APPENDIX

TWO RECENT RESEARCH TOPICS

We point out here (mostly by indicating the appropriate references) two interesting recent directions of research which are closely related to the topics discussed in this book. Moreover, it is quite likely that these directions will yield some major new results in a not too distant future.

The first of these topics consists in investigating the simplest *non-isolated* singularities of function germs $f:(\mathbb{C}^n,0) \to (\mathbb{C},0)$, namely those germs f for which the associated hypersurface singularity $X:f=0$ has a 1-dimensional singular space (i.e. $\dim \operatorname{Sing}(X)=1$).

This direction was initiated by D. Siersma and some useful references here are the following.

1. Siersma, D.: Isolated line singularities, PROC. SYMP. PURE MATH. 40 (ARCATA, PART II), Amer. Math. Soc. (1983), 485--496.
2. Siersma, D.: Quasihomogeneous singularities with transversal type A_1 , preprint Utrecht University, 1987.
3. Pellikaan, R.: Hypersurface singularities and resolutions of Jacobi modules, Thesis Utrecht University, 1985.

In addition, the very recent preprint [2] contains a more extensive reference list for this topic. The second recent research direction consists in passing from the study of function germs $f:(\mathbb{C}^n,0) \to (\mathbb{C},0)$ to the more general study of function germs $f:(X,0) \to (\mathbb{C},0)$ *defined on an analytic space germ* $(X,0)$. There are many motivations for this extension. One of them can be found in [D3], some others in the next references.

The simplest case to handle is when $(X,0)$ is an isolated hypersurface (or, more generally, complete intersection singularity) and it was V.I. Arnold who has originally insisted upon the necessity of such a study. As references, we mention

4. Lyashko, O.V.: Classification of critical points of functions on a manifold with singular boundary, Funct. Anal. Appl. 17 (1983), 187-193.

5. Dimca, A.: Function germs defined on isolated hypersurface singularities, Compositio Math. 53 (1984), 245-258. See also [D3].

The treatment in [5] is very close to that in this book and the reader will encounter no difficulties to understand all the details. Moreover, he will find out there that the $\mathcal{R}$-simple function germs do no longer coincide to the $\mathcal{K}$-simple ones in this more general setting (and that even the existence of simple function germs is a problem!).

Another interesting case to consider is when (X,0) is a quotient singularity and this is done in

6. Wall, C.T.C.: Functions on quotient singularities, preprint Liverpool University, 1986.

The general case when (X,0) is a complex analytic set germ was recently treated with more sophisticated tools in

7. Bruce, J.W. and Roberts, R.M.: Critical points of functions on analytic varieties, preprint Warwick University, 1986.

This last preprint contains also a more complete reference list on the subject.

There are many possibilities now for starting a research work in singularities. We hope that our suggestions above may help a little a beginner who misses a competent advisor.

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The reader can find additional useful references in [AGV], [Du], [Ln] and [W2], the first one giving a good idea about the substantial contributions of the Russian School in singularities.

LIST OF NOTATIONS

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