion remains about the time trajectories even when the number of economic agents reaches infinity. This implies that focus on the mean path behavior of macroeconomic variables is not justified, which, in turn, means that sophisticated optimization exercises that provide us with information on means have little value. We call an urn state **self-averaging** if the number of balls in each urn could eventually be convergent.

Definition A.1. Non-self-averaging means that a size-dependent (i.e., *extensive* in physics) random variable X of the model has a coefficient of variation that does not converge to zero as the model size tends to infinity. The coefficient of variation of an extensive random variable, defined by

$$C.V.(X) = \frac{\text{variance}(X)}{\text{mean}(X)}, \tag{A.9}$$

is normally expected to converge to zero as model size (e.g., the number of economic agents) tends to infinity. In this case, the model is said to be *self-averaging*.

A.2.1 Avatamsaka Stochastic Process Under Various Payoff Matrices

Next we introduce various payoff matrices into our Avatamsaka process. Let various types of different sized payoff matrices be $1, \cdots, K$. We then have similar gain-ratios of players with different payoff types for each number of agent with gain i. Every player who encounters an opponent must draw lots from any lottery urn. A couple of players can draw a lot arbitrarily, and then return them to the urn. It then holds that the greater spillover does not change Avatamsaka characteristics. However, different spillovers may change the players' inclinations and reactions as shown in Fig. A.3. It is noted that we can get a PD game by extending the PR line to the north-east.

Suppose there are a number of different urns with various different payoff matrices, each of which has its own spillover size. A different spillover in our Avatamsaka game may change the inclinations of the players, but these inclinations are *not* necessarily symmetrical. An urn with a greater spillover might sometimes be more attractive for a highly cooperative player, because the players could earn greater gains in size. A less cooperative player might prefer to enter an urn with

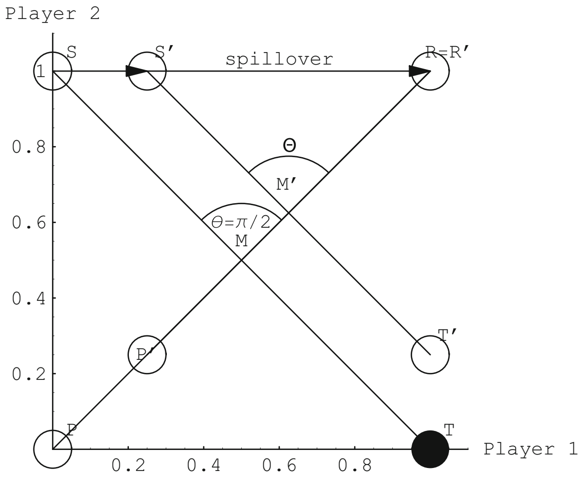

Fig. A.3 Spillovers in Avatamsaka payoff matrix. See Aruka and Akiyama (2009, Fig. 9)

a smaller spillover in a much higher level of cooperation, but he can change his mind and defect. Players may therefore have various plans. Any player depends on the state in which he remains or enters, while an urn, i.e., a payoff matrix, occurs *stochastically*.

By the time the i-th cooperation occurs, the total of K_n payoff urns are formed in the whole game space, where the i-th payoff urn has experienced cooperation for $i = 1, 2, \cdots, K_n$. If we replace "innovations" with "increases in cooperation", then by definition the following equality holds:

$$n_1 + n_2 + \cdots + n_k = n \tag{A.10}$$

when $K_n = k$. If the n-th cooperation creates a new payoff matrix (urn), then $n_k = 1$, and

$$n = \sum_j j a_j(n) \tag{A.11}$$

So there are a finite number of urns into which various types of payoff matrices are embedded for $1, 2, \cdots, K_n$.

In this new environment, we can have n inventions to increase cooperation. In other words, the amount of cooperation x_i in urn i may grow, because of stochastic multiple inventions occurring within it.

Assumption 3 (Aruka and Akiyama 2009, p. 157; Aruka 2011, p. 257) Cooperation accelerates cooperation, i.e., the greater the cooperation, the larger the total gain in the urn will be.

Because of an Avatamsaka property, we can also impose another assumption.

Assumption 4 (Aruka and Akiyama 2009, p. 157; Aruka 2011, p. 257) A player can compare situations between the urns by normalizing his own gain.

Under the new assumptions, we can prove the *non-self-averaging* property in our Avatamsaka game as

$$X_n K_n + \beta \sum_{j}^{n} j a_j(n) \tag{A.12}$$

Here $\beta = \log(\gamma) > 0$. It turns out that X_n depends on how cooperation occurs. Finally, according to Aoki and Yoshikawa (2007), the next proposition also follows:

Proposition A.1. *In the two-parameter Poisson-Dirichlet model, the aggregate cooperation behavior X_n is non-self-averaging.*

Appendix B
The JAVA Program of URandom Strategy

In order to be familiar with implementing a new strategy in the U-Mart, we need to construct a java program for a particular strategy. Here we merely demonstrate the URandom strategy.[4] This strategy premises the next institutional properties as its default setting[5]:

```
ReferenceBrand=Futures
TradingBrand=Futures
TradingType=NORMAL

#Price
Price.Buy.Average=Ask
Price.Buy.StandardDeviation=2.5Tick
Price.Sell.Average=Bid
Price.Sell.StandardDeviation=2.5Tick

#Volume
Volume.Buy.Max=10
Volume.Buy.Min=1
Volume.Sell.Max=10
Volume.Sell.Min=1

#Frequency
OrderFrequency=Fixed
```

(continued)

[4] See src/startegyv4/URandomStrategy.java in the **U-Mart ver.4** system.

[5] See src/startegyv4/URandomStrategyProperty.txt in the **U-Mart ver.4** system.

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```
OrderFrequency.Frequency=0
#OrderFrequency.Average=5
#OrderFrequency.StandardDeviation=2

#OrderLimit
OrderLimit.Position=-1
OrderLimit.Cash=-1
```

It is trivial that the institutional setting may be easy to be changed by assigning a different value in the file. As for "#Frequency", it may change the frequency framing by either removing # or changing the given numerical values.

```
package strategyV4;
p
import java.io.IOException;

import server.UBrandInformationManager;
import server.UMachineAgentInformation;
import server.UMartTime;

public class URandomStrategy extends UStandardAgent
{
private static final double CASH_LIMIT = 1 / 3;
private int fCountOfOrderFrequency;
private int fOrderFrequency;

public URandomStrategy(int dummyParam) {
super(dummyParam);
}

@Override
public void setParameters(String parameters)
  throws IOException
{
super.setParameters(parameters);
fCountOfOrderFrequency = 0;
fOrderFrequency = getOrderfrequency();
}
```

(continued)

```
@Override
public void action(UMartTime time,
  UMachineAgentInformation machineAgentInfo,
UBrandInformationManager brandInfoManager) {
super.action(time, machineAgentInfo,
  brandInfoManager);

if(isOrder()){
//If the random value is zero,  buy; if 1, sell.
if(getRandomInteger(2) == 0){
buyOrder(brandInfoManager);
}
else{
sellOrder(brandInfoManager);
}
}
}

/**
 * Check the timing whether order should be given
    or not.
 * @return
 */
private boolean isOrder() {
if (fOrderFrequency == fCountOfOrderFrequency) {
fCountOfOrderFrequency = 0;
fOrderFrequency = getOrderfrequency();
return true;
}
fCountOfOrderFrequency++;
return false;
}

/**
 * Decide the frequency of giving orders.
 * If the frequency is fixed, orders are given
 by a constant frequency.
 If it is randomly generated, orders are given
 by the frequency generated by a normal random
  number.
 * @return
```

(continued)

```
 */
private int getOrderfrequency() {
int count;

if(fSystemParameters.getProperty
  ("OrderFrequency").equals("Fixed")){
count = fSystemParameters.getIntProperty
  ("OrderFrequency.Frequency");
}
else{
count = (int)getRandomGaussian
(fSystemParameters.getDoubleProperty
  ("OrderFrequency.Average"),
  fSystemParameters.getDoubleProperty
  ("OrderFrequency.StandardDeviation"));
}

if (count < 0) {
return 0;
} else {
return count;
}
}
}
```

Appendix C
An Elementary Derivation of the One-Dimensional Central Limit Theorem from the Random Walk

A particle starting at 0 jumps by moving randomly towards just $+1$ or -1 on the coordinate $(-\infty, +\infty)$. Let k be the number of jumps in the positive direction, and l the number of jumps in the negative direction. We then denote these by $N and m$:

$$N = k + l, m = k - l \tag{C.1}$$

It then holds that m is odd *if* N is odd. The probability that a particle jumps n in the positive direction is set as $P(n)$. The probability in the opposite direction is set as $1 - P$. The generated path will accompany its probability $P^k \times (1 - P)^l$. There are also combinatory pairs of different jumps from $k = 0$ to $k = N/2$:

$$\binom{N}{k} = \frac{N!}{k!l!} = \frac{N!}{k!(N-k)!} \tag{C.2}$$

The probability that the particle jumped k times for $+1$ each, and l times for -1 each, is set as:

$$\Pi_N(k) = \binom{N}{k} P^k (1 - P)^l \tag{C.3}$$

Taking into account that $m = 2k - N$, it holds that:

$$k = \frac{N+m}{2}, l = \frac{N-m}{2} \tag{C.4}$$

We have now calculated the probability of net jumps. This is verified by noting that (Fig. C.1 and Table C.1):

$$m = 2k - N \tag{C.5}$$

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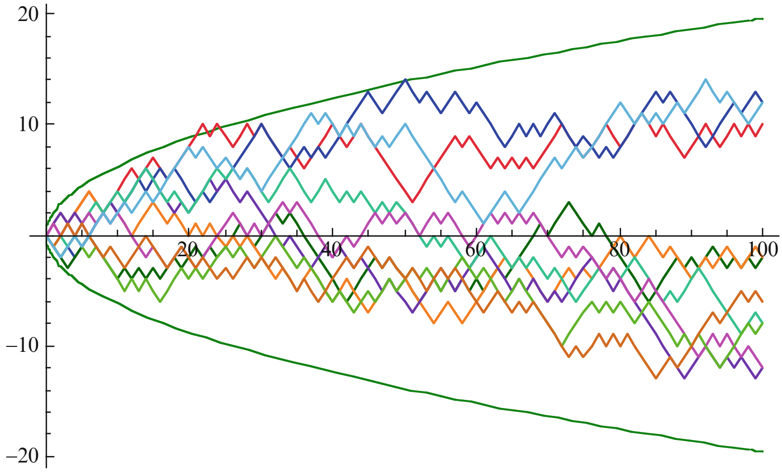

Fig. C.1 Simple random walks

Table C.1 The probability distribution of the one-dimensional random walk

Steps	−5	−4	−3	−2	−1	0	1	2	3	4	5
0	0	0	0	0	0	1	0	0	0	0	0
1	0	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0
2	0	0	0	$\frac{1}{4}$	0	$\frac{2}{4}$	0	$\frac{1}{4}$	0	0	0
3	0	0	$\frac{1}{8}$	0	$\frac{3}{8}$	0	$\frac{3}{8}$	0	$\frac{1}{8}$	0	0
4	0	$\frac{1}{16}$	0	$\frac{4}{16}$	0	$\frac{6}{16}$	0	$\frac{4}{16}$	0	$\frac{1}{16}$	0
5	$\frac{1}{32}$	0	$\frac{5}{32}$	0	$\frac{10}{32}$	0	$\frac{10}{32}$	0	$\frac{5}{32}$	0	$\frac{1}{32}$

$$\Pi_N(m) = \binom{N}{\frac{N}{\frac{N+m}{2}}} P^{\frac{N+m}{2}} (1-P)^{\frac{N-m}{2}} \tag{C.6}$$

For a rudimentary confirmation, we give the next formula. Let p_i be the probability that event i occurs. Taking a normalization rule $\sum_{i=1}^{n} p_i(x_i) = 1$, we reach the weighted arithmetic mean of $x = (x_1, \cdots, x_n)$:

$$E(x) = \frac{\sum_{i=1}^{n} p_i(x_i)x_i}{\sum_{i=1}^{n} p_i(x_i)} = \sum_{i=1}^{n} p_i(x_i)x_i \tag{C.7}$$

The deviation from this mean value is defined as:

$$\triangle x_i = x_i - E(x) \tag{C.8}$$

It then follows that:

$$E(\triangle x_i) = \sum_i^n (p_i(x_i) - E(x)) = 0 \tag{C.9}$$

The second moment is then defined as:

$$E(\triangle x_i^2) = \sum_k^n (p_i(x_i) - E(x))^2 \tag{C.10}$$

$$= E(x^2) - E(x)^2 \tag{C.11}$$

C.1 The Derivation of the Density Function of the Normal Distribution[6]

In the following, we will derive the density function of the normal distribution. We give a Taylor expansion around the maximum $k = \nu$ on the function $\ln \Pi(k)$, for convenience of calculation, instead of $\Pi(k)$.

$$\ln \Pi(k) = \ln \Pi(\nu) + \frac{d \ln \Pi(\nu)}{dk}(k-\nu) + \frac{d^2 \ln \Pi(\nu)}{dk^2}(k-\nu)^2 + \cdots \tag{C.12}$$
$$+ \frac{1}{k!}\frac{d^k \ln \Pi(\nu)}{dk^k}(k-\nu)^k + \cdots$$

We replace $\frac{1}{k!}\frac{d^k \ln \Pi(\nu)}{dk^k}(k-\nu)^k$ with b_k, and $k-\nu$ with λ:

$$\ln \Pi(k) = \ln \Pi(\nu) + b_1\lambda + \frac{b_2}{2}\lambda^2 + \frac{b_3}{6}\lambda^3 + \cdots \tag{C.13}$$

As the maximum is attained at $k = \nu$, the necessary condition of the first derivative and the sufficient one of the second are fulfilled as:

$$b_1 = \ln \Pi(\nu)' = 0;\ b_2 = \ln \Pi(\nu)'' < 0 \tag{C.14}$$

[6]The next proof is inspired by the idea of Prof. Yoshihiro Nakajima, Osaka City University, who believes the necessity of a rudimentary understanding of the CLT by some self-contained proof (Nakajima 2011).

b_2 becomes negative, and hence may be rewritten as:

$$\frac{b_2}{2}\lambda^2 = -\frac{|b_2|}{2}\lambda^2 \tag{C.15}$$

We then assume that $\lambda = k - \nu$ is small enough to ignore $\lambda^3, \cdots, \lambda^k$, and we may then reach the next approximation:

$$\ln \Pi(k) \approx \ln \Pi(\nu) - \frac{|b_2|}{2}\lambda^2, \; i.e., \tag{C.16}$$

$$\Pi(k) \approx \Pi(\nu) e^{-\frac{|b_2|}{2}\lambda^2} \tag{C.17}$$

Finally, we transform $\sum_k^N \Pi(k)$ into a continuous integral form:

$$\sum_k^N \Pi(k) = \int_{-\infty}^{\infty} \Pi(k-\nu) d\lambda \tag{C.18}$$

We note that:

$$\lambda = k - \nu, \; \sum_k^N \Pi(k) = 1 \tag{C.19}$$

It then by the famous integral formula on a Gaussian function holds that:

$$\sum_k^N \Pi(k) = \int_{-\infty}^{\infty} \Pi(\nu) e^{-\frac{|b_2|}{2}\lambda^2} d\lambda \tag{C.20}$$

$$= \Pi(\nu)\sqrt{\frac{2\pi}{|b_2|}} = 1 \tag{C.21}$$

And then:

$$\Pi(\nu) = \sqrt{\frac{|b_2|}{2\pi}} \tag{C.22}$$

We examine each item $\Pi(k)$.

$$\Pi(k) = \frac{N!}{k!(N-k)!} P^k (1-P)^{N-k}$$

Use the next approximation: $\frac{\ln (k+1)!-\ln k!}{(k+1)-k}$. As k becomes large enough, $\frac{d \ln k}{dk} \approx \ln k$.

$$\frac{d \ln \Pi(k)}{dk} = -\ln k - \ln(N-k) + \ln P - \ln(1-P)$$
$$\frac{d^2 \ln \Pi(k)}{dk^2} = -\frac{1}{k} - \frac{1}{N-k} \tag{C.23}$$

If $\frac{d \ln \Pi(k)}{dk} = 0$ at $k = \nu$, it then follows that:

$$\nu = PN \tag{C.24}$$

Then, at $k = \nu$:

$$b_2 = -\frac{1}{NP(1-P)} \tag{C.25}$$

We then have:

$$E(k^2) = \sum \Pi(k) k^2 \tag{C.26}$$
$$= \sum_k^N \binom{N}{k} P^k (1-P)^l k^2 \tag{C.27}$$

Taking into account the expression:

$$P \frac{d}{dP} P^k = PkP^{k-1} = kP^k, \left(P \frac{d}{dP}\right)^2 P^k = k^2 P^k \tag{C.28}$$

It follows:

$$E(k^2) = \left(P \frac{d}{dP}\right)^2 \sum_k^N \binom{N}{k} P^k (1-P)^l k^2 \tag{C.29}$$
$$= \left(P \frac{d}{dP}\right)^2 (P + (1-P))^N \tag{C.30}$$
$$= \left(P \frac{d}{dP}\right) \left(PN(P + (1-P))^{N-1}\right) \tag{C.31}$$
$$= (PN)^2 + NP(1-P) \tag{C.32}$$

It then holds:

$$E(k^2) = \nu^2 + NP(1-P) \tag{C.33}$$

Since:

$$E(\triangle k^2) = E(k^2) - E(k)^2 \tag{C.34}$$

It then holds:

$$E(\triangle k^2) = E(k^2) - E(k)^2 \tag{C.35}$$
$$= \nu^2 + NP(1-P) - \nu^2 \tag{C.36}$$
$$= NP(1-P) \tag{C.37}$$

We denote $E(\triangle k^2)$ by σ^2. This creates **the next formula for the normal distribution**:

$$\Pi(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-\nu)^2}{2\sigma^2}} \tag{C.38}$$

This is the density function of the normal distribution.

$$\Pi(\nu) = \sqrt{\frac{|b_2|}{2\pi}}$$
$$b_2 = \frac{1}{NP(1-P)}$$
$$\sigma^2 = E((\triangle k)^2) = 4NP(1-P)$$

If $p = 0.5$ in the last equation, it holds that:

$$\sigma^2 = E((\triangle k)^2) = N, i.e.,$$
$$\sigma = E(\triangle k) = \sqrt{N}$$

By $\sigma^2 = E((\triangle k)^2) = N$, it also holds that:

$$b_2 = \frac{1}{NP(1-P)}$$
$$\sigma = E(\triangle k) = \sqrt{N}$$

QED

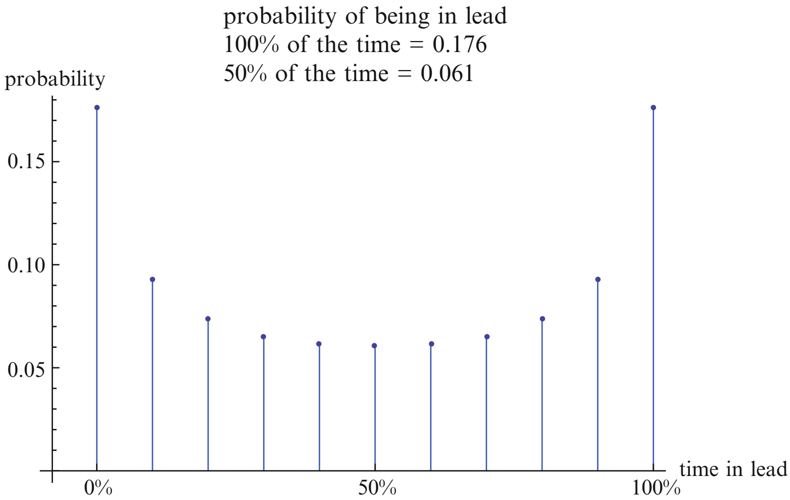

Fig. C.2 The probability of long leads

C.2 A Heuristic Finding in the Random Walk

We finally give an interesting result in the random walk. Even now, the research on random walk is under development. As we showed in Section 6.1.1.1, we can find self-similarity in the random walk. We moreover show a heuristic finding of the **probability of long leads** in a random walk.[7] Consider a game in which a fair coin is tossed repeatedly. When the cumulative number of heads is greater than the cumulative number of tails, heads is in the lead. It retains that position until the cumulative number of tails is greater. The probability distribution indicates the probability of one side being in the lead for different percentages of the duration of the game. For example, if the coin is tossed 20 times, the probability that heads will be in the lead during the entire course of the game is 0.176, the same as the probability that it will never be in the lead. Surprisingly, the least likely situation is for the two sides to be in the lead equal amounts of time (Fig. C.2).

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