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Index of Definitions

additive quadratic form : **8.4**
almost nondegenerate : (2.ex.14)
almost orthogonal (or orthonormal) basis : **8.2**
anisotropic : **2.7**, **8.4**
Arf invariant, − subalgebra : **3.7**, (3.ex.25)
associated bilinear mapping : **2.1**
Azumaya algebra : (3.5.1)

balanced grading : (3.5.2)
bilinear module, − space : **2.5**
Brauer (-Wall) group : **3.5**

cancellation : (2.7.7)
canonical scalar product : (4.8.8)
Cartan-Chevalley mapping : (7.2.3)
Cartan-Chevalley criterion : (7.4.1)
central simple algebra (graded −) : **6.6**
centralizer (graded −) : **6.5**
Clifford algebra : **3.1**
Clifford group : **5.1**
cliffordian quadratic form : (4.8.1)
coalgebras, comodules : **4.1**
comultiplication, counit,. . . : **4.1**
conjugation : **3.1**, before (3.6.8)
covariant or contravariant functor : **1.5**

decomposable : **4.5**
defective : (2.ex.6), (5.7.1), **8.4**
definite (positive or negative −) : **2.8**
deformation : (4.7.1)
derivation : (4.3.4), (4.4.4), (6.5.9)
determinant : **3.6**
direct (or inductive) limit : (1.ex.27)
direct sum, − product : **1.3**
direct summand : **1.13**
discriminant module : **3.4**, **3.8**
divided powers : **4.6**
divided trace (complex, twisted −−) : **6.8**
division algebra (graded −) : (3.5.20), before (6.6.2)
dual category : **1.3**

equivalence of categories : **6.4**
exact sequence, − functor : **1.6**
exponential : **4.5**
extension of ring, − of module : **1.8**
exterior algebra : **4.3**

faithful functor : **6.2**
faithful module : (1.13.3)
filtration : **3.1**, **5.2**
finitely presented : **1.8**
flat, faithfully flat : **1.7**, **1.9**
fractions : **1.10**
freely generated module : **1.3**
functor : **1.5**

. . . -g-linear : **6.2**
generator of modules : **6.1**, (6.2.9)
grade automorphism : **3.1**, **3.2**
graded center : (3.5.2)
grading (or gradation) : **4.2**

half determinants : (2.ex.13)
homogeneous : **3.2**, **4.2**
hyperbolic space : **2.5**

indefinite : **2.8**
infinitesimal. . . : **5.4**
integral domain : **1.10**
interior multiplication : **4.3**, **4.4**
invariance property : (5.4.1)
invertible module : (1.12.10)
involution of an algebra : (1.13.7)
irreducible module : **6.3**
invertible module : (1.12.10)
invertible submodule : (1.ex.25), after (5.1.12)
isotropic : **2.5**

Leibniz formula : (4.1.3), (4.3.8), (4.4.9), (4.4.10)
Lie algebra : **5.4**
Lipschitz monoid, − group : (5.3.1)
lipschitzian : (5.3.1)
local ring, localization : **1.10**
local property : **1.11**
localization of q : **2.2**

maximal ideal : **1.10**
metabolic space : **2.5**
Morita context (graded −) : (6.4.1)
multiplicative subset : **1.10**

nondegenerate : **2.3**
norm : (1.13.7)

opposite algebra : **3.1**
orthogonal : **2.1**, **2.3**
orthogonal basis : **2.6**
orthogonal group : **5.1**
orthogonal sum : **2.4**
orthogonal summand : **2.6**
orthogonal transformation : **5.1**

parity grading : **4.2**
parity of a submodule T : **7.5**
pfaffian : after (5.9.7)
Picard group : after (1.12.10)
prime ideal : **1.10**
projective (or inverse) limit : (1.ex.27)
projective module : **1.7**, (6.2.8)
Prüfer ring : **8.3**
purely inseparable : **8.5**

quadratic extension : **3.4**
quadratic form, − mapping : **2.1**
quadratic module, − space : (2.5.1)
quaternion algebra : **3.3**, end of **3.6**, **3.8**
quotient module : **1.3**

radical of a ring : (1.10.2)
rank : **1.12**
reduced center : after (5.1.5)
reflection : **5.5**
regular filtration : **5.2**
regular grading : (3.5.2)
residue field : **1.10**
reversion : (3.1.4)

scalar component : end of **4.8**
scalar product (admissible −) : (4.8.6)
scalar product (on a module) : (6.8.10), **7.3**
semi-simple module (or algebra) : **5.3**
separable (graded −) : **6.5**
shifted grading : **6.2**
signature : after (2.8.1)
simple algebra (graded −) : **6.6**
spectrum of a ring : **1.11**
spinor : (6.2.2), end of **7.3**
spinorial group : (5.ex.24)
spinorial norm : (5.ex.21)
splitting exact sequence : **1.6**
standard involution : (1.13.7)
support : before (7.6.4)
symmetric algebra : **1.4**
swap automorphism : **3.2**

tamely degenerate : before (5.6.7)
tensor algebra : **1.4**
tensor product : **1.3**, **2.4**
totally isotropic : **2.5**
trace : (1.13.7), **3.6**
twisted algebra : **3.2**

twisted module : **6.2**
twisted inner automorphism : **5.1**,
especially (5.1.5)
twisted opposite algebra : **3.2**
twisted opposite bilinear form : **4.7**
twisted tensor product : **3.2**
type (even or odd) : **2.8**, (3.5.14)

universal object : **1.2**

Villamayor group : (3.ex.18)

Weyl algebra : (4.ex.18)
Witt rings : **2.7**
Witt-Grothendieck rings : **2.7**

Zariski extensions : (1.10.6)
Zariski topology : after (1.11.1)

Index of Notation

Here every algebra is denoted by A (or B), every module by M (or N),.... Only notations that are important, or that are used several times, are recalled here. According to the context, single letters like β, γ, ε, φ,... may also be used with another meaning than the one recalled here.

$A^\times$: **1.1**
A^o , A^t , A^{to} : **3.1**, **3.2**
$(A^2)^g$: **3.5**
$(A^{\leq k})_{k\in\mathbb{Z}}$: **3.1** and **5.2**
A^{ng} : **3.8**
A_D : **3.8**
$A \hat{\otimes}_K B$: **3.2**
$[a]$, $(\!(a)\!)$: **8.1**
$[a,b]$, $(\!(a,b)\!)$: **8.2**
$\langle a\rangle$, $\langle a_1, a_2, \dots, a_n\rangle$: **2.6**
$\langle a_1, b_1; a_2, b_2; \dots; a_n, b_n\rangle$: **8.2**
$\mathcal{A}lg(K)$: **1.1**
$\mathrm{Aut}(M,q)$: **5.1**
b_q : **2.1**
$\mathrm{Bil}_K(M,N)$: **2.1**
$\mathrm{BLip}(V,q)$: **7.2**
$\mathrm{Br}(K)$, $\mathrm{Br}^g(K)$: **3.5**
$\mathrm{Br}_2(K)$: **8.6**
β^{to} , $\beta_{//}$, $\beta_{//\cdot}$, $[\beta]$,... : **4.7**
$\mathbb{C}$: **1.1**
$\mathcal{C}_K(N)$: **2.4**
$\mathcal{C}om(K)$: **1.1**
$\mathrm{C}\ell_K(M,q)$, $\mathrm{C}\ell^{\leq k}(M,q)$: **3.1**
$\mathrm{C}\ell_0(M,q)$, $\mathrm{C}\ell_1(M,q)$: **3.2**
$\mathrm{C}\ell(M,q;\beta)$: **4.7**
$\mathrm{C}\ell^k(M,q)$: **4.8**
$\mathrm{C}\ell(M,q;V)^{\leq k}$, $\mathrm{C}\ell(M,q;U,V)^k$: **5.2**
$\mathrm{C}\ell_0^{\leq 2}(M,q)$: **5.4**
$\mathrm{cp.dv.tr}(\tau)$: **6.8**
∂x : **3.1** and **4.2**
d_q , d_φ , d_β , d_β^{to} ,... : **2.3** and **4.7**
$\det(f)$: **3.6**
$\mathrm{Disc}(K)$: **3.8**, also (3.ex.27)
$\mathrm{Der}^g(A,M)$: **6.5**
$\mathrm{End}_K(M)$: **1.1**
$\mathrm{End}_A^g(M)$: **6.2**
$\mathrm{Exp}(x)$: **4.5**
ε_A , ε'_A : **4.1**
ε , ε' : **4.3**
$f\wedge g$, $x\rfloor f$, $f\rfloor x$: **4.3** and **4.4**
F_y : **5.5**
G_x , G_X : **5.3**
$\mathrm{GC}\ell(M,q)$, $\mathrm{G'C}\ell(M,q)$, $\mathrm{G''C}\ell(M,q)$: **5.1**
$\mathrm{GLip}(M,q)$, $\mathrm{G'Lip}(M,q)$, $\mathrm{G''Lip}(M,q)$: **5.3**
$\mathrm{GO}(M,q)$: **5.1**
$\mathrm{Gr}(A)$, $\mathrm{Gr}^k(A)$: **3.1**
$\Gamma^2_K(M)$, γ : **2.1**
$\mathbb{H}$: **1.1**
$\mathcal{H}(K)$: **8.6**
$\mathbf{H}[M]$, $\mathbf{H}(M)$: **2.5**
$\mathrm{Hom}_K(M,N)$: **1.1**
$\mathrm{Hom}^\wedge(A,B)$: **4.2**
$\mathrm{Hom}_A^g(M,N)$, $\mathrm{Hom}_{A,0}(M,N)$: **6.2**
id_M : **1.2**

$\mathrm{Id}_{\mathcal{C}}$: **2.4**
$\mathrm{Im}(f)$: **1.6**
$\mathrm{Ip}(K)$, $\mathrm{Ip}'(K)$: **3.4**
$K_{\mathfrak{p}}$: **1.10**
$\mathrm{Ker}(f)$: **1.6**
$\mathrm{Ker}(q)$, $\mathrm{Ker}(\mathrm{b}_q) = \mathrm{Ker}(\mathrm{d}_q)$: **2.2**
$\mathrm{Lip}(M,q)$, $\mathrm{Lip}(M)$, $\mathrm{Lip}^*(M)$: **5.3**
$\ell ip(M)$: **5.9**
$\bigwedge_K(M)$: **3.1** and **4.3**
$\bigwedge^*(M) = \mathrm{Hom}(\bigwedge(M),\ K)$: **4.3**
$\bigwedge(M;\beta)$: **4.7**
$\bigwedge^{\max}(U)$: (3.2.6)
M^* : **1.7**
$M^\perp$: **2.3**
$M_{\mathfrak{p}}$: **1.10**
M^c , M^t , M^s,. . . : **6.2**
$\mathbf{M}(M,\varphi)$: **2.5**
$M \otimes_K N$: **1.3** and **2.4**
$M \otimes_A N$: **6.4**
$M \perp N$: **2.4**
$\mathcal{M}(m,A)$, $\mathcal{M}(m,n;A)$: **6.6**
$\mathcal{M}od(A)$: **1.1**
$\mathcal{M}od^g(A)$, $\mathcal{M}od_0(A)$, $\mathcal{M}od(A^{to})$: **6.4**
$\mu_2(K)$, $\mu_4(K)$, $\mu_8(K)$: (3.4.14), **6.8**
$\mathbb{N}$: **1.1**
$\mathcal{N}(x)$: (1.13.7)
$\mathrm{par}(\mathfrak{p};U,T)$: **7.6**
$\mathrm{Pic}(K)$: **1.12**, also (3.ex.27)
π , π_* , π' , π^* : **4.3**
π_A , π_M , π'_A , π'_M : **4.1**
π_q , π'_q : **4.4**
φ : often as in **2.4** or (1.13.7)
$\mathbb{Q}$: **1.1**
$\mathcal{Q}$: **3.8**, **8.6**
$\mathrm{Q}(K)$, $\mathrm{Q}^g(K)$: **3.4**
$\mathrm{Quad}_K(M,N)$: **2.1**
$\mathrm{QZ}(A)$: **3.5**
$\mathrm{QZ}(M,q)$: **3.7**
$\mathbb{R}$: **1.1**
R_x : **7.2**
$\mathrm{rk}(\mathfrak{p},M)$: **1.12**
$\mathrm{Rad}(K)$: (1.10.2)
ρ : **3.1**
$\mathrm{S}_K(M)$: **1.4**
$S^{-1}K$, $S^{-1}M$: **1.10**
$\mathrm{Scal}(x)$: **4.8**
$\mathrm{SO}(M,q)$: **5.6**
$\mathrm{Spec}(K)$: **1.10**
$\mathrm{Spin}(M,q)$: end of **7.3**
$\mathrm{Spin}^{\pm}(M,q)$: (5.ex.24)
σ : **3.1** and **4.3**
$\top$, $\top^\wedge$,. . . : **4.1** and **4.2**
$\mathrm{T}_K(M)$: **1.4**
$\mathcal{T}(M,q)$: **7.2**
$\mathrm{tr}(x)$: (1.13.7), **3.6**
$\mathrm{tr}(f)$: **3.6**
$\mathrm{tw.dv.tr}(\tau)$: **6.8**
Θ_x , Θ_X : **5.1**
τ : (3.1.4)
$\mathcal{V}(\mathfrak{a})$: **1.10**
$\mathrm{WB}(K)$, $\mathrm{WQ}(K)$, $\mathrm{W}(K)$: **2.7**
$\mathrm{WGQ}(K)$, $\mathrm{WGQ}(K)$, $\mathrm{WG}(K)$: **2.7**
$\mathrm{WIQ}(K)$, $\mathrm{WIQ}(K)$, $\mathrm{WI}(K)$: **2.7**
$\mathbb{Z}$: **1.1**
$\mathrm{Z}(A)$, $\mathrm{Z}^g(A)$, $\mathrm{Z}(A_0,A)$: (3.5.2)
$\mathrm{Z}^g(\theta)$, $\mathrm{Z}^r(\theta)$, $\mathrm{Z}^r(A)$: **5.1**
$\mathrm{Z}^g(A,M)$: **6.5**